Math 102
Final Exam
Second
Saturday, June 11, 2011

# EXAM COVER

Number of versions: 4 Number of questions: 28 Number of Answers: 5 per question

Math 102
Final Exam
Second
Saturday, June 11, 2011
Net Time Allowed: 180 minutes

# **MASTER VERSION**

- 1. If  $F(x) = \int_x^{x^2} \frac{dt}{\sqrt{1-t^2}}$ , then  $F'\left(\frac{1}{2}\right)$  is
  - (a)  $\frac{4-2\sqrt{5}}{\sqrt{15}}$
  - (b)  $\frac{4+2\sqrt{5}}{\sqrt{15}}$
  - (c)  $\frac{4}{\sqrt{15}} \frac{1}{\sqrt{3}}$
  - (d)  $\frac{2 2\sqrt{5}}{\sqrt{15}}$
  - (e)  $\frac{2+2\sqrt{5}}{\sqrt{15}}$

- 2. The value of the integral  $\int_0^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) dx$  is
  - (a) -1/2
  - (b) 1/2
  - (c)  $-\sqrt{2}/2$
  - (d) e/2
  - (e) -e/2

- 3. The value of the integral  $\int_{-1}^{0} x^2 \sqrt{1+x} \ dx$  is
  - (a)  $\frac{16}{105}$
  - (b)  $\frac{19}{105}$
  - (c)  $\frac{8}{105}$
  - (d)  $\frac{22}{105}$
  - (e)  $\frac{102}{105}$

- 4. The value of the integral  $\int_0^1 x\sqrt{1-x^4} \, dx$  is
  - (a)  $\frac{\pi}{8}$
  - (b)  $\frac{\pi}{16}$
  - (c)  $\frac{\pi}{3}$
  - (d)  $\frac{\pi}{6}$
  - (e)  $\frac{\pi}{2}$

- 5. The area of the region bounded by the curves  $4x + y^2 = 12$  and x = y is
  - (a)  $\frac{64}{3}$
  - (b) 21
  - (c)  $\frac{44}{3}$
  - (d)  $\frac{52}{3}$
  - (e) 17

- 6. The volume of the solid obtained by rotating the region bounded by the curves  $y = e^{x^2}$ , y = e and y = 0 (in the first quadrant) about y-axis is
  - (a)  $\pi$
  - (b)  $2\pi$
  - (c)  $e^2\pi$
  - (d)  $\pi e$
  - (e)  $\pi e^2$

7. The volume of the solid obtained by rotating the region bounded by the curves  $x^2 - y^2 = 1$  and x = 3 about the line x = -2 is given by

(a) 
$$\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 - (2 + \sqrt{1 + y^2})^2] dy$$

(b) 
$$\int_0^{2\sqrt{2}} \pi [25 - (2+y)^2] dy$$

(c) 
$$\int_{-3}^{3} \pi [25 - (2 + \sqrt{x^2 - 1})^2] dx$$

(d) 
$$\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 - (1 + y^2)] dy$$

(e) 
$$\int_{-3}^{3} \pi [9 - (x^2 - 1)] dx$$

8. If f(x) = f'(x) + 3, f(0) = 1 and f(1) = 4, then the value of the integral  $\int_0^1 e^x f'(x) dx$  is

(a) 
$$\frac{e+2}{2}$$

(b) 
$$\frac{e-2}{2}$$

(c) 
$$\frac{2e-1}{2}$$

(d) 
$$\frac{e+1}{2}$$

(e) 
$$\frac{e-1}{2}$$

- 9. The value of the integral  $\int_0^{\pi/4} \tan^4 x \sec^4 x \ dx$  is
  - (a)  $\frac{12}{35}$
  - (b)  $\frac{2}{35}$
  - (c)  $-\frac{2}{35}$
  - (d)  $\frac{1}{5}$
  - (e)  $\frac{5}{12}$

- 10. The value of the integral  $\int_0^1 x^3 \sqrt{1+x^2} dx$  is
  - (a)  $\frac{2}{15}\sqrt{2} + \frac{2}{15}$
  - (b)  $\frac{\sqrt{2}}{3} \frac{1}{15}$
  - (c)  $\sqrt{2} 1$
  - (d)  $\frac{1}{3} \sqrt{2}$
  - (e)  $\frac{1}{6} \frac{\sqrt{2}}{2}$

- 11. The value of the integral  $\int_0^1 \frac{3x+2}{x^2-4} dx$  is
  - (a)  $\ln 3 3 \ln 2$
  - (b)  $\ln 2 2 \ln 3$
  - (c)  $3 \ln 2 \ln 3$
  - (d)  $2 \ln 3 \ln 2$
  - (e)  $\ln 3 \ln 2$

