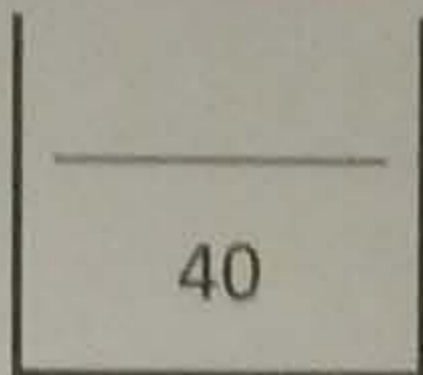


Key



- 1- Does the sequence  $\{a_n\}$  converge or diverge? Find the limit if it converges, where

$$a_n = \frac{\ln(n+1)}{\sqrt{n}}$$

10.1/Q49

$$\lim_{n \rightarrow \infty} \left( \frac{\ln(n+1)}{\sqrt{n}} \right) = \lim_{n \rightarrow \infty} \left( \frac{\frac{1}{n+1}}{\frac{1}{2\sqrt{n}}} \right) \quad \text{by L'Hopital Rule}$$

$$= \lim_{n \rightarrow \infty} \left( \frac{2\sqrt{n}}{n+1} \right) = \lim_{n \rightarrow \infty} \left( \frac{\frac{2}{\sqrt{n}}}{1 + \frac{1}{n}} \right)$$

$$= 0 \Rightarrow \{a_n\} \text{ converges.}$$

- 2- Find the sum of this series

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$$

$$\frac{6}{(2n-1)(2n+1)} = \frac{3}{2n-1} - \frac{3}{2n+1} \quad \text{by partial fraction.}$$

10.2/Q42

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \sum_{n=1}^{\infty} \left( \frac{3}{2n-1} - \frac{3}{2n+1} \right) = 3 \left( 1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \dots \right)$$

$$S_k = 3 \left( 1 - \frac{1}{2k+1} \right)$$

$$\text{The sum} = \lim_{k \rightarrow \infty} S_k = 3.$$

- 3- Express this number as the ratio of two integers.

$$0.\overline{7} = 0.7777\dots$$

$$0.\overline{7} = \sum_{n=1}^{\infty} \frac{7}{10} \left( \frac{1}{10} \right)^{n-1}$$

10.2/Q21

$$= \frac{\frac{7}{10}}{1 - \frac{1}{10}} = \frac{7}{9}$$

4- For what values of  $a$ , if any, do the series converge?

$$\sum_{n=3}^{\infty} \left( \frac{1}{n-1} - \frac{2a}{n+1} \right)$$

Use the integral test:

$$\int_3^{\infty} \left( \frac{1}{x-1} - \frac{2a}{x+1} \right) dx = \lim_{b \rightarrow \infty} \left( \ln \left| \frac{x-1}{(x+1)^{2a}} \right| \right)_3^b$$

$$= \lim_{b \rightarrow \infty} \ln \left( \frac{b-1}{(b+1)^{2a}} \right) - \ln \left( \frac{2}{4^{2a}} \right),$$

$$\lim_{b \rightarrow \infty} \left( \frac{b-1}{(b+1)^{2a}} \right) = \lim_{b \rightarrow \infty} \left( \frac{1}{2a(b+1)^{2a-1}} \right) = \begin{cases} 1, & a = \frac{1}{2} \quad \text{conv.} \\ \infty, & a < \frac{1}{2} \quad \text{div.} \end{cases}$$

The series converges to  $-\ln \left( \frac{2}{4^{2(\frac{1}{2})}} \right) = \ln 2$  if  $a = \frac{1}{2}$ , otherwise diverges. Check when  $a > \frac{1}{2}$ .

5- Test this integral for convergence

$$\int_1^{\infty} \frac{dx}{x^3+1}$$

$$0 < \frac{1}{x^3+1} < \frac{1}{x^3} \quad (x > \infty)$$

and  $\int_1^{\infty} \frac{1}{x^3} dx$  converges.

Then  $\int_1^{\infty} \frac{dx}{x^3+1}$  converges by the Comparison Test.