Math 102 Sec# 4

Quiz#3

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1. Evaluate $I = \int_0^9 \frac{2\log_{10}(s+1)}{s+1} ds$.

(Q43, page 426)

$$I = 2\int_0^9 \frac{\frac{\ln(s+1)}{\ln 10}}{s+1} ds$$

$$= \frac{2}{\ln(10)} \int_0^9 \frac{\ln(s+1)}{s+1} ds$$

$$= \frac{\cancel{2}}{\ln 10} \frac{(\ln(s+1))^2}{\cancel{2}} \Big]_0^9$$

$$= \ln 10.$$

2pts

2. Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cosh(\tan\theta) \sec^2\theta \, d\theta = I$

(Q55, page 442)

let $\boxed{u = \tan\theta}$, $\boxed{du = \sec^2\theta \, d\theta}$

when $\begin{cases} \theta = -\frac{\pi}{4}, & u = \tan(\frac{-\pi}{4}) = -1 \\ \theta = \frac{\pi}{4}, & u = \tan\frac{\pi}{4} = 1 \end{cases}$

$$I = \int_{-1}^{1} \cosh u \, du$$

$$= \sinh u \Big]_{-1}^{1}$$

$$= \sinh(1) - \sinh(-1)$$

$$= \frac{e^1 - e^{-1}}{2} - \left(\frac{e^{-1} - e^1}{2}\right)$$

$$= e - e^{-1}.$$

2.5pts

3. Evaluate $\int \sec^3 x \tan^3 x\, dx = I$ (Q36, page 466)

$I = \int \sec^2 x \tan^2 x \cdot \sec x \tan x\, dx$

$= \int \sec^2 x (\sec^2 x - 1) \cdot \sec x \tan x\, dx$ 2.5 pts

Let $\boxed{u = \sec x}$ $\boxed{du = \sec x \tan x\, dx}$,

So,

$I = \int u^2 (u^2 - 1)\, du$

$= \int (u^4 - u^2)\, du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C.$

4. Find $\int \sin(\ln x^2)\, dx = I$

Let $u = \sin(\ln x^2)$, $dv = dx$

$du = \cos(\ln x^2) \frac{2x}{x^2} dx$, $v = x$

$I = x \sin(\ln x^2) - 2 \int \cos(\ln x^2)\, dx$

Let $u = \cos(\ln x^2)$ $dv = dx$ 3 pts

$du = -\sin(\ln x^2) \frac{2x}{x^2} dx$ $v = x$

$I = x \sin(\ln x^2) - 2\left[x \cos(\ln x^2) + 2 \int \sin(\ln x^2)\, dx\right]$

$I = \frac{1}{5}\left[x \sin(\ln x^2) - 2x \cos(\ln x^2)\right] + C.$