



Q	MM	V1	V2	V3	V4
1	a	d	d	e	b
2	a	e	d	e	a
3	a	d	b	e	d
4	a	e	a	c	a
5	a	a	d	e	d
6	a	e	b	b	d
7	a	d	a	b	e
8	a	e	e	b	b
9	a	c	c	a	c
10	a	c	c	e	a
11	a	a	a	b	b
12	a	d	b	e	d
13	a	d	a	d	d
14	a	e	e	d	c
15	a	c	d	b	e
16	a	a	e	c	e
17	a	c	b	d	c
18	a	b	d	b	e
19	a	c	e	d	c
20	a	e	d	a	d

Detailed
Solutions →

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 162
Wednesday 15/3/2017
Net Time Allowed: 120 minutes

MASTER VERSION

1. Using **three** rectangles and **midpoints**, the area under the graph of $f(x) = 3x - x^2$ from $x = 0$ to $x = 3$ is approximately equal to

$$\Delta x = \frac{3-0}{3} = 1$$

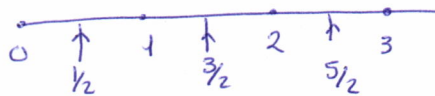
(a) $\frac{19}{4}$

(b) $\frac{17}{4}$

(c) $\frac{17}{2}$

(d) 9

(e) $\frac{8}{3}$



$$\begin{aligned} A &\approx f\left(\frac{1}{2}\right) \Delta x + f\left(\frac{3}{2}\right) \Delta x + f\left(\frac{5}{2}\right) \Delta x \\ &= \left(\frac{3}{2} - \frac{1}{4}\right) + \left(\frac{9}{2} - \frac{9}{4}\right) + \left(\frac{15}{2} - \frac{25}{4}\right) \\ &= \frac{5}{4} + \frac{9}{4} + \frac{5}{4} \\ &= \frac{19}{4} \end{aligned}$$

2. $\int \left(\frac{1-x}{x}\right)^2 dx = \int \frac{1-2x+x^2}{x^2} dx = \int \frac{1}{x^2} - \frac{2}{x} + 1 dx$

$$= -\frac{1}{x} - 2 \ln|x| + x + C$$

(a) $-\frac{1}{x} - 2 \ln|x| + x + C$

(b) $-\frac{1}{3} \left(\frac{1-x}{x}\right)^3 + C$

(c) $-\frac{1}{x} + x + C$

(d) $\frac{2}{x} + \ln|x| - x + C$

(e) $\frac{1}{x^2} + \frac{2}{x} + C$

3. $\int \frac{6}{x(\ln x)^4} dx =$

(a) $\frac{-2}{(\ln x)^3} + C$

(b) $\frac{-3}{(\ln x)^3} + C$

(c) $\frac{x}{(\ln x)^2} + C$

(d) $\frac{1}{3(\ln x)^3} + C$

(e) $\frac{6}{(\ln x)^2} + C$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$= \int \frac{6}{u^4} du = \int 6u^{-4} du$$

$$= 6 \cdot \frac{u^{-3}}{-3} + C$$

$$= -2 \cdot \frac{1}{u^3} + C$$

$$= \frac{-2}{(\ln x)^3} + C$$

4. $\int_0^1 x(\sqrt[3]{x} + 3x^2 \sqrt{x}) dx = \int_0^1 x x^{1/3} + 3x^3 \cdot x^{1/2} dx$

$$= \int_0^1 x^{4/3} + 3x^{7/2} dx$$

(a) $\frac{23}{21}$

(b) $\frac{21}{5}$

(c) $\frac{1}{2}$

(d) $\frac{3}{7}$

(e) $\frac{8}{5}$

$$= \int_0^1 x^{4/3} + 3x^{7/2} dx$$

$$= \left[\frac{3}{7} x^{7/3} + \frac{6}{9} x^{9/2} \right]_0^1$$

$$= \left(\frac{3}{7} + \frac{2}{3} \right) - 0$$

$$= \frac{9+14}{21} = \frac{23}{21}$$

5. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \cos \left(1 + \frac{i}{n} \right)^2 =$