- 12. The improper integral  $\int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx$  is
  - (a) Convergent and its value is  $e^{-2}$
  - (b) Convergent and its value is  $e^{+2}$
  - (c) Convergent and its value is  $-e^{-2}$
  - (d) Convergent and its value is 0
  - (e) Divergent

- 13. The sequence  $\left\{\frac{\ln n}{n^2}\right\}_{n\geq 3}$  is
  - (a) Decreasing and convergent
  - (b) Decreasing and divergent
  - (c) Increasing and convergent
  - (d) Increasing and divergent
  - (e) Neither increasing, nor decreasing and convergent

- 14. The series  $\sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}}$  is
  - (a) Convergent and its sum is  $\frac{e^3}{2e^2-4}$
  - (b) Convergent and its sum is  $\frac{1}{e^3}$
  - (c) Convergent and its sum is  $\frac{e}{2}$
  - (d) Convergent and its sum is  $\frac{e^2}{e^2-2}$
  - (e) Divergent

- 15. For which values of p, the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$  is convergent
  - (a) p > 1
  - (b) p < 1
  - (c) p > e
  - (d) converges for all p
  - (e) diverges for all p

- 16. The series  $\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n}$  is
  - (a) Conditionally convergent
  - (b) Divergent
  - (c) Absolutely convergent
  - (d) Divergent by Ratio Test
  - (e) Divergent by Comparison Test

- 17. If the sum of the first n terms of a series  $\sum_{n=1}^{+\infty} a_n$  is given by  $S_n = \frac{2n}{n+1}$ , then
  - $(a) \quad a_n = \frac{2}{n^2 + n}$
  - (b)  $a_n = \frac{1}{n^2 + n}$
  - $(c) \quad a_n = \frac{4}{n^2 + n}$
  - (d)  $a_n = \frac{2}{n^2 + 2n}$
  - (e)  $a_n = \frac{2}{n^2 + 1}$

- 18. The minimum number of terms needed to estimate the sum of the series  $\sum_{n=1}^{+\infty} \frac{(-1)^n}{(n+1)^4}$  within 0.0001 (error) is
  - (a) n = 10
  - (b) n = 13
  - (c) n = 12
  - (d) n = 11
  - (e) n = 8

- 19. The series  $\sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n$  is
  - (a) Convergent
  - (b) Divergent by Divergence Test
  - (c) Divergent by Ratio Test
  - (d) Conditionally convergent
  - (e) Divergent by Comparison Test

- 20. The series  $\sum_{n=0}^{+\infty} \frac{2^n \sin\left(n\frac{\pi}{2}\right)}{n!}$  is
  - (a) Convergent by Alternating Series Test
  - (b) Divergent by Ratio Test
  - (c) Divergent by Divergence Test
  - (d) Divergent by Comparison Test
  - (e) Convergent by Limit Comparison Test

- 21. The radius of convergence of the series  $\sum_{n=0}^{+\infty} \frac{(n!)^2 x^n}{(2n)!}$  is
  - (a) 4
  - (b)  $e^2$
  - (c) 0
  - (d) k
  - (e) 1

- 22. The interval of convergence of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n4^n}$  is
  - (a) (-6, 2]
  - (b) (-6,2)
  - (c) [-6, 2)
  - (d) [-6, 2]
  - (e) [-2, 2]

- 23. A power series representing the function  $f(x) = \frac{3x^3}{(x-3)^2}$  is [Hint: You may consider the function:  $\frac{d}{dx} \left(\frac{1}{3-x}\right) = \frac{1}{(3-x)^2}$ ]
  - (a)  $\sum_{n=1}^{+\infty} \frac{nx^{n+2}}{3^n}$
  - (b)  $\sum_{n=1}^{+\infty} \frac{x^{n+3}}{3^n}$
  - (c)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^{n+2}}$
  - (d)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^n}$
  - (e)  $\sum_{n=1}^{+\infty} \frac{x^n}{3^n}$

- 24. The value of the integral  $\int_0^{1/3} \frac{x^2 dx}{1 + x^7}$  is
  - (a)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+3) \cdot 3^{7n+3}}$
  - (b)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+1}}$
  - (c)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}}$
  - (d)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+2) \cdot 3^{7n+2}}$
  - (e)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+2}}$

