Math 102 | | Name:
Quiz#4 Key | 40 | ID No.:
| | Serial No.:

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1- Does the sequence $\{a_n\}$ converge or diverge? Find the limit if it converges. where

$$a_n = \frac{\ln(n+1)}{\sqrt{n}}$$

10.1/Q49

$$\lim_{n\to\infty}\left(\frac{\ln(n+1)}{\sqrt{n}}\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{n+1}}{\frac{1}{2\sqrt{n}}}\right)$$ by l'Hopitml Rule

$$= \lim_{n\to\infty}\left(\frac{2\sqrt{n}}{n+1}\right) = \lim_{n\to\infty}\left(\frac{\frac{2}{\sqrt{n}}}{1+\frac{1}{n}}\right)$$

$$= 0 \Rightarrow$$ $\{a_n\}$ converges.

2- Find the sum of this series

$$\sum_{n=1}^{\infty}\frac{6}{(2n-1)(2n+1)}$$

10-2/Q42

$$\frac{6}{(2n-1)(2n+1)} = \frac{3}{2n-1} - \frac{3}{2n+1}$$ by paritial fraction.

$$\sum_{n=1}^{\infty}\frac{6}{(2n-1)(2n+1)} = \sum_{n=1}^{\infty}\frac{3}{(2n-1)} - \frac{3}{2n+1} = 3\left(1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}+\cdots\right)$$

$$S_K = 3\left(1-\frac{1}{2K+1}\right)$$

The sum $= \lim_{K\to\infty} S_K = 3.$

3- Express this number as the ratio of two integers.

$$0.\overline{7} = 0.7777\ldots$$

10.2/Q21

$$0.\overline{7} = \sum_{n=1}^{\infty}\frac{7}{10}\left(\frac{1}{10}\right)^{n-1}$$

$$= \frac{\frac{7}{10}}{1-\frac{1}{10}} = \frac{7}{9}$$

②

10.3/Q42

4- For what values of a, if any, do the series converge?

$$\sum_{n=3}^{\infty}\left(\frac{1}{n-1}-\frac{2a}{n+1}\right)$$

use the integral test:

$$\int_3^{\infty}\left(\frac{1}{x-1}-\frac{2a}{x+1}\right)dx = \lim_{b\to\infty}\left(\ln\left|\frac{x-1}{(x+1)^{2a}}\right|\right)_3^b$$

$$= \lim_{b\to\infty} \ln\left(\frac{b-1}{(b+1)^{2a}}\right) - \ln\left(\frac{2}{4^{2a}}\right),$$

$$\lim_{b\to\infty}\left(\frac{b-1}{(b+1)^{2a}}\right) = \lim_{b\to\infty}\left(\frac{1}{2a(b+1)^{2a-1}}\right) = \begin{cases} 1, & a=\frac{1}{2} \quad \text{conv.} \\ \infty, & a<\frac{1}{2} \quad \text{div.}\end{cases}$$

The series converges to $-\ln\left(\frac{2}{4^{2(\frac{1}{2})}}\right) = \ln 2$ if $\boxed{a=\frac{1}{2}}$,

otherwise diverges. Check when $\boxed{a>\frac{1}{2}}$.

5- Test this integral for convergence

8.7/Q.47

$$\int_1^{\infty}\frac{dx}{x^3+1}$$

$$0 \leq \frac{1}{x^3+1} \leq \frac{1}{x^3} \qquad (1\leq x<\infty)$$

and $\int_1^{\infty}\frac{1}{x^3}\,dx$ converges.

Then $\int_1^{\infty}\frac{dx}{x^3+1}$ converges by the Comparison Test.