Solution

DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 301 Methods of Applied Mathematics Term 061
QUIZ #5(a)

Name ______________________ ID # __________ Section # ____

Q1) Find the ***Fourier cosine*** and ***Fourier sine*** series in $0 < x < 4$.

$$f(x) = \begin{cases} 1, & 0 < x < 2 \\ -1, & 2 \le x < 4 \end{cases}$$

Fourier Cosine Series : $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{4} x \quad , \quad 0 < x < 4$

$$a_0 = \frac{2}{4}\int_0^4 f(x)\,dx = \frac{1}{2}\left\{ \int_0^2 dx + \int_2^4 - dx \right\} = \frac{1}{2}\left\{ [x]_0^2 \; [x]_2^4 \right\}$$

$$= \frac{1}{2}\left\{ 2 - 4 + 2 \right\} = 0.$$

$$a_n = \frac{2}{4}\int_0^4 f(x) \cos \frac{n\pi}{4} x \, dx = \frac{1}{2}\left\{ \int_0^2 \cos \frac{n\pi}{4} x \, dx - \int_2^4 \cos \frac{n\pi}{4} x \, dx \right\}$$

$$= \frac{1}{2}\left\{ \frac{4}{n\pi}\left(\sin \frac{n\pi}{4} x \Big|_0^2 - \sin \frac{n\pi}{4} x \Big|_2^4 \right)\right.$$

$$= \frac{2}{n\pi}\left\{ 2\sin \frac{n\pi}{2} - \underbrace{\sin n\pi}_{=0} \right\} = \frac{4}{n\pi} \sin \frac{n\pi}{2}$$

So, $f(x) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos \frac{n\pi}{4} x$

Fourier Sine Series : $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{4} x\right)$

$$b_n = \frac{2}{4}\int_0^4 f(x) \sin \frac{n\pi}{4} x \, dx$$

$$b_n = \frac{1}{2}\left\{ \int_0^2 \sin \frac{n\pi}{4} x \, dx - \int_2^4 \sin \frac{n\pi}{4} x \, dx \right\} = \frac{1}{2}\left\{ \frac{4}{n\pi}\left[\left(-\cos \frac{n\pi}{4} x \right)_0^2 + \left(\cos \frac{n\pi}{4} \right)_2^4 \right] \right\}$$

$$= \frac{2}{n\pi}\left[-2 \cos \frac{n\pi}{2} + 1 + (-1)^n \right]$$

Thus, $f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(-2\cos \frac{n\pi}{2} + 1 + (-1)^n \right) \sin\left(\frac{n\pi}{4} x\right).$

Solution

DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 301 Methods of Applied Mathematics   Term 061

QUIZ #5(b)

Name ____________________ ID # __________ Section # ____

Q1) Find the ***Fourier cosine*** and ***Fourier sine*** series in $0 < x < \pi$.

$$f(x) = \begin{cases} -\dfrac{1}{2}, & 0 < x < \dfrac{\pi}{2} \\ \dfrac{1}{2}, & \dfrac{\pi}{2} \le x < \pi \end{cases}$$

Fourier cosine series : $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad , \quad 0<x<\pi$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi/2} -\frac{1}{2}\,dx + \frac{2}{\pi}\int_{\pi/2}^{\pi} \frac{1}{2}\,dx = \frac{1}{\cancel{2}}\frac{\cancel{2}}{\pi}\left\{[-x]_0^{\pi/2} + [x]_{\pi/2}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2} + \pi - \frac{\pi}{2}\right] = 0$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{\cancel{2}}{\pi}\times\frac{1}{\cancel{2}}\left[\int_0^{\pi/2} -\cos nx\,dx + \int_{\pi/2}^{\pi}\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\frac{1}{n}\left\{[-\sin nx]_0^{\pi/2} + [\sin nx]_{\pi/2}^{\pi}\right\} = \frac{1}{n\pi}\left\{-\sin\frac{n\pi}{2} + \sin n\pi - \sin\frac{n\pi}{2}\right\}$$

$$= \frac{-2}{n\pi}\sin\frac{n\pi}{2}.$$

So, $f(x) = \sum_{n=1}^{\infty} \frac{-2\sin\left(\frac{n\pi}{2}\right)}{n\pi}\cos nx$

