

1. Evaluate $\lim_{x \rightarrow 1} \frac{3x + 5x^2 - 8}{x^2 + 4x - 5} = \lim_{x \rightarrow 1} \frac{(5x+8)(x-1)}{(x+5)(x-1)} \Big|_{x=1} = \frac{13}{6}$

- (a) 2
- (b) 3
- (c) 4
- (d) $\frac{7}{3}$
- (e) 5

2. Evaluate $\lim_{t \rightarrow 9} \frac{\sqrt{t} - 3}{t - 9} = \lim_{t \rightarrow 9} \frac{\sqrt{t} - 3}{(\sqrt{t} - 3)(\sqrt{t} + 3)} = \frac{1}{6}$

- (a) $\frac{1}{3}$
- (b) $\frac{1}{6}$
- (c) $\frac{1}{12}$
- (d) 3
- (e) $\frac{1}{18}$

3. Let

$$f(x) = \begin{cases} \sqrt{6-x}, & x < 2, \\ x^3 + k(x-1) + 1, & x \geq 2. \end{cases}$$

Find k so that f is continuous at $x = 2$.

- (a) $k = 3$
- (b) $k = -2$
- (c) $k = -7$
- (d) $k = 0$
- (e) $k = 2$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \sqrt{6-x} = 2$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x^3 + k(x-1) + 1 = 8 + k + 1 = f(2)$$

$$\text{f is cont if } 2 = 8 + k + 1 \Rightarrow k = -7$$

4. Using the definition of derivative, for $f(x) = \frac{1}{x+a}$

(a) $f'(x) = \lim_{h \rightarrow 0} \frac{x - x + h}{(ah + xh)(x + h + a)}$

(b) $f'(x) = \lim_{h \rightarrow 0} \frac{x - x - h}{(ah + xh)(x + h + a)}$

(c) $f'(x) = \lim_{h \rightarrow 0} \frac{x - x - h}{(a + x)(x + h + a)}$

(d) $f'(x) = \lim_{h \rightarrow 0} \frac{h}{(a + x)(x + h + a)}$

(e) $f'(x) = \lim_{h \rightarrow 0} \frac{x - x - h}{(ah + xh)}$

$f'(x) = \lim_{h \rightarrow 0} \frac{1}{x+h+a} - \frac{1}{x+a}$

$= \lim_{h \rightarrow 0} \frac{x+a - x-a-h}{h(x+a)(x+h+a)}$

$= \lim_{h \rightarrow 0} \frac{-h}{(ah + xh)(x+h+a)}$

5. For $y = 3x - 6\sqrt{x}$, the slope of the tangent line at $x = 9$ is:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 6

$y' = 3 - \frac{6}{2\sqrt{x}} = 3(1 - \frac{1}{\sqrt{x}})$

$y'|_{x=9} = 3(1 - \frac{1}{3}) = 3(\frac{2}{3}) = 2$

6. The average cost (in dollars per unit) of producing q units is

$$\bar{c}(q) = 0.01q + 12 + \frac{900}{q}$$

What is the marginal cost at $q = 20$?

(a) 11.6

(b) 12.4

(c) 12.5

(d) 13.0

(e) 12.5

$C(q) = q \bar{c}(q)$

$C(q) = 0.01q^2 + 12q + 900$

marginal cost $\frac{dC}{dq} = 0.02q + 12$

$\frac{dC}{dq}|_{q=20} = 0.4 + 12$

7. If $p(q) = \frac{q+7}{q+2}$, find the marginal revenue $R'(q)$ at $q = 2$, where $R(q) = p(q)q$.

(a) 1

(b) 26/16

(c) 24/16

(d) 20/18

(e) 22/18

$\frac{dR}{dq} = 12' = \left(\frac{q^2 + 7q}{q+2} \right)'$

$\frac{dR}{dq} = \frac{(2q+7)(q+2) - (q^2+7q) \cdot 1}{(q+2)^2}$

$\frac{dR}{dq}|_{q=2} = \frac{11(4) - 18}{16} = \frac{26}{16}$

8. For $y = 5x^2 + 1$, the relative rate of change $\frac{y'}{y}$ at $x = 5$ equals:

(a) $\frac{50}{126}$

(b) $\frac{5}{13}$

(c) $\frac{10}{51}$

(d) $\frac{5}{12}$

(e) $\frac{10}{25}$

$$\frac{10x}{5x^2+1} = \frac{50}{126}$$

$$y' = 5(2)x^{2-1} = 10x$$

9. For $x^2 + y^2 = 25$, find $\frac{dy}{dx}$ at the point $(-4, 3)$.

(a) $\frac{3}{4}$

(b) $-\frac{3}{4}$

(c) $\frac{4}{3}$

(d) $-\frac{4}{3}$

(e) 0

implicit derivative

$$2x + 2y y' = 0$$

$$y' = -\frac{2x}{2y} = -\frac{x}{y} = \frac{4}{3}$$

10. Let $F(x) = (x + g(x))^3$ with $g(2) = -1$ and $g'(2) = 4$. Then $F'(2) =$

(a) 36

(b) 54

(c) 15

(d) 90

(e) 17

$$F' = 3(x + g(x))^2 (1 + g'(x))$$

$$F'(2) = 3(2 + (-1))^2 (1 + 4)$$

$$= 3(5)$$