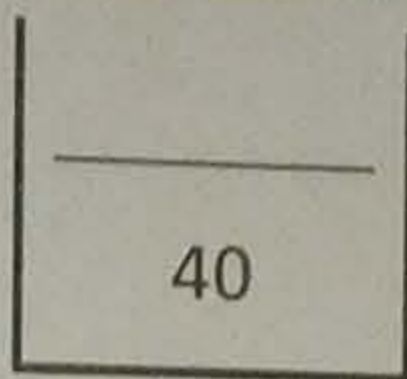


Key



Name:

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①

- 1- Does the sequence $\{a_n\}$ converge or diverge? Find the limit if it converges, where

$$a_n = \frac{n}{2^n}$$

10.1/Q47

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2^n} \right) = \lim_{n \rightarrow \infty} \left(\frac{1}{2^n \ln 2} \right) \quad \text{using l'Hopital Rule}$$

$$= 0 \Rightarrow \{a_n\} \text{ converges.}$$

- 2- Find the sum of this series

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

$$\frac{4}{(4n-3)(4n+1)} = \frac{1}{4n-3} - \frac{1}{4n+1} \quad \text{by partial fraction}$$

10.2/Q41

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{4n-3} - \frac{1}{4n+1} \right) = \left(1 - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{13}\right) + \left(\frac{1}{13} - \dots\right)$$

$$S_k = 1 - \frac{1}{4k+1} \Rightarrow \text{The sum } \lim_{k \rightarrow \infty} S_k = 1.$$

- 3- Express this number as the ratio of two integers.

$$0.\overline{23} = 0.232323\dots$$

10.2/Q19

$$0.\overline{23} = \sum_{n=1}^{\infty} \frac{23}{100} \left(\frac{1}{100} \right)^{n-1}$$

$$= \frac{\frac{23}{100}}{1 - \frac{1}{100}} = \frac{23}{99}$$

4- For what values of a , if any, do the series converge?

(2)

$$\sum_{n=1}^{\infty} \left(\frac{a}{n+2} - \frac{1}{n+4} \right)$$

10.3/Q4

use the integral test:

$$\int_1^{\infty} \left(\frac{a}{x+2} - \frac{1}{x+4} \right) dx = \lim_{b \rightarrow \infty} \left\{ a \ln|x+2| - \ln|x+4| \right\}_0^b$$

$$= \lim_{b \rightarrow \infty} \left[\ln \left(\frac{(b+2)^a}{b+4} \right) - \ln \left(\frac{3^a}{5} \right) \right]$$

$$\lim_{b \rightarrow \infty} \frac{(b+2)^a}{b+4} = a \lim_{b \rightarrow \infty} (b+2)^{a-1} = \begin{cases} \infty, & a > 1 \\ 1, & a = 1 \end{cases} \quad \begin{matrix} \text{div} \\ \text{conv.} \end{matrix}$$

The series converges to $-\ln\left(\frac{3^a}{5}\right) = \ln\left(\frac{5}{3^a}\right)$ when $a=1$.

Note if $a < 1$, the series terms are negatives, we can not apply the integral test. You can check that for $a < 1$ the \sum div.

5- Test this integral for convergence

$$\int_0^{\pi} \frac{dt}{\sqrt{t} + \sin t}$$

8.7/Q.41

$$0 \leq t \leq \pi$$

$$\Leftrightarrow 0 \leq \frac{1}{\sqrt{t} + \sin t} \leq \frac{1}{\sqrt{t}}$$

and $\int_0^{\pi} \frac{1}{\sqrt{t}} dt$ converges.

Then $\int_0^{\pi} \frac{dt}{\sqrt{t} + \sin t}$ converges by the Comparison Test.