Math101 Key Name:

Quiz#2 ID No:

Serial No:

1- *Evaluate the limit if it exists*

a. $\lim_{x\to\frac{1}{2}}(x-[\![2x]\!])$, *where* $[\![\quad]\!]$ *denotes the greatest integer function.*

$$\left.\begin{aligned}\lim_{x\to\frac{1}{2}^-}(x-[\![2x]\!]) &= \lim_{x\to\frac{1}{2}^-}(x-0) = \frac{1}{2}\\ \lim_{x\to\frac{1}{2}^+}(x-[\![2x]\!]) &= \lim_{x\to\frac{1}{2}^+}(x-1) = -\frac{1}{2}\end{aligned}\right\} \Rightarrow \lim_{x\to\frac{1}{2}^+}(x-[\![2x]\!]) \neq \lim_{x\to\frac{1}{2}^-}(x-[\![2x]\!])$$

$\Rightarrow$ limit DNE.

b. $\lim_{x\to1^-}\left(\frac{x^2-|x-1|-1}{|x-1|}\right)$

$$= \lim_{x\to1^-}\left[\frac{x^2-(-(x-1))-1}{-(x-1)}\right] = \lim_{x\to1^-}\left(\frac{x^2+(x-1)-1}{-(x-1)}\right)$$

5

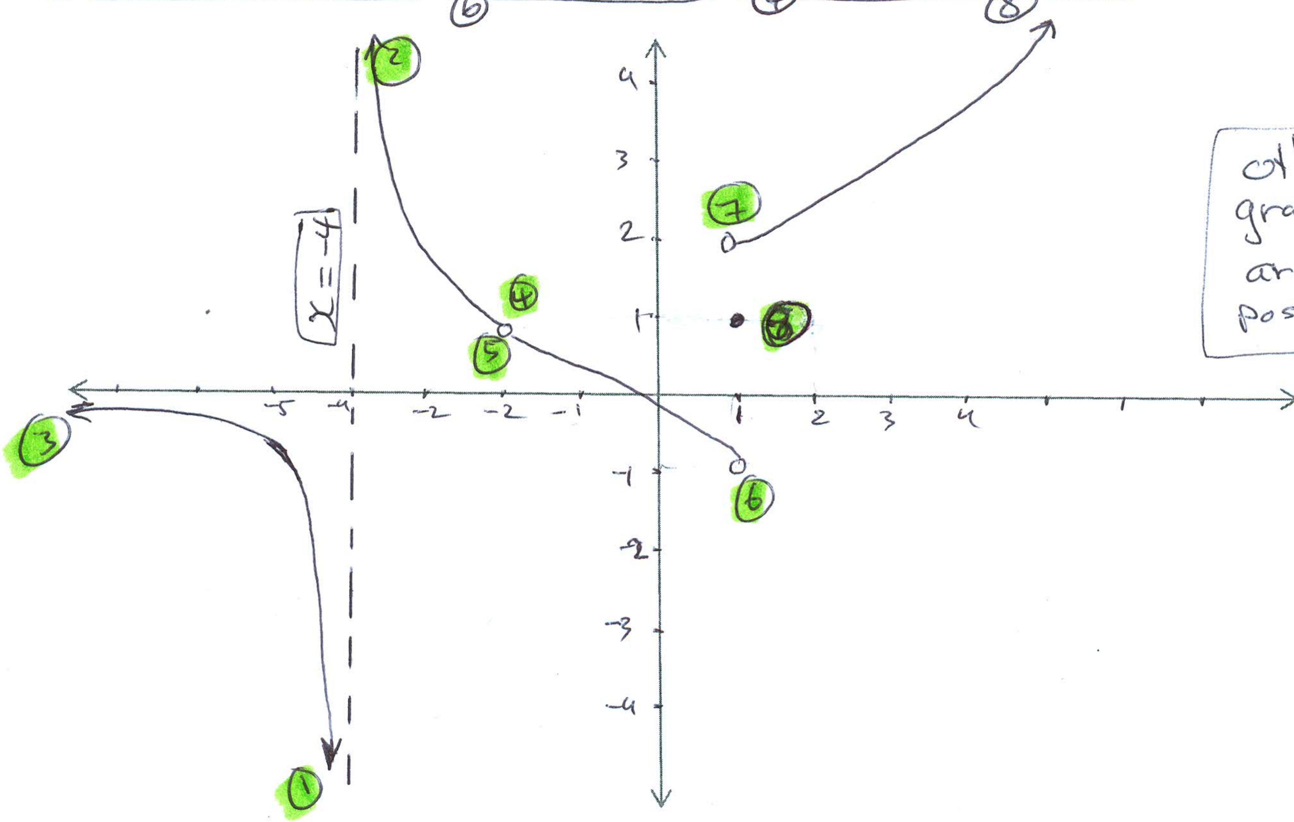
$$= \lim_{x\to1^-}\left(\frac{x^2+x-2}{-(x-1)}\right) = \lim_{x\to1^-}\left(\frac{\cancel{(x-1)}(x+2)}{-\cancel{(x-1)}}\right), \quad (x\neq1)$$

$$= \lim_{x\to1^-} -(x+2) = \boxed{-3}.$$

2- *Sketch the graph of a function* f *that satisfies all of the following conditions:*

(1) $\lim_{x\to-4^-} f(x) = -\infty$; (2) $\lim_{x\to-4^+} f(x) = \infty$; (3) $\lim_{x\to-\infty} f(x) = 0$; (4) $\lim_{x\to-2} f(x) = 1$;

(5) $f(x)$ is undefined at -2; (6) $\lim_{x\to1^-} f(x) = -1$; (7) $\lim_{x\to1^+} f(x) = 2$; (8) $f(1) = 1$



other graphs are also possible.

5

3- Let $f(x) = \begin{cases} \frac{6a}{x+1} & if\ x > 1 \\ 9 & if\ x = 1 \\ a^2 & if\ x < 1 \end{cases}$, *Find the values of a so that:*

(a) $f(x)$ *is continuous everywhere.*

We need $\lim_{x\to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1)$.

$\lim_{x\to 1^+} f(x) = \frac{6a}{2} = 3a$ and $\lim_{x\to 1^-} f(x) = a^2$

So, $f(x)$ is continuous at $\boxed{x=1}$, if $f(1) = 9 = 3a = a^2$

Then $\boxed{a=3}$.

5

(b) $f(x)$ *has a removable discontinuity.*

$f(x)$ has a removable discontinuity at $\boxed{x=1}$, if

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$ but $\boxed{f(1) \neq \lim_{x\to 1} f(x)} \Rightarrow a=0$.

For $\boxed{a=0}$ we have a removable discontinuity

4- *Use the Intermediate Value Theorem to show that the equation* $\cos x = \sqrt{x}$ *has at least one real root in the interval* $\left(0, \frac{\pi}{2}\right)$.

Let $\boxed{f(x) = \cos x - \sqrt{x}}$

check $f(0) = 1 > 0$

$f\left(\frac{\pi}{2}\right) = -\sqrt{\frac{\pi}{2}} < 0$

5

So, ① $f(x)$ is continuous on $\left[0, \frac{\pi}{2}\right]$

② $f\left(\frac{\pi}{2}\right) < 0 < f(0)$

by (IVT) $\exists\ c \in \left(0, \frac{\pi}{2}\right)$ s.t. $\boxed{f(c) = 0}$.