1. $\int_1^e \frac{1}{x} \cdot \frac{\ln x}{1+(\ln x)^2}\,dx = I$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$I = \int \frac{u}{1+u^2}\,du = \frac{1}{2}\int \frac{2u\,du}{1+u^2}$

$= \frac{1}{2}\ln(1+u^2) + c$

$= \frac{1}{2}\ln(1+(\ln x)^2) + c$

$\int_1^e = \left[\frac{1}{2}\ln(1+(\ln x)^2)\right]_1^e$

$= \frac{1}{2}\ln 2 - 0 = \ln\sqrt{2}$

(a) $\ln\sqrt{2}$

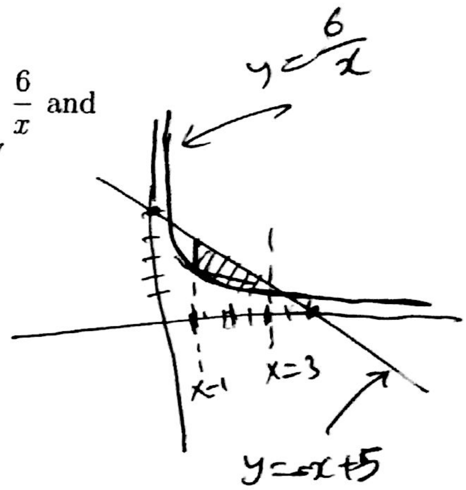
(b) $\ln 4$

(c) $\frac{1}{2}$

(d) $\frac{1}{2} + \ln\sqrt{2}$

(e) $\ln\sqrt{3}$

2. The area of the region bounded by the curve $y = \frac{6}{x}$ and the line $y = -x+5$ from $x = 1$ to $x = 3$ is given by



(a) $\int_1^2 \frac{x^2-5x+6}{x}\,dx + \int_2^3 \frac{-x^2+5x-6}{x}\,dx$

(b) $\int_1^3 \frac{-x^2+5x-6}{x}\,dx$

(c) $\int_1^2 \frac{-x^2+5x+6}{x}\,dx + \int_2^3 \frac{x^2-5x+6}{x}\,dx$

(d) $\int_1^3 \frac{x^2-5x+6}{x}\,dx$

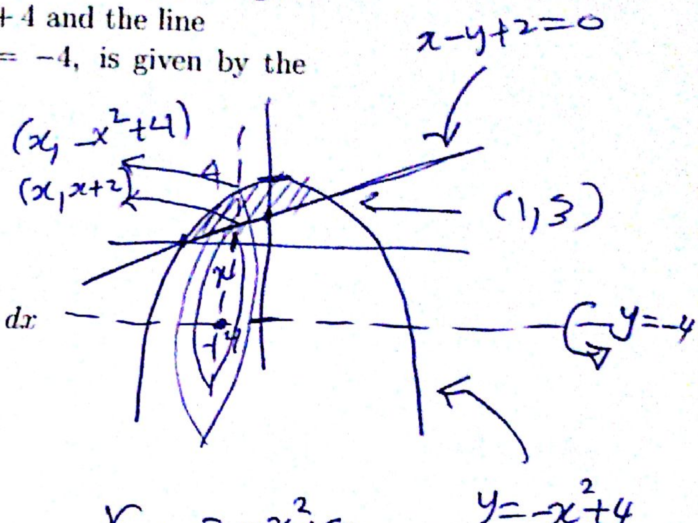
(e) $\int_1^2 \frac{6}{x}\,dx - \int_2^3 (-x+5)\,dx$

$A = \int_1^3 \left[(-x+5) - \frac{6}{x}\right]dx$

$= \int_1^3 \frac{-x^2+5x-6}{x}\,dx$

$= \int_1^2 \frac{-x^2+5x-6}{x}\,dx + \int_2^3 \frac{-x^2+5x-6}{x}\,dx$

3. The volume of the solid generated by revolving the region bounded by the parabola $y = -x^2 + 4$ and the line $x - y + 2 = 0$, about the line $y = -4$, is given by the definite integral