- 25. For x > 0 (but close to 1), the sum of the series  $\sum_{n=0}^{+\infty} \frac{(-2)^n (\ln x)^n}{n!}$  is
  - (a)  $\frac{1}{x^2}$
  - (b)  $x^2$
  - (c)  $e^x$
  - (d)  $e^{x^2}$
  - (e)  $e^{-2x}$

26. The first four terms of the Taylor series of the function  $f(x) = \ln(3+x)$  at a=1 are:

(a) 
$$\ln 4 + \frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

(b) 
$$\ln 4 - \frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192}$$

(c) 
$$\frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192} - \frac{(x-1)^4}{256}$$

(d) 
$$-\frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192} + \frac{(x-1)^4}{256}$$

(e) 
$$\ln 4 + \frac{(x-1)}{4} + \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

- 27. The length of the curve  $y = \ln(\sec x)$ ,  $0 \le x \le \frac{\pi}{4}$  is
  - (a)  $\ln(1+\sqrt{2})$
  - (b)  $\ln(\sqrt{2})$
  - (c)  $1 + \sqrt{2}$
  - (d)  $\ln(\sqrt{2} + \sqrt{3})$
  - (e)  $\ln(2 + \sqrt{2})$

- 28. The areas of the surface of the solid obtained by rotating the curve  $y = 2\sqrt{x}$ ,  $8 \le x \le 15$  about x-axis is
  - (a)  $\frac{296\pi}{3}$
  - (b)  $\frac{148\pi}{3}$
  - (c)  $\frac{74\pi}{3}$
  - (d)  $\frac{48\pi}{3}$
  - (e)  $\frac{96\pi}{3}$

#### CODE 001

# Math 102 Final Exam Second

#### CODE 001

Saturday, June 11, 2011 Net Time Allowed: 180 minutes

Name:		
ID:	Sec:	 

Check that this exam has 28 questions.

#### **Important Instructions:**

- 1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
- 2. Use HB 2.5 pencils only.
- 3. Use a good eraser. DO NOT use the erasers attached to the pencil.
- 4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
- 5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
- 6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
- 7. When bubbling, make sure that the bubbled space is fully covered.
- 8. When erasing a bubble, make sure that you do not leave any trace of penciling.

- 1. The volume of the solid obtained by rotating the region bounded by the curves  $x^2 y^2 = 1$  and x = 3 about the line x = -2 is given by
  - (a)  $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 (1 + y^2)] dy$
  - (b)  $\int_0^{2\sqrt{2}} \pi [25 (2+y)^2] dy$
  - (c)  $\int_{-3}^{3} \pi [25 (2 + \sqrt{x^2 1})^2] dx$
  - (d)  $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 (2 + \sqrt{1 + y^2})^2] dy$
  - (e)  $\int_{-3}^{3} \pi [9 (x^2 1)] dx$

- 2. The interval of convergence of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n4^n}$  is
  - (a) (-6,2)
  - (b) [-6,2)
  - (c) [-6, 2]
  - (d) [-2, 2]
  - (e) (-6, 2]

- 3. If the sum of the first n terms of a series  $\sum_{n=1}^{+\infty} a_n$  is given by  $S_n = \frac{2n}{n+1}$ , then
  - (a)  $a_n = \frac{2}{n^2 + 2n}$
  - (b)  $a_n = \frac{1}{n^2 + n}$
  - $(c) \quad a_n = \frac{4}{n^2 + n}$
  - (d)  $a_n = \frac{2}{n^2 + 1}$
  - (e)  $a_n = \frac{2}{n^2 + n}$

- 4. The value of the integral  $\int_0^{1/3} \frac{x^2 dx}{1 + x^7}$  is
  - (a)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+1}}$
  - (b)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}}$
  - (c)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+2}}$
  - (d)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+3) \cdot 3^{7n+3}}$
  - (e)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+2) \cdot 3^{7n+2}}$

- 5. If f(x) = f'(x) + 3, f(0) = 1 and f(1) = 4, then the value of the integral  $\int_0^1 e^x f'(x) dx$  is
  - (a)  $\frac{2e-1}{2}$
  - (b)  $\frac{e+1}{2}$
  - (c)  $\frac{e-1}{2}$
  - (d)  $\frac{e+2}{2}$
  - (e)  $\frac{e-2}{2}$

- 6. The areas of the surface of the solid obtained by rotating the curve  $y = 2\sqrt{x}$ ,  $8 \le x \le 15$  about x-axis is
  - (a)  $\frac{74\pi}{3}$
  - (b)  $\frac{148\pi}{3}$
  - (c)  $\frac{96\pi}{3}$
  - (d)  $\frac{48\pi}{3}$
  - (e)  $\frac{296\pi}{3}$