Fourier Sine Series $\quad f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad , \quad 0<x<\pi$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx = \frac{1}{2}\cdot\frac{\cancel{2}}{\pi}\left[\int_0^{\pi/2} -\sin nx\,dx + \int_{\pi/2}^{\pi}\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\cdot\frac{1}{n}\left\{[\cos nx]_0^{\pi/2} - [\cos nx]_{\pi/2}^{\pi}\right\}$$

$$= \frac{1}{n\pi}\left\{\cos\frac{n\pi}{2} - 1 - (-1)^n + \cos\frac{n\pi}{2}\right\} = \frac{1}{n\pi}\left\{2\cos\frac{n\pi}{2} - 1 - (-1)^n\right\}$$

So, $f(x) = \sum_{n=1}^{\infty} \frac{1}{n\pi}\left(2\cos\frac{n\pi}{2} - 1 - (-1)^n\right)\sin nx$

Solution

DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 301 Methods of Applied Mathematics Term 061

QUIZ #5(c)

Name ____________________ ID # __________ Section # ____

Q1) Find the ***Fourier cosine*** and ***Fourier sine*** series in $0<x<3$.

$$f(x)=\begin{cases}-1, & 0<x<1\\ 0, & 1\le x<2\\ 1, & 2\le x<3.\end{cases}$$

<u>Fourier Cosine Series</u> $\quad f(x)=\dfrac{a_0}{2}+\displaystyle\sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{3}x \quad , \quad 0<x<3$

$$a_0=\frac{2}{3}\int_0^3 f(x)\,dx=\frac{2}{3}\left\{\int_0^1 -dx+\int_1^2 0\,dx+\int_2^3 dx\right\}$$

$$=\frac{2}{3}\left\{\left[-x\right]_0^1+\left[x\right]_2^3\right\}=\frac{2}{3}\left\{-1+3-2\right\}=0$$

$$a_n=\frac{2}{3}\int_0^3 f(x)\cos\frac{n\pi}{3}x\,dx=\frac{2}{3}\left\{\int_0^1 -\cos\frac{n\pi}{3}x\,dx+\int_2^3\cos\frac{n\pi}{3}x\,dx\right\}$$

$$=\frac{2}{3}\left\{\frac{3}{n\pi}\left(-\sin\frac{n\pi}{3}x\Big|_0^1+\sin\frac{n\pi}{3}x\Big|_2^3\right)\right.$$

$$=\frac{2}{n\pi}\left\{-\sin\frac{n\pi}{3}+\sin\frac{2n\pi}{3}\right\}$$

Thus $\quad f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{2}{n\pi}\left(-\sin\frac{n\pi}{3}+\sin\frac{2n\pi}{3}\right)\cos\frac{n\pi}{3}x$

<u>Fourier Sine Series</u>: $\quad f(x)=\displaystyle\sum_{n=1}^{\infty} b_n\sin\frac{n\pi}{3}x \quad , \quad 0<x<3$

$$a_n=\frac{2}{3}\int_0^3 f(x)\sin\frac{n\pi}{3}x\,dx=\frac{2}{3}\left\{\int_0^1 -\sin\frac{n\pi}{3}x\,dx+\int_2^3\sin\frac{n\pi}{3}x\,dx\right\}$$

$$=\frac{2}{3}\left\{\frac{3}{n\pi}\left(\cos\frac{n\pi}{3}x\Big|_0^1-\cos\frac{n\pi}{3}x\Big|_2^3\right)\right\}=\frac{2}{n\pi}\left[\cos\frac{n\pi}{3}-1-(-1)^n+\cos\frac{2n\pi}{3}\right]$$

Thus, $\quad f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{2}{n\pi}\left(\cos\frac{n\pi}{3}-1-(-1)^n+\cos\frac{2n\pi}{3}\right)\sin\frac{n\pi}{3}x.$