(a) $\int_{-2}^{1} \pi(x^4 - 17x^2 - 12x + 28)\,dx$

(b) $\int_{-2}^{1} \pi(x^4 + 2x^3 - 4x^2 - 4x - 12)\,dx$

(c) $\int_{-2}^{1} \pi(x^4 + 18x^2 + 14x - 28)\,dx$

(d) $\int_{-2}^{1} \pi(x^4 + 18x^2 + 14x - 28)\,dx$

(e) $\int_{-2}^{1} \pi(x^4 - 19x^2 + 12x + 28)\,dx$

$r_{out} = -x^2 + 8$

$r_{in} = x + 6$

$V = \int_{-2}^{1} \pi\left((-x^2+8)^2 - (x+6)^2\right)dx$

$= \int_{-2}^{1} \pi\left[x^4 + 64 - 16x^2 - x^2 - 36 - 12x\right]dx$

$= \int_{-2}^{2} \pi(x^4 - 17x^2 - 12x + 28)\,dx$

4. If the length of the curve $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = b$ is equal to $\frac{14}{3}$, then $b =$

(a) 3

(b) 2

(c) 1

(d) 0

(e) 4

$y' = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right)x^{1/2} = x^{1/2}$

$\sqrt{1+(y')^2} = \sqrt{1+x}$

$\text{Length} = \int_0^b \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}\Big]_0^b$

$= \frac{2}{3}(1+b)^{3/2} - \frac{2}{3}$

$\text{Length} = \frac{2}{3}\left[(1+b)^{3/2} - 1\right] = \frac{14}{3}$

$\Rightarrow (1+b)^{3/2} - 1 = 7 \Rightarrow (1+b)^{3/2} = 8$

$\Rightarrow (1+b) = (8)^{2/3} \Rightarrow 1+b = 4 \Rightarrow b = 3$

5. $\int x(\ln(2x))^2\,dx = I$

(a) $\frac{1}{2}(x\ln(2x))^2 - \frac{1}{2}x^2\ln(2x) + \frac{1}{4}x^2 + c$

(b) $\frac{1}{2}(x\ln(2x))^2 + \frac{1}{2}x^2\ln(2x) + x^2 + c$

(c) $\frac{(\ln(2x))^3}{3} + x + c$

(d) $\frac{(\ln(2x))^3}{3} - x + c$

(e) $(x\ln(2x))^2 + \frac{1}{4}x^2 - \ln(2x) + c$

$u = (\ln(2x))^2 \qquad dv = x\,dx$

$du = 2(\ln(2x))\frac{dx}{x} \qquad v = \frac{1}{2}x^2$

$I = \frac{1}{2}x^2(\ln(2x))^2 - \int x(\ln(2x))\,dx$

$u = \ln(2x) \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{2}x^2$

$\int x(\ln(2x))\,dx = \frac{1}{2}x^2\ln(2x) - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2\ln(2x) - \frac{1}{4}x^2$