(a) $\int_1^2 \cos(x^2) dx$

(b) $\int_1^2 \cos(1+x^2) dx$

(c) $\int_1^2 \cos^2 x dx$

(d) $\int_0^1 \cos(x^2) dx$

(e) $\int_0^1 \cos(1+x^2) dx$

$$x_i^* = 1 + \frac{i}{n}, \quad \Delta x = \frac{1}{n}$$

$$= a + i \Delta x$$

$$\Rightarrow a=1, \quad \Delta x = \frac{b-a}{n} = \frac{1}{n}$$

$$\Rightarrow a=1, \quad b=2$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \cos(x_i^{*2}) \Delta x$$

$$= \int_1^2 \cos(x^2) dx$$

6. The **volume** of the solid obtained by rotating the region bounded by the curves $y = 2\sqrt{x}$, $y = 0$, $x = 2$ about the x -axis is

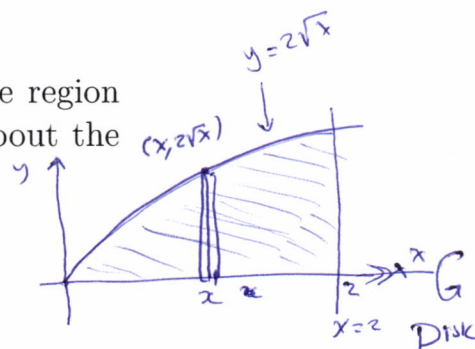
(a) 8π

(b) $\frac{5\pi}{2}$

(c) 3π

(d) 6π

(e) $\frac{\pi}{2}$



$$V = \pi \int_0^2 (2\sqrt{x})^2 dx$$

$$= 4\pi \int_0^2 x dx$$

$$= 4\pi \cdot \left[\frac{x^2}{2} \right]_0^2$$

$$= 4\pi \cdot (2 - 0)$$

$$= 8\pi$$

7. If $F(x) = \int_x^{x^2} e^{t^2} dt$ then $F'(x) =$ $e^{(x^2)^2} \cdot \frac{d}{dx}(x^2) - e^{x^2} \cdot \frac{d}{dx}(x)$
 $= 2x e^{x^4} - e^{x^2}$

(a) $2x e^{x^4} - e^{x^2}$

(b) $e^{x^4} - e^{x^2}$

(c) $e^{x^4 - x^2}$

(d) $2e^{x^2} - e^x$

(e) $2e^{x^4} - x e^{x^2}$

8. $\int \frac{\tan \theta}{\sec \theta (\sec \theta - \cos \theta)} d\theta = \int \frac{\tan \theta}{\sec^2 \theta - 1} d\theta = \int \frac{\tan \theta}{\tan^2 \theta} d\theta$

(a) $\ln |\sin \theta| + C$

(b) $\ln |\sec \theta - \cos \theta| + C$

(c) $\sin \theta + \tan \theta + C$

(d) $-\tan \theta + \ln |\sin \theta| + C$

(e) $\cot \theta + \cos \theta + C$

$$= \int \frac{1}{\tan \theta} d\theta$$

$$= \int \cot \theta d\theta, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$= \ln |\sin \theta| + C$$

9. An equation for the **tangent line** to the curve $y = \int_x^{\sqrt{3}} \sqrt{1+t^2} dt$ at the point with x -coordinate $\sqrt{3}$ is given by

• point: $x = \sqrt{3} \Rightarrow y = \int_{\sqrt{3}}^{\sqrt{3}} \sqrt{1+t^2} dt = 0$

(a) $y = -2x + 2\sqrt{3}$

(b) $y = 2x - 2\sqrt{3}$

(c) $y = \sqrt{3}x - 3$

(d) $y = 3x - 3\sqrt{3}$

(e) $y = -\sqrt{3}x + 2\sqrt{3}$

• slope: $y = - \int_{\sqrt{3}}^x \sqrt{1+t^2} dt$

$\frac{dy}{dx} = -\sqrt{1+x^2}$

slope = $\frac{dy}{dx} \Big|_{x=\sqrt{3}} = -\sqrt{1+3} = -2$

• Equation: $y - 0 = -2(x - \sqrt{3})$
 $\Rightarrow y = -2x + 2\sqrt{3}$

10. If $f(x) = \begin{cases} 2 + \sqrt{4-x^2} & \text{if } x < 2 \\ |x-4| & \text{if } x \geq 2 \end{cases}$,

then $\int_{-2}^4 f(x) dx =$

(a) $2\pi + 10$

(b) $\pi - 6$

(c) $2\pi + 2$

(d) $\pi - 2$

(e) $6 + \frac{\pi}{2}$

$$= \int_{-2}^2 (2 + \sqrt{4-x^2}) dx + \int_2^4 |x-4| dx$$

$$= 2x \Big|_{-2}^2 + \frac{1}{2} \pi (2)^2 + \int_2^4 4-x dx$$

$$= 8 + 2\pi + \left(4x - \frac{1}{2}x^2 \right) \Big|_2^4$$

$$= 8 + 2\pi + (16-8) - (8-2)$$

$$= 8 + 2\pi + 8 - 6$$

$$= 2\pi + 10$$

11. The **area** of the region enclosed by the curves $y = |x|$ and $y = x^2 - 2$ is

By symmetry about the y-axis:

