King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math-321 , Semester (991)
Instructor : Dr. Faisal Fairag
HOMEWORK(4)

Due October 27,1999

PLEASE SUBMIT CLEAR,CLEAN AND NEAT FORM.

(please use A4 size paper for computer output)

Problem 1 Consider the following formula:

$$f''(x) = \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2} \tag{1}$$

(where $f_k = f(x + kh)$ for $k = -2, -1, 0, +1, +2$)

a) Derive the error term of formula (1). What is the order?

b) If $\phi(h) = f''(x)$+error terms (obtained from part a), then what combination of $\phi(h)$ and $\phi(h/2)$ should give an accurate estimate of $f''(x)$? (write the formula and the error term).

Problem 2

a) Write a Matlab function **[d]= derive(x,n,h)** which computes the derivative of a function f at a point x by using equation(5) page 153 and Richardson extrapolation.

b) Use **derive.m** to approximate the derivative of

$$f(x) = \tan(\cos(\frac{\sqrt{5} + \sin(x)}{1 + x^2})) \quad \text{at} \quad x = \frac{1 + \sqrt{5}}{3}.$$

Approximations should be accurate to 13 decimal places. Print out all *D(i,i)*.

c) Use the Matlab command **diff** and compare the answer with your result in part (b).

Problem 3 Partial differentiation formulas. The partial derivative $u_x(x, y)$ of $u(x, y)$ with respect to x is obtained by holding y fixed and differentiating with respect to x. Similarly, $u_y(x, y)$ is found by holding x fixed and differentiating with respect to y. The centeral difference formula can be adapted to partial derivatives

$$\begin{aligned} u_x(x, y) &= \frac{u(x + h, y) - u(x - h, y)}{2h} + O(h^2), \\ u_y(x, y) &= \frac{u(x, y + h) - u(x, y - h)}{2h} + O(h^2). \end{aligned} \tag{2}$$

Let $u(x, y) = \frac{xy}{(x+y)}$. Calculate approximations to $u_x(2, 3)$ and $u_y(2, 3)$ using formulas in (2) with $h = 0.1, 0.01$, and 0.001. Compare values obtained by differentiating $u(x, y)$ partially.

. (please turn over)

<u>Problem 4</u> Do problem # 26 page 186

<u>Problem 5</u>

a) Write Matlab function [**value**]= **trap(a,b,n)** to calculate $\int_a^b f(x)dx$ using the trapezoid with n equal subintervals.

b) Use **trap.m** to compute π from an integral of the form $c\int_a^b \frac{dx}{1+x^2}$. Print a table which contains values of approximations for n=10,20,30,40,....,300