So, $I = \frac{1}{2}x^2(\ln(2x))^2 - \frac{1}{2}x^2\ln(2x) + \frac{1}{4}x^2 + c$

6. $\int 4\tan^3 x\,dx = I$

(a) $2\tan^2 x + 4\ln|\cos x| + c$

(b) $2\tan^2 x + \ln|\cos x| + c$

(c) $\tan^4 x + c$

(d) $2\tan^2 x + \cot x\cos^2 x + c$

(e) $-4\tan^2 x + \ln|\cos x| + c$

$I = 4\int \tan^3 x\,dx$

$= 4\int \tan^2 x \cdot \tan x\,dx$

$= 4\int (\sec^2 x - 1)\cdot \tan x\,dx$

$= 4\int \sec^2 x \cdot \tan x\,dx - 4\int \tan x\,dx$

$= 4\left(\frac{1}{2}\tan^2 x\right) - 4\ln|\sec x| + c$

$= 2\tan^2 x - 4\ln|\sec x| + c$

$= 2\tan^2 x + 4\ln|\cos x| + c$

$\int \sec^2 x \cdot \tan x\,dx = \int u\,du$

let $u = \tan x$ $\qquad = \frac{1}{2}u^2 + c$

$du = \sec^2 x\,dx \qquad = \frac{1}{2}\tan^2 x + c$

7. $\int x\sqrt{1-x^4}\,dx = I$

(a) $\frac{1}{4}(x^2\sqrt{1-x^4}+\sin^{-1}(x^2))+c$

(b) $\frac{1}{4}(x^2\sqrt{1-x^4}-3\sin^{-1}(x^2))+c$

(c) $x+x^2\sqrt{1-x^4}+c$

(d) $\sqrt{1-x^4}+\sin^{-1}(x^2)+c$

(e) $\frac{1}{2}\sqrt{1-x^4}+2\sin^{-1}(x^2)+c$

$u = x^2 \rightarrow du = 2x\,dx$

$I = \frac{1}{2}\int 2x\sqrt{1-x^4}\,dx$

$= \frac{1}{2}\int\sqrt{1-u^2}\,du$

$u = \sin\theta \rightarrow du = \cos\theta\,d\theta$

$I = \frac{1}{2}\int \cos\theta(\cos\theta\,d\theta)$

$= \frac{1}{2}\int\cos^2\theta\,d\theta$

$= \frac{1}{2}\int\frac{1}{2}(1+\cos 2\theta)\,d\theta$

$= \frac{1}{4}\theta + \frac{1}{8}\sin 2\theta + c$

$= \frac{1}{4}\sin^{-1}u + \frac{1}{8}(2\sin\theta\cos\theta) + c$

1, u, θ, $\sqrt{1-u^2}$

$= \frac{1}{4}\sin^{-1}u + \frac{1}{4}(u)\sqrt{1-u^2}+c$

$= \frac{1}{4}\sin^{-1}u + \frac{1}{4}u\sqrt{1-u^2}+c$

$= \frac{1}{4}\sin^{-1}x^2 + \frac{1}{4}x^2\sqrt{1-x^4}+c$

8. $\int \frac{3x^3-3x^2+4}{x^2-x}\,dx = I$

(a) $\frac{3}{2}x^2+4\ln\left|\frac{x-1}{x}\right|+c$

(b) $\frac{3}{2}x^2+2\ln|x^2-x|+c$

(c) $\frac{3}{2}x^2+8\ln\left|\frac{x}{x-1}\right|+c$

(d) $3x^2+2\ln\left|\frac{x-1}{x}\right|+c$

(e) $3x^2+2\ln|x^2-x|+c$

$$\begin{array}{r|l} & 3x \\ \hline x^2-x & 3x^3-3x^2+4 \\ & \mp 3x^3 \pm 3x^2 \\ \hline & 4 \end{array}$$

$I = \int\left(3x + \frac{4}{x^2-x}\right)dx$

$= \frac{3}{2}x^2 + \int\frac{4}{x(x-1)}\,dx$

$\frac{1}{x(x-1)} = \frac{-1}{x} + \frac{1}{x-1}$

$\int\frac{4}{x(x-1)}\,dx = \int -\frac{1}{x}\,dx + \int\frac{1}{x-1}\,dx$

$= -\ln|x| + \ln|x-1|$

$= \ln\left|\frac{x-1}{x}\right|$

$I = \frac{3}{2}x^2 + 4\ln\left|\frac{x-1}{x}\right| + C$

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9. $\int (x^2+1)\,\text{sech}(\ln x)\,dx = I$

$\text{sech}(\ln x) = \frac{1}{\cosh(\ln x)}$

$= \frac{2}{e^{\ln x} + e^{-\ln x}} = \frac{2}{x + \frac{1}{x}}$

$= \frac{2x}{x^2+1} \Rightarrow$

$(x^2+1)\,\text{sech}(\ln x) = 2x$

$I = \int 2x\,dx = x^2 + c$

(a) $x^2 + c$

(b) $x^2 \ln x + \tanh(\ln x) + c$

(c) $\left(\frac{x^3}{3} + x\right) \text{sech}(\ln x) + c$

(d) $\text{sech}(\ln x) + x^2 \text{sech}(\ln x) + c$

(e) $x^3 + c$

10. The area of the surface generated by revolving the curve $y = \frac{x^3}{3}$, $0 \le x \le 1$, about the x-axis is equal to

