Solution

DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 301 Methods of Applied Mathematics   Term 061

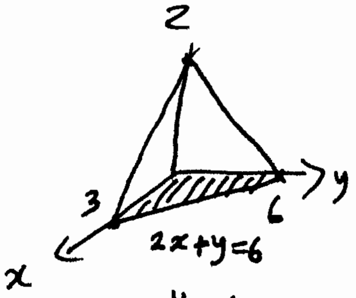
QUIZ # 3(a)

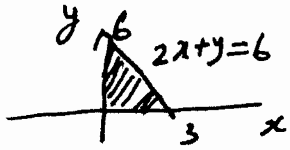
Name ____________________ ID # __________ Section # ____

Q1) In the following, use the *Divergence Theorem* to write the given surface integral as a triple integral over region **D**, showing correct integral limits. Then **evaluate** the RHS you obtain.

$\iint_S (2x\underline{i} + 4xe^z \underline{j} + 3z\underline{k}) \bullet \underline{n} ds$

where **D** is the region bounded by $2x + y + z = 6$ and the coordinate planes.





By divergence theorem

$$\iint_S (2x\underline{i} + 4xe^z\underline{j} + 3z\underline{k}) \cdot \underline{n}\, ds$$

$$= \iiint_D \operatorname{div}(2x\underline{i} + 4xe^z\underline{j} + 3z\underline{k})\, dV$$

$$= \iiint_D 5\, dV = \int_0^3 \int_0^{6-2x} \int_0^{6-2x-y} 5\, dz\, dy\, dx$$

$$= 5 \int_0^3 \int_0^{6-2x} (6-2x-y)\, dy\, dx$$

$$= 5 \int_0^3 \left[6y - 2xy - \tfrac{1}{2} y^2\right]_0^{6-2x} dx$$

$$= 5 \int_0^3 6(6-2x) - 2x(6-2x) - \tfrac{1}{2}(36 + 4x^2 - 24x)\, dx$$

$$= 5 \int_0^3 (18 + 2x^2 - 12x)\, dx = 5\left[18x + \tfrac{2}{3}x^3 - 6x^2\right]_0^3$$

$$= 5[54 + 18 - 54] = 90.$$

Solution

DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 301 Methods of Applied Mathematics Term 061
QUIZ # 3(b)

Name ______________________ ID # __________ Section # ____

Q1) In the following, use the *Divergence Theorem* to write the given surface integral as a triple integral over region **D**, showing correct integral limits. Then **evaluate** the RHS you obtain.

$\iint_S (x^3\underline{i} + y^3\underline{j} + z^3\underline{k}) \bullet \underline{n}ds$

where **D** is the region bounded by hemisphere in the upper half-space $x^2 + y^2 + z^2 = 9$ and $z = 0$.

The given integral, by divergence theorem gives

$$I = \iint_S (x^3\underline{i} + y^3\underline{j} + z^3\underline{k}) \cdot \underline{n}\, ds = \iiint_D \operatorname{div}(x^3\underline{i} + y^3\underline{j} + z^3\underline{k})\, dV$$

$$= \iiint_D 3(x^2+y^2+z^2)\, dV \quad , \text{ where } D: x^2+y^2+z^2=0,\ z \geq 0$$

using spherical coordinates

$$I = \int_0^{2\pi}\int_0^{\pi/2}\int_0^3 3\rho^2 \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \frac{3}{5}\left[\rho^5\right]_0^3 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \frac{3 \times 243}{5} \left[-\cos\phi\right]_0^{\pi/2} \left(\theta\right)_0^{2\pi}$$

$$= \frac{1458}{5}\pi.$$

Solution

DEPARTMENT OF MATHEMATICAL SCIENCES

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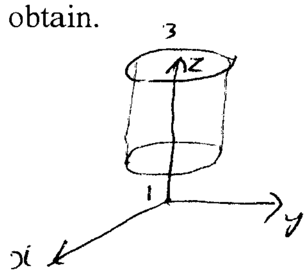
QUIZ # 3(b)

Name ______________________ ID # __________ Section # ____

Q1) In the following, use the *Divergence Theorem* to write the given surface integral as a triple integral over region **D**, showing correct integral limits. Then **evaluate** the RHS you obtain.

$\iint_S (x^3 \underline{i} + y^3 \underline{j} + 2x\underline{k}) \bullet \underline{n} ds$

where **D** is the region bounded by cylinder $x^2 + y^2 = 4$ and $z = 1$ to $z = 3$.



By divergence theorem

$$\iint_S \underline{F} \cdot \underline{n} \, ds = \iiint_D \text{div } \underline{F} \, dV$$

Here $\text{div } \underline{F} = \frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial y}(y^3) + \frac{\partial}{\partial z}(2x) = 3(x^2+y^2)$

D is shown. Thus

R.H.S $\iiint_D 3(x^2+y^2) dV$. In cylindrical coordinates

$$= \int_1^3 \int_0^{2\pi} \int_0^2 3 r^2 r \, dr \, d\theta \, dz = \int_1^3 \int_0^{2\pi} \frac{3}{4} \left[r^4 \right]_0^2 d\theta \, dz$$

$$= \frac{3}{\cancel{4}} \cancel{16}^{4} \, 2\pi (3-1) = 48\pi.$$