Math 202

(1.2)

2) $y = \dfrac{1}{1+c_1 e^{-x}}$

Given $y(-1) = 2$ i.e.

At $x=-1$, $y=2$.

This gives $2 = \dfrac{1}{1+c_1 e}$

$\Rightarrow 2 + 2c_1 e = 1 \Rightarrow c_1 = -\dfrac{1}{2e}$

Thus IVP has solution

$$y = \frac{1}{1-\frac{1}{2e}e^{-x}} = \frac{2}{2-e^{-(x+1)}}.$$

12) $y = c_1 e^{x} + c_2 e^{-x}$; $y' = c_1 e^{x} - c_2 e^{-x}$

$y(1)=0 \Rightarrow c_1 e + c_2 e^{-1} = 0$ —(1)

$y'(1) = e \Rightarrow c_1 e - c_2 e^{-1} = e$ —(2)

Solving (1) and (2) for c_1, c_2

(1) $\Rightarrow c_1 e = -c_2 e^{-1} \Rightarrow c_1 = -c_2 e^{-2}$

Put in (2)

$-c_2 e^{-2} e - c_2 e^{-1} = e \Rightarrow c_2 = -\dfrac{e^2}{2}$

Thus $c_1 = -\,-\dfrac{e^2}{2} e^{-2} = \dfrac{1}{2}$.

So, $y = \dfrac{1}{2} e^{x} - \dfrac{e^2}{2} e^{-x}$

22) $(1+y^3)y' = x^2$

$$y' = \frac{x^2}{1+y^3} = f(x,y)$$

$$\frac{\partial f}{\partial y} = \frac{-3y^2x^2}{(1+y^3)^2}.$$

$f(x,y)$ and $\dfrac{\partial f}{\partial y}$ are not continuous in domain containing $y=-1$. Thus unique solution may exist in $y<-1$ or $y>-1$.

27) $y' = \sqrt{y^2-9} = f(x,y)$

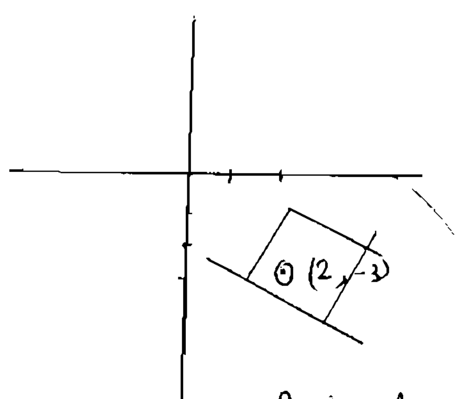
$$f(x,y) = \sqrt{y^2-9}$$

$$\frac{\partial f}{\partial y} = \frac{y}{2}\cdot 2\,(y^2-9)^{-1/2} = \frac{y}{\sqrt{y^2-9}}.$$

Thus region should not include $y = \pm 3$ or $-3<y<3$.

The given point $(2,-3)$.

Any rectangle containing $(2,-3)$ will include $y=-3$, thus the condition of the theorem will be violated. No unique solution is guaranteed.



Rectangle containing (2, -3).