(a) $\pi\left(\frac{\sqrt{8}-1}{9}\right)$

(b) $\frac{2\pi\sqrt{3}}{9}$

(c) 2π

(d) $\pi(\sqrt{7}-1)$

(e) $2\pi\left(\frac{\sqrt{8}-2}{9}\right)$

11. If $f(x) = \int_{e^x}^{1} \sin(\ln t)\, dt$, then $f'\left(\frac{\pi}{2}\right) =$

Fundamental Thm of calculus

$f'(x) = 0 - \sin(\ln e^x) \cdot e^x$

$= - e^x \sin(x)$

$f'(\frac{\pi}{2}) = - e^{\pi/2} \cdot \sin(\frac{\pi}{2}) = -e^{\pi/2}$

(a) $-e^{\frac{\pi}{2}}$

(b) 0

(c) -1

(d) $\sin(\ln 2)$

(e) $-e^{\frac{\pi}{2}} \sin\left(\ln \frac{\pi}{2}\right)$

12. Express $\int e^{x^2}\, dx$ as a power series

$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \text{——}$

$e^{x^2} = 1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 + \frac{1}{24}x^8 + \cdots$

$\int e^{x^2} dx = x + \frac{1}{3}x^3 + \frac{1}{10}x^5 + \frac{1}{42}x^7 + \frac{1}{(24)(9)}x^9 + \cdots + C$

(a) $c + x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \ldots$

(b) $c + x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \ldots$

(c) $c + 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$

(d) $c + 1 - x^2 + x^3 - x^4 + \ldots$

(e) $c + x^2 + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots$

13. If A, B and C are the undetermined coefficients of the partial fractions decomposition of the rational function $\frac{x}{x^3-1}$, then $A^2+B^2+C^2$ is equal to

(a) $\frac{1}{3}$

(b) $\frac{1}{9}$

(c) $\frac{2}{3}$

(d) $\frac{2}{9}$

(e) 1

$$\frac{x}{x^3-1} = \frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

multiply by $(x-1)(x^2+x+1)$

$$x = A(x^2+x+1) + (Bx+C)(x-1)$$

$\underline{x=1}$: $1 = 3A \rightarrow A = 1/3$

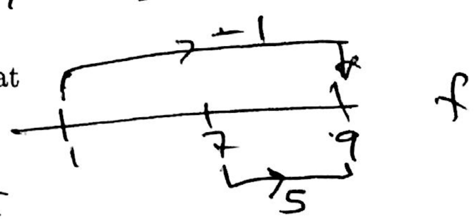
$\underline{x=0}$: $0 = A - C \rightarrow C = A = 1/3$

$\underline{\text{Coeff of } x^2}$: $0 = A+B \rightarrow B = -A = -1/3$

$A = 1/3$, $B = -1/3$, $C = 1/3$

$A^2+B^2+C^2 = \frac{1}{9}+\frac{1}{9}+\frac{1}{9} = 1/3$

14. If f and h are integrable functions such that $\int_1^9 f(x)dx = -1$, $\int_7^9 f(x)dx = 5$ and $\int_1^7 h(x)dx = 4$, then $\int_7^1 [h(x)-f(x)]dx = I$



