King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math-321 , Semester (991)
Instructor : Dr. Faisal Fairag
HOMEWORK(5)
Due November 8,1999

PLEASE SUBMIT CLEAR,CLEAN AND NEAT FORM.

(please use A4 size paper for computer output)

Problem #1

a) Find the truncation error term for the two-point gaussian quadrature formula on the closed interval [-1,1]. [Hint: Read page 199-201]

b) The truncation error for Simpson's rule on [-1,1] is $-\frac{1}{90}f^{(4)}(\xi)$ for some $\xi \in (a,b)$. Compare the truncation error terms in part (a). Which method do you think is best? why?

Problem #2

a) Trapezoid rule is used to compute $\int_0^4 e^{-2x^2}dx$ with error 10^{-4}. How many point should be used.

b) Find an approximate value of $\int_2^3 \frac{\cos(x)}{x^2}dx$ using Simpson's method. give a bound on the error.

Problem #3

Consider a formula of the form

$$\int_{-1}^{1} f(x)dx \approx w_0 f(x_0) + w_1 f(x_1) + w_2 f(x_2)$$

where x_0, x_1 and x_2 are the chebyshev nodes.

a) Use the method of undetermined coefficient to determine w_0, w_1 and w_2.

b) Use Lagrange polynomials to determine w_0, w_1 and w_2.

Problem #4

a) Write a Matlab function **[quad]=gauss(f,a,b,x,w)** which computes $\int_a^b f(x)dx$ using Gaussain quadrature with x as nodes vector and w as weight vector.

b) Use six-point Gaussian quadrature formula to approximate $\int_0^1 \frac{\sin x}{x}dx$. Can we use Simpson's or Trapezoid?

Problem #5

a) Use adapt.m to approximate $\int_{10}^{30} f(x)dx$ with tolerance 0.00001.
$f(x) = \frac{13}{5}(x - x^2/25 - 6)e^{3-3x/10}$

b) Graph the function $y = f(x)$ and all subintervals used in adapt.m .[Hint: the first column of SRmat contains the endpoints (except for b) of the subintervals from the adaptive process. T=SRmat(:,1); m=length(T);T(m+1)=30;z=zeros(m+1);plot(T,z,'.');]

c) Use the Matlab commands **quad** , **quad8** to approximate the above integral.

Problem #6

The flow rate of an incompressible fluid in a pipe of radius 1 is given by

$$Q = \int_0^1 2\pi r V dr,$$

where r is the distance from the center of the pipe and V is the velocity of the fluid. Find Q if only the following tabulated velocity measurements V are available:

$$r = [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]$$
$$V = [1.0, 0.99, 0.96, 0.91, 0.84, 0.75, 0.64, 0.51, 0.36, 0.19, 0.0]$$

Problem #7

approximate the following integrals:

a) $\int_{\frac{1}{2}}^{\infty} x^{-2}\sin(\frac{\pi}{2x^2})dx$ [Hint: use change of variables]

b) The error function is defined as $Erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$. Find $Erf(2)$.

c) The exponential-integral is defined as $Ei(x) = \int_1^x \frac{e^{-t}}{t}dt$. Find $Ei(2)$.