$$A = 2 \cdot \int_0^2 [x - (x^2 - 2)] dx$$

$$= 2 \cdot \int_0^2 x - x^2 + 2 dx$$

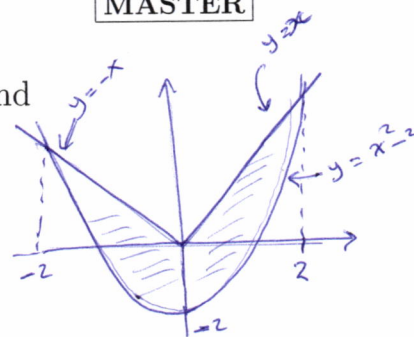
$$= 2 \cdot \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x \right]_0^2$$

$$= 2 \cdot \left[\left(2 - \frac{8}{3} + 4 \right) - 0 \right]$$

$$= 2 \cdot \left(6 - \frac{8}{3} \right)$$

$$= 2 \cdot \frac{18 - 8}{3} = 2 \cdot \frac{10}{3} = \frac{20}{3}$$

(e) $\frac{17}{12}$



$$\Delta x = \frac{5-2}{n} = \frac{3}{n}$$

$$x_i^* = a + i \Delta x = 2 + \frac{3i}{n}$$

12. Using n subintervals with **right endpoints**, we get

$$\int_2^5 (x^2 - 4) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(2 + \frac{3i}{n} \right)^2 - 4 \right] \frac{3}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(4 + \frac{12i}{n} + \frac{9i^2}{n^2} - 4 \right) \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{36i}{n^2} + \frac{27i^2}{n^3} \right)$$

(a) $\lim_{n \rightarrow \infty} \left[\frac{18(n+1)}{n} + \frac{9(n+1)(2n+1)}{2n^2} \right]$

$$= \lim_{n \rightarrow \infty} \frac{36}{n^2} \cdot \frac{n(n+1)}{2} + \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

(b) $\lim_{n \rightarrow \infty} \left[\frac{9(n+1)(2n+1)}{n^2} - 4n \right]$

$$= \lim_{n \rightarrow \infty} 18 \frac{(n+1)}{n} + \frac{9}{2} \frac{(n+1)(2n+1)}{n^2}$$

(c) $\lim_{n \rightarrow \infty} \left[\frac{9(n+1)}{2n} - \frac{9(n+1)(2n+1)}{2n^2} \right]$

(d) $\lim_{n \rightarrow \infty} \left[\frac{12(n+1)}{n} + \frac{15(n+1)(2n+1)}{2n^2} \right]$

(e) $\lim_{n \rightarrow \infty} \left[\frac{18(n+1)}{n} + \frac{9(n+1)(2n+1)}{n^2} \right]$

13. The **volume** of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $x = -1$ is given by

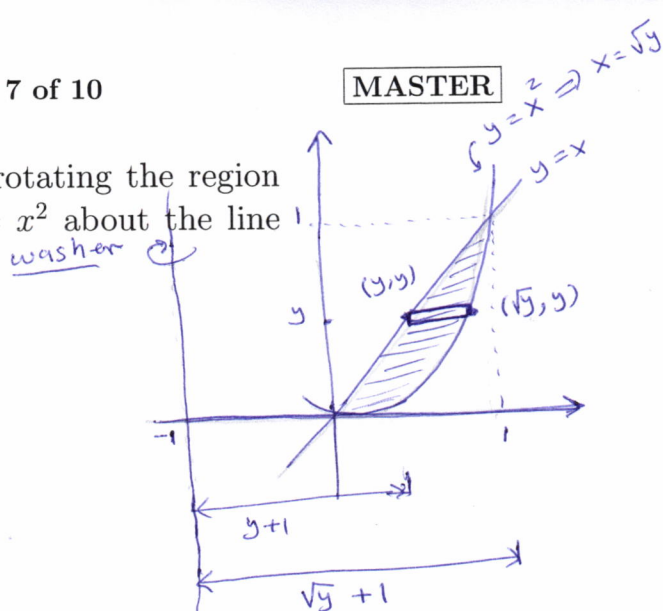
(a) $\pi \int_0^1 [(1 + \sqrt{y})^2 - (1 + y)^2] dy$

