**King Fahd University of Petroleum and Minerals**
**Department of Mathematical Sciences**
**Math 574, ME 510, Semester 992**
**Instructor: Dr. Faisal Fairag**
**Assignment (1)**
**Due Monday 31/1/2000**
**PLEASE SUBMIT CLEAR, CLEAN AND NEAT FORM.**
**(please use A4 size paper for computer output)**

---

**(1)** Classify each equation (Elliptic,Parabolic, Hyperbolic)

$$\text{a)} x u_{xx} + y u_{xy} + u_{yy} = 0$$

$$\text{b)} 3u_{xx} + u_{xy} + u_{yy} = e^{xy}$$

$$\text{c)} u_{xx} = u_y + u - u_x + y$$

$$\text{d)} u_{xx} + 2u_{xy} + u_{yy} = 0$$

**(2)** Use the two-dimentional Taylor's theorem to develop the following finite difference approximation

$$(u_{xy})_{ij} = \frac{u_{i+1,j+1} - u_{i-1,j+1} - u_{i+1,j-1} + u_{i-1,j-1}}{4h^2}$$

and show it is of error $O(h^2)$.

**(3)** Consider the Laplace equation

$$\begin{aligned} u_{xx} + u_{yy} = 0 \qquad & \text{in} \qquad \Omega \\ u = \phi \qquad & \text{on} \qquad \partial\Omega \end{aligned}$$

$$\Omega = \{(x, y) \in R : \quad 0 < x, y < 1\}$$

and $\phi$ given by

$$\begin{aligned} \phi(x, 0) = x \qquad & \phi(x, 1) = 1 \\ \phi(0, y) = y \qquad & \phi(1, y) = 1 \end{aligned}$$

Take $h = k = \frac{1}{4}$. Show that the difference equation is equivalent to a linear system of equations and write the system and then solve it.

**(4)** Consider the potential equation

$$\begin{aligned} u_{xx} + u_{yy} &= 1 \qquad \text{in} \qquad \Omega \\ u &= 0 \qquad \text{on} \qquad \partial\Omega \end{aligned}$$

where $\Omega = \{(x, y) \in R, \quad 0 < x, y < 1\}$
(a) write an m-file which solve the equation using the 5 point finite-difference method with $h = k = 0.01$.
(b) what is the approximation of $u(\frac{1}{2}, \frac{1}{2})$.
(c) use the Matlab command *contour* to graph the contour lines of $u(x, y)$.
(d) use the Matlab command *mesh* or *surf* to graph the surface of $u(x, y)$ in $x, y, u$ plane.
(Note: send an e-mail to (ffairag@kfupm.edu) with attached m-files and write the following as the subject: (MATH574HW1ID) where ID is your ID #).