King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 574, ME 510, Semester 992
Instructor: Dr. Faisal Fairag
Assignment (2)
Due Monday 7/2/2000
PLEASE SUBMIT CLEAR, CLEAN AND NEAT FORM.
(please use A4 size paper for computer output)

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(1) Consider the elliptic equation ($b^2 < ac$)

$$\begin{aligned} -(au_{xx} + 2bu_{xy} + cu_{yy}) = f \qquad & \text{in} \qquad \Omega \\ u = g \qquad & \text{on} \qquad \partial\Omega \end{aligned} \tag{1}$$

when $\Omega = (0,1) \times (0,1)$¿
Describe a diffrence scheme for (1) using $\frac{\partial^2}{\partial x dy} = \frac{1}{2}(D_{x+}D_{y+} + D_{x-}D_{y-})$
(a) What is the order of accuracy attained?
(b) Under what condition is this of positive type?

(2) Consider the heat transfer problem on the irregular region shown in the figure below The mathematical statement of this problem is as follows:

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad & \text{inside} \\ u = 0 \qquad & \text{top} \\ u = 100 \qquad & \text{bottom} \\ \frac{\partial u}{\partial x} = 0 \qquad & \text{sides} \end{aligned}$$

Establish that the insulated boundaries act as mirrors so that we can assume the temperature is the same as at an adjacent interior grid point. Determine the associated linear system and solve for the temperature $u_{i,j}$
Solve the resulting linear system with (a)Jacobi (b)Gauss Seidel (c)SOR $w = 1.2$
Compare # of iterations or use the Matlab commands <u>*tic*</u> and <u>*toc*</u> to compare cpu time.

(3) Consider the 2-dimentional convection diffusion equation with the discretization

$$[\phi = \phi(x,y), \phi_{ij} = \phi(x_i, y_j)]$$

$$L\phi = -\epsilon(\phi_{xx} + \phi_{yy}) + a\phi_x + b\phi_y = f$$

$$L_0^h\phi = -\epsilon[\frac{\phi_{i+1,j} - 2\phi_{ij} + \phi_{i-1,j}}{h^2} + \frac{\phi_{i,j+1} - 2\phi_{ij} + \phi_{i,j-1}}{h^2}] + +a[\frac{\phi_{i+1,j} - \phi_{i-1,j}}{2h}] + b[\frac{\phi_{i,j+1} - \phi_{i,j-1}}{2h}]$$

Under what condition is L_0^h a positive type scheme?

(4) Consider the biharmonic equation

$$\triangle^2 u = u_{xxxx} + 2u_{xxyy} + u_{yyyy} = f(x,y) \qquad (2)$$

it occurs in the study of elastic stress and fluid dynamics (see also plate bending problem). We approximate (2) with $f(x,y) = 0$ by

where $\alpha_0 = -\frac{20}{h^4}, \alpha_1 = \frac{-8}{h^4}, \alpha_2 = \frac{2}{h^4}, \alpha_3 = \frac{1}{h^4}$
Use the above finite difference scheme to solve the following problem

$$\begin{aligned} \epsilon\,\triangle^2\, u &= 0 \qquad \text{in} \qquad \Omega = (0,1)\times(0,1) \\ u &= 0 \qquad \text{on} \qquad \partial\Omega \\ \frac{\partial u}{\partial n} &= 0 \qquad \text{on}\partial\Omega - \Gamma_1 \\ \frac{\partial u}{\partial n} &= 1 \qquad \text{on} \qquad \Gamma_1 \end{aligned}$$

with $\epsilon = 1, 0.01, h = \frac{1}{8}, \frac{1}{64}, \frac{1}{100}$
Use the Matlab command *pcg* to solve the resulting linear system.
(a) with the m-file which solves the problem (Note: send an e-mail to ffairag@kfupm.edu.sa with attached m-files and write the following as the subject: (MATH574HW1ID) where ID is your ID #).)
(b) use the Matlab command *contour* to graph the contour lines.