King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math-321 , Semester (991)
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HOMEWORK(6)
Due November 27,1999

PLEASE SUBMIT CLEAR,CLEAN AND NEAT FORM.

(please use A4 size paper for computer output)

1. Solve the following linear system using Gaussian elimination with scaled partial pivoting. What is the scaled vector? What is the index vector? [Show all your work.]

$$\begin{aligned}
x_1 + 20x_2 \quad & -x_3 + 0.001x_4 = 0 \\
2x_1 - 5x_2 \quad & +30x_3 - 0.1 \quad x_4 = 1 \\
5x_1 + x_2 \quad & -100x_3 - 10 \quad x_4 = 0 \\
2x_1 - 100x_2 \quad & -x_3 + \qquad x_4 = 0
\end{aligned}$$

2. Consider the linear system $Ax = b \qquad (*) \qquad$ where

$$A = \begin{bmatrix} 2 & -1 & 0 & \dots \\ -1 & 2 & -1 & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & 2 & -1 \\ \dots & 0 & -1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ n \end{bmatrix}$$

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(a) Write a Matlab script file which solves $(*)$ by using gauss-pivot.m. for any integer n.

(b) Write a Matlab script file which solves $(*)$ by using tri.m for any integer n.

(c) Use the Matlab script in (a) to solve $(*)$ when $n = 400$. print $\tilde{x}[100 : 100 : n]$ where $\tilde{x}$ is the computed solution. Use the Matlab command tic and toc to find the cpu time.

(d) Use the Matlab script in (b) to solve $(*)$ when $n = 400$. Print $\hat{x}[100 : 100 : n]$ where $\hat{x}$ is the computed solution. Use the Matlab command tic and toc to find the cpu time. Compare the cpu time with the one in part (c).

(e) [Without using tri.m] can you solve $(*)$ when $n = 1000$. [try b/a, $\text{inv}(a) * b$, gauss-pivot, $\ell u \ldots$]. If yes print $x[1000 : 1000 : n]$. If no why?

(f) [Without using tri.m] can you solve $(*)$ when $n = 1000$. If you print $x[100 : 100 : n]$. If no why?

3. The Hilbert matrix is a classical ill-conditioned matrix and small changes in its coefficients will produce a large change in the solution to the perturbed system.

(a) Find the exact solution of $Ax = b$. [Hint: do the computation by hand or in Matlab use the command format rat].

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

(b) Solve $\tilde{A}\tilde{X} = b$ where

$$\begin{bmatrix} 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ 0.5000 & 0.3333 & 0.2500 & 0.2000 \\ 0.3333 & 0.2500 & 0.2000 & 0.1667 \\ 0.2500 & 0.2000 & 0.1667 & 0.1420 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compare your solution with the exact solution obtained in part a. [Note: The coefficient matrix in $\tilde{A}$ is an approximation to the coefficient matrix in A].

4. Determine a, b, c, d, e so that S is a cubic spline function.

$$S(x) = \begin{cases} 5x^3 - 60x^2 + ax + b & 3 \le x \le 4 \\ -2x^3 + 3x^2 + cx + 26 & 0 \le x \le 3 \\ -x^3 + 3x^2 + 19x + d & -1 \le x \le 0 \\ x^3 + 9x^2 + ex + 28 & -3 \le x \le -1 \end{cases}.$$

5. [Chemical Reaction Product]

Consider the observed concentration of the product of a simple chemical reaction, as a function of time.

Time:	0.00 0.10 0.40 0.50 0.60 0.90 1.00 1.10 1.40 1.50 1.60 1.90 2.00
Product:	0.00 0.06 0.17 0.19 0.21 0.25 0.26 0.27 0.29 0.29 0.30 0.31 0.31

Use the Matlab command spline to find values for time = 1.30, 0.70 and 0.80; then plot the curve.

6. Consider Figure 1. Reproduce Figure 1 by spline functions as follows: Select 30 points on the curve and label them $t = 1, 2, 3, \ldots, 30$. For each value of t, read off the x- and y-coordinates of the point, thus producing a table:

t	1	$2 \ldots$	30
x	x_1	$x_2 \ldots$	x_{30}
y	y_1	$y_2 \ldots$	y_{30}

Now, let $tt = 1 : 0.1 : 25; xx = \text{spline}(t, x, tt);$

$yy = \text{spline}(t, y, tt); \text{plot}(xx, yy).$