(b) $\pi \int_0^1 [y - (1 + y)^2] dy$

(c) $\pi \int_0^1 y - y^2 dy$

(d) $\pi \int_0^1 [(x^2 + 1)^2 - (x + 1)^2] dx$

(e) $\pi \int_0^1 (\sqrt{y} - y) dy$



$$V = \pi \int_0^1 [(\sqrt{y} + 1)^2 - (y + 1)^2] dy$$

14. $\int_{-1}^1 \frac{\sin^3 t}{2 + \sin^2 t} dt =$

0

Since $f(t) = \frac{\sin^3 t}{2 + \sin^2 t}$ is an odd function (check?)

(a) 0

(b) $\ln 2$

(c) $2 \ln(2 + \sin 1)$

(d) $-\ln(\sin 1)$

(e) $\ln \left(\frac{2 + \sin 1}{2 - \sin 1} \right)$

15. If $\int_{-5}^7 f(x) dx = -17$, $\int_{-5}^{11} f(x) dx = 32$, and $\int_8^7 f(x) dx = 5$, then $\int_{11}^8 f(x) dx =$

(a) -54

(b) 19

(c) -60

(d) 44

(e) -50

$$\int_{-5}^{11} f(x) dx = \int_{-5}^7 f(x) dx + \int_7^8 f(x) dx + \int_8^{11} f(x) dx$$

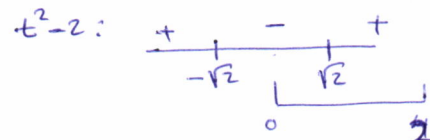
$$32 = -17 - 5 + \int_8^{11} f(x) dx$$

$$\int_8^{11} f(x) dx = 32 + 17 + 5 = 32 + 22 = 54$$

$$\int_{11}^8 f(x) dx = -54$$

16. The velocity (in m/s) of a particle moving along a line is given by

$$v(t) = t^2 - 2$$



The **distance** traveled by the particle during the time interval $0 \leq t \leq 2$ is

(a) $\frac{8\sqrt{2}-4}{3} m$

(b) $\frac{4\sqrt{2}+2}{3} m$

(c) $\frac{4+\sqrt{2}}{3} m$

(d) $\frac{8-2\sqrt{2}}{3} m$

(e) $\frac{5-3\sqrt{2}}{3} m$

$$\begin{aligned} \text{Distance} &= \int_0^2 |v(t)| dt = \int_0^2 |t^2 - 2| dt \\ &= \int_0^{\sqrt{2}} |t^2 - 2| dt + \int_{\sqrt{2}}^2 |t^2 - 2| dt \\ &= \int_0^{\sqrt{2}} (2 - t^2) dt + \int_{\sqrt{2}}^2 (t^2 - 2) dt \\ &= \left[2t - \frac{t^3}{3} \right]_0^{\sqrt{2}} + \left[\frac{t^3}{3} - 2t \right]_{\sqrt{2}}^2 \\ &= 2\sqrt{2} - \frac{2\sqrt{2}}{3} - 0 + \left(\frac{8}{3} - 4 \right) - \left(\frac{2\sqrt{2}}{3} - 2\sqrt{2} \right) \\ &= \frac{4\sqrt{2}}{3} + \frac{8-12}{3} - \left(\frac{-4\sqrt{2}}{3} \right) \\ &= \frac{8\sqrt{2}}{3} + \frac{-4}{3} = \frac{8\sqrt{2}-4}{3} \end{aligned}$$

17. If f is a continuous function and

$$2 \leq f(x) \leq 5 \text{ for } 3 \leq x \leq 9,$$

$$\begin{aligned} &\Rightarrow f(x) > 0 \text{ for } 3 \leq x \leq 9 \\ &\Rightarrow |f(x)| = f(x) \text{ for } 3 \leq x \leq 9 \end{aligned}$$

then which one of the following statements is in general

FALSE:

$$(a) \int_3^9 (1 - 2|f(x)|) dx \geq -10$$

$$-10 \leq -2f(x) \leq -4$$

$$-9 \leq 1 - 2f(x) \leq -3$$

$$-54 \leq \int_3^9 (1 - 2f(x)) dx \leq -18$$

$$(b) \int_3^9 (3 - f(x)) dx \geq -12$$

$$(c) \int_3^9 |f(x)| dx \geq 12$$

$$(d) \int_3^9 -2f(x) dx \leq -24$$

$$(e) \int_3^9 (f(x))^2 dx \geq 24$$

18. $\int x^3 \sqrt{x^2+1} dx = \int x^2 \sqrt{x^2+1} \cdot x dx$; $u = x^2+1 \Rightarrow du = 2x dx$

$$(a) \frac{1}{5}(x^2+1)^{3/2} \left(x^2 - \frac{2}{3}\right) + C$$

$$(b) \frac{1}{3}(x^2+1)^{3/2} \left(x^2 - \frac{3}{5}\right) + C$$

$$(c) \frac{1}{5}(x^2+1)^5 + \frac{1}{3}(x^2+1)^3 + C$$

$$(d) \frac{1}{5}(x^2+1)^{3/2} \left(x^2 - \frac{4}{3}\right) + C$$

$$(e) \frac{1}{3}(x^2+1)^{3/2} \left(\frac{3}{5}x^2 - 3\right) + C$$

$$\begin{aligned} &\downarrow \\ &= \int (u-1) \sqrt{u} \cdot \frac{1}{2} du = \frac{1}{2} \int u^{3/2} - u^{1/2} du \\ &= \frac{1}{2} \left[\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right] + C \\ &= \frac{1}{5} (x^2+1)^{5/2} - \frac{1}{3} (x^2+1)^{3/2} + C \\ &= \frac{1}{5} (x^2+1)^{3/2} \left[(x^2+1) - \frac{5}{3} \right] + C \\ &= \frac{1}{5} (x^2+1)^{3/2} \left(x^2 - \frac{2}{3} \right) + C \end{aligned}$$

19. If $f(x) = x^{-1} \left[\cos \left(\frac{\pi}{4} \ln x \right) \right]^{-2} \left[4 + 5 \tan \left(\frac{\pi}{4} \ln x \right) \right]^{-1/2}$,

then $\int_1^e f(x) dx =$

(a) $\frac{8}{5\pi}$

(b) 4

(c) $\frac{6}{5\pi}$

(d) 4π

(e) $\frac{30}{\pi}$

$$u = 4 + 5 \tan \left(\frac{\pi}{4} \ln x \right) \Rightarrow du = 5 \sec^2 \left(\frac{\pi}{4} \ln x \right) \cdot \frac{\pi}{4} \frac{1}{x} dx$$

$$\Rightarrow \frac{4}{5\pi} du = \frac{1}{x \cos^2 \left(\frac{\pi}{4} \ln x \right)} dx$$

$$x=1 \Rightarrow u=4 \quad \& \quad x=e \Rightarrow u=4+5=9$$

$$\int_1^e f(x) dx = \int_4^9 u^{-1/2} \cdot \frac{4}{5\pi} du$$

$$= \frac{4}{5\pi} \cdot 2 u^{1/2} \Big|_4^9$$

$$= \frac{8}{5\pi} (3-2)$$

$$= \frac{8}{5\pi}$$

20. The base of a solid is the triangular region with vertices $(0,0)$, $(1,0)$, and $(0,1)$. If the cross sections of the solid perpendicular to the x -axis are **semi-circles**, then the **volume** of the solid is

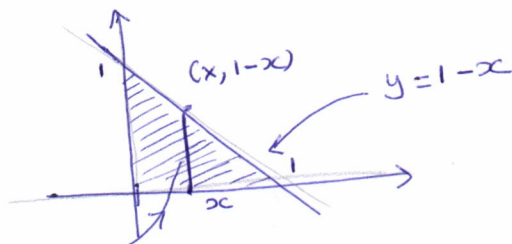
(a) $\frac{\pi}{24}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{4}$

(e) $\frac{\pi}{3}$



$$\text{Diameter} = (1-x)$$

$$\text{radius} = \frac{1-x}{2}$$

$$A(x) = \frac{1}{2} \pi \left(\frac{1-x}{2} \right)^2 = \frac{\pi}{8} (1-x)^2$$

$$V = \int_0^1 A(x) dx$$

$$= \frac{\pi}{8} \int_0^1 (1-x)^2 dx = -\frac{\pi}{8} \cdot \frac{(1-x)^3}{3} \Big|_0^1$$

$$= -\frac{\pi}{8} \left(0 - \frac{1}{3} \right)$$

$$= \frac{\pi}{24}$$