King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 572 , Term: 081
Instructor: Dr. Faisal Fairag
Assignment (1)
Due Sunday 19/10/2008

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(1) Consider the difference equation :

$$\frac{U_{i+1,j}+U_{i-1,j}-4U_{i,j}+U_{i,j+1}U_{i,j-1}}{h^2}=0 \qquad (**)$$

Which approximate the partial differential equation: $u_{xx}+u_{yy}=0$

Show that the truncation error is of $O(h^2)$.

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(2) Use the two-dimensional Taylor's theorem to show that the following finite difference approximation is of $O(h^2)$.

$$(u_{xy})_{ij}\approx\frac{U_{i+1,j+1}-U_{i-1,j+1}-U_{i+1,j-1}U_{i-1,j-1}}{h^2}$$

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(3) [Problem 4.5 Page 49]
Consider the Dirichlet problem (4.9)

$$-\Delta u=f \quad in \quad \Omega, \quad with \quad u=0 \quad on \quad \partial\Omega \qquad (4.9)$$

with $f(x,y)=\sin(\pi x)\sin(\pi y)+\sin(\pi x)\sin(2\pi y)$

in $\Omega=(0,1)\times(0,1)$. Compute the approximate solution by the finite difference method (**) with $h=1/4$ and find the error at $(0.5,0.5)$ using that the exact solution is

$$u(x,y)=(2\pi^2)^{-1}\sin(\pi x)\sin(\pi y)+(5\pi^2)^{-1}\sin(\pi x)\sin(2\pi y)$$
