Math 102

Quiz#4

Key

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Name:

ID No.:

Serial No.:

1- Does the sequence $\{a_n\}$ converge or diverge? Find the limit if it converges, where

$$a_n = \frac{n}{2^n}$$

10.1/Q47

$$\lim_{n\to\infty}\left(\frac{n}{2^n}\right) = \lim_{n\to\infty}\left(\frac{1}{2^n \ln 2}\right)$$ using l'Hopital Rule.

$$= 0 \Rightarrow \{a_n\} \text{ converges.}$$

2- Find the sum of this series

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

10.2/Q41

$$\frac{4}{(4n-3)(4n+1)} = \frac{1}{4n-3} - \frac{1}{4n+1}$$, by partial fraction

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)} = \sum_{n=1}^{\infty}\left(\frac{1}{4n-3} - \frac{1}{4n+1}\right) = \left(1-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{9}\right)+\left(\frac{1}{9}-\frac{1}{13}\right)+\left(\frac{1}{13}-\right)\cdots$$

$$S_k = 1 - \frac{1}{4k+1} \Rightarrow \text{The sum } \lim_{k\to\infty} S_k = 1.$$

3- Express this number as the ratio of two integers.

$$0.\overline{23} = 0.23\ 23\ 23\ldots$$

10.2/Q19

$$0.\overline{23} = \sum_{n=1}^{\infty} \frac{23}{100}\left(\frac{1}{100}\right)^{n-1}$$

$$= \frac{\frac{23}{100}}{1-\frac{1}{100}} = \frac{23}{99}$$

4- For what values of a, if any, does the series converge? ②

$$\sum_{n=1}^{\infty}\left(\frac{a}{n+2}-\frac{1}{n+4}\right)$$

10.3/Q41

use the integral test:

$$\int_1^{\infty}\left(\frac{a}{x+2}-\frac{1}{x+4}\right)dx = \lim_{b\to\infty}\left\{\Big[a\ln|x+2| - \ln|x+4|\Big]_0^b\right\}$$

$$= \lim_{b\to\infty} \ln\left[\frac{(b+2)^a}{b+4}\right] - \ln\left(\frac{3^a}{5}\right)$$

$$\lim_{b\to\infty}\frac{(b+2)^a}{b+4} = a\lim_{b\to\infty}(b+2)^{a-1} = \begin{cases} \infty, & a>1 \quad \text{div} \\ 1, & \boxed{a=1} \quad \text{conv.} \end{cases}$$

The series converges to $-\ln\left(\frac{3^1}{5}\right) = \ln\frac{5}{3}$ when $\boxed{a=1}$.

Note: if $a<1$, the series terms are negatives, we can not apply the integral test. You can check that for $a<1$ the Σ div.

5- Test this integral for convergence

$$\int_0^{\pi}\frac{dt}{\sqrt{t}+\sin t}$$

8.7/Q.41

$$0 \le t \le \pi$$

$$\Longleftrightarrow \quad 0 \le \frac{1}{\sqrt{t}+\sin t} \le \frac{1}{\sqrt{t}}$$

and $\displaystyle\int_0^{\pi}\frac{1}{\sqrt{t}}\,dt$ converges.

Then $\displaystyle\int_0^{\pi}\frac{dt}{\sqrt{t}+\sin t}$ converges by the Comparison Test.