(a) -10

(b) 8

(c) 6

(d) -8

(e) 12

$$I = \int_7^1 h\,dx - \int_7^1 f\,dx$$

$$= -\int_1^7 h\,dx + \int_1^7 f\,dx$$

$$= -(4) + (-1-5)$$

$$= -4 - 6 = -10$$

15. The improper integral $\int_0^{3\pi/2} \frac{\sin x}{1+\cos x}\, dx$ is $= I$

Note that: $x = \pi$ is a discont point

$1+\cos(\pi) = 0$

$I = \int_0^{\pi} dx + \int_{\pi}^{3\pi/2} dx$

(a) divergent

(b) covergent and its value is $\ln \frac{1}{2}$

(c) covergent and its value is $\ln 2$

(d) covergent and its value is 0

(e) covergent and its value is $\frac{1}{2}$

$\int \frac{\sin x\, dx}{1+\cos x} = -\int \frac{du}{1+u}$ let $u = \cos x$, $du = -\sin x\, dx$

$= -\ln|1+u| + c$

$= -\ln|1+\cos x| + C$

$\int_0^{\pi} dx = \lim_{t\to\pi^-} \left[-\ln|1+\cos x|\right]_0^t$

$= \lim_{t\to\pi^-} \left[-\ln|1+\cos t| + \ln 2\right]$

$= +\infty \Rightarrow$ diverge

16. $\int \frac{\sin^{-1}(e^{-x})}{\sqrt{e^{2x}-1}}\, dx =$

(a) $-\frac{1}{2}(\sin^{-1}(e^{-x}))^2 + c$

(b) $e^{-x}\sin^{-1}(e^{-x}) + c$

(c) $-\frac{3}{2}(\sin^{-1}(e^{-x}))^2 + c$

(d) $-\frac{1}{2}e^{-x}(\sin^{-1}(e^{-x}))^2 + c$

(e) $(\sin^{-1}(e^{-x}))^2 + c$

17. The sequence $\{2n - \sqrt{4n^2 - n}\}$ is $\times \dfrac{2n+\sqrt{4n^2-n}}{2n+\sqrt{4n^2-n}}$

$$= \frac{4n^2-(4n^2-n)}{2n+\sqrt{4n^2-n}}$$

$$= \frac{n}{2n+\sqrt{4n^2-n}}$$

(a) converges to $\frac{1}{4}$

(b) converges to 1

(c) converges to 0

(d) converges to $\frac{1}{2}$

(e) diverges

$$\lim \text{seq} = \lim_{n\to\infty} \frac{n}{2n+\sqrt{4n^2-n}}$$

$$= \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{2n}{n}+\sqrt{\frac{4n^2}{n^2}-\frac{n}{n^2}}} = \frac{1}{2+\sqrt{4-0}}$$

$$= 1/4$$

18. The series

$$\frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \frac{1}{(5)(6)} + \frac{1}{(6)(7)} + \cdots \quad \left\{\frac{1}{(n)(n+1)}\right\}_{n=3}^{\infty}$$

is

$$\sum_{n=3}^{\infty} \frac{1}{n(n+1)}$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$= \sum_{n=3}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

it is convg telescoping

$$\Rightarrow \text{sum} = \frac{1}{3}$$

(a) convergent and its sum is $\frac{1}{3}$

(b) convergent and its sum is 0

(c) convergent and its sum is $\frac{3}{4}$

(d) convergent and its sum is $\frac{1}{5}$

(e) divergent

19. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 3^{n-1}}{5^{n+1}} =$ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 3^n \cdot 3^{-1}}{5^n \cdot 5} = \sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{3}{5}\right)^n \frac{1}{15}$

geometric with $r = 3/5$, $a = \left(\frac{3}{5}\right)\left(\frac{1}{15}\right)$

(a) 0.025

(b) 0.01

(c) 0.5

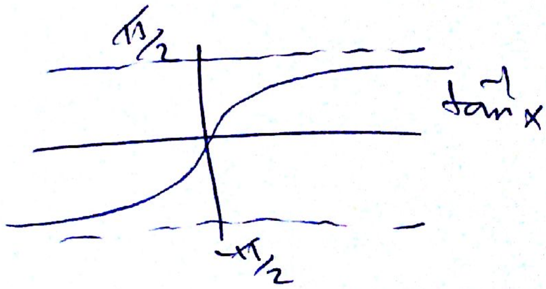
(d) 0.005

(e) 2.5

$$\text{Sum} = \frac{a}{1-r} = \frac{\left(\frac{3}{5}\right)\left(\frac{1}{15}\right)}{1-\frac{3}{5}} = \frac{(3)\left(\frac{1}{15}\right)}{5-3}$$

$$= \frac{\frac{3}{15}}{2} = \frac{3}{30} = 1/10$$

20. The series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^3}$ is



(a) convergent by the Comparison Test

(b) divergent by the nth-Term Test for Divergence

(c) divergent by the Ratio Test

(d) divergent by the Integral Test

(e) convergent by the Ratio Test

$\tan^{-1} n < \frac{\pi}{2}$

$$\frac{\tan^{-1} n}{n^3} < \frac{\pi/2}{n^3}$$

$n = 1, 2, 3, \ldots$

but $\sum_{n=1}^{\infty} \frac{\pi/2}{n^3} = \frac{\pi}{2} \sum_{n=1}^{\infty} \frac{1}{n^3}$

Convergent p-series

By comparison test $\sum \frac{\tan^{-1} n}{n^3}$ Conv

21. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}+\sqrt{n+3}}$

(a) converges conditionally

(b) diverges by the Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$

(c) converges absolutely

(d) diverges by the Ratio Test

(e) diverges by the nth-Term Test for Divergence.

22. The series $\sum_{n=1}^{\infty} \frac{3 \cdot 2^{2n}}{3^{n+1} n^n}$ is

$$\frac{3 \cdot 2^{2n}}{3^{n+1} \cdot n^n} = \frac{\cancel{3} \cdot 4^n}{3^n \cdot \cancel{3} \cdot n^n} = \frac{4^n}{3^n \cdot n^n} = \left(\frac{4}{3n}\right)^n$$

(a) convergent by the Root Test

(b) divergent by the Root Test

(c) a series for which the Root Test is inconclusive

(d) a divergent geometric series

(e) a convergent p-series.

$$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \frac{4}{3n} = 0 < 1$$

Conv by root test

23. The series $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^3}{(3n+1)!}$ is

First study $\sum \frac{(n!)^3}{(3n+1)!}$

use ratio test:

(a) absolutely convergent

(b) conditionally convergent

(c) divergent by the Ratio test

(d) a divergent p-series.

(e) a series for which the Ratio test is inconclusive

$$\frac{a_{n+1}}{a_n} = \frac{[(n+1)!]^3}{(3n+4)!} \cdot \frac{(3n+1)!}{[n!]^3}$$

$$= \frac{[(n+1)\cancel{n!}]^3 \cdot \cancel{(3n+1)!}}{(3n+4)(3n+3)(3n+2)\cancel{(3n+1)!} \cdot \cancel{[n!]^3}}$$

$$= \frac{(n+1)^3}{(3n+4)(3n+3)(3n+2)}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{27} \Rightarrow \text{Abso. Conv.}$$

24. The interval of convergence I and the radius of convergence R of the series $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^{2n} (x-3)^n$, are given by

✓ (a) $I = (-6, 12)$, $R = 9$

(b) $I = (-3, 3)$, $R = 3$ × center 0

(c) $I = (-6, 12]$, $R = 6$

(d) $I = (-3, 3)$, $R = 6$ × center 0

✓ (e) $I = [-6, 12)$, $R = 9$.

let $x = -6 \Rightarrow \sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^{2n} (-9)^n = \sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^{2n} (-3^2)^n$

$$= \sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^{2n} \cdot (-1)^n \cdot (3)^{2n} = \sum_{n=1}^{\infty} (-1)^n \left(\frac{3n}{3n+1}\right)^{2n}$$

it is diverge

$(-6, 12)$

25. The Taylor polynomial of order 3 generated by $f(x) = \ln(2+x)$ at $a = -1$ is

$f = \ln(2+x)$ $f(-1) = 0$

$f' = \frac{1}{2+x}$ $f'(-1) = 1$

$f'' = \frac{-1}{(2+x)^2}$ $f''(-1) = -1$

$f''' = \frac{2}{(2+x)^3}$ $f'''(-1) = 2$

(a) $P_3(x) = (x+1) - \frac{1}{2}(x+1)^2 + \frac{1}{3}(x+1)^3$

(b) $P_3(x) = 1 + (x+1) + \frac{1}{2}(x+1)^2 - \frac{1}{3}(x+1)^3$

Center 1 ✗ (c) $P_3(x) = (x-1) + \frac{1}{2}(x-1)^2 - (x+1)^3$

(d) $P_3(x) = (x+1) + \frac{1}{2}(x+1)^2 + \frac{1}{6}(x+1)^3$

Center 1 ✗ (e) $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

$$\text{Taylor} = f(-1) + \frac{f'(-1)}{1!}(x+1) + \frac{f''(-1)}{2!}(x+1)^2 + \frac{f'''(-1)}{3!}(x+1)^3 + \cdots$$

$$= 0 + (x+1) - \frac{1}{2}(x+1)^2 + \frac{2}{6}(x+1)^3 + \cdots$$

26. let $f(x) = \frac{1}{2-x}$, $|x| < 2$, then the power series representation of $f''(x)$ is

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$f(x) = \frac{1}{2-x} = \frac{1}{2} \frac{1}{1-\frac{x}{2}}$$

replace each $x \to \frac{x}{2}$

(a) $\sum_{n=2}^{\infty} \frac{n(n-1)x^{n-2}}{2^{n+1}}$

(b) $\sum_{n=2}^{\infty} n(n-1)\left(\frac{x}{2}\right)^{n-1}$

(c) $\sum_{n=0}^{\infty} \frac{n(n-1)x^{n-2}}{2^n}$

(d) $\sum_{n=2}^{\infty} n(n-1)\left(\frac{x}{2}\right)^{n-2}$

(e) $\sum_{n=3}^{\infty} n(n-1)\left(\frac{x}{2}\right)^{n-3}$

$$f(x) = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \frac{x^n}{2^n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}}$$

$$f''(x) = \sum_{n=2}^{\infty} \frac{n(n-1)x^{n-2}}{2^{n+1}} \longleftarrow f'(x) = \sum_{n=1}^{\infty} \frac{n x^{n-1}}{2^{n+1}}$$

27. The coefficient of x^5 in the product of the Maclaurin series of $\sin x$ and $\frac{1}{1-x}$ is equal to

(a) $\frac{101}{120}$

(b) $\frac{97}{120}$

(c) $\frac{17}{60}$

(d) $\frac{131}{120}$

(e) $\frac{1}{120}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots$$

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \cdots$$

$$\left(\frac{1}{1-x}\right)\sin x = (\quad)(\quad)$$

Coeff of x^5: $\frac{1}{120} - \frac{1}{6} + 1$

$$= \frac{1 - 20 + 120}{120} = \frac{101}{120}$$

28. For $-1 < x < 1$, the Maclaurian series generated by $f(x) = \sqrt[3]{(1-x)^2}$ is

(a) $1 - \frac{2}{3}x - \frac{x^2}{9} - \frac{4x^3}{81} + \ldots$

(b) $1 - \frac{2}{3}x + \frac{x^2}{6} - \frac{4x^3}{81} + \ldots$

(c) $1 - \frac{2}{3}x - \frac{x^2}{9} + \frac{4x^3}{27} + \ldots$

(d) $1 - \frac{2}{3}x + \frac{x^2}{9} + \frac{4x^3}{63} + \ldots$

(e) $1 + \frac{2}{3}x - \frac{x^2}{6} - \frac{4}{81}x^3 + \ldots$

$$f(x) = \sqrt[3]{(1-x)^2} = (1-x)^{2/3}$$

$k = 2/3$

binomial:

$$(1-x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k}{3!} + \cdots$$

$$(1-x)^{2/3} = 1 + \frac{2}{3}x + \frac{\frac{2}{3}\left(\frac{2}{3}-1\right)}{2!}x^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\left(\frac{2}{3}-2\right)}{3!}x^3 + \cdots$$

$$= 1 - \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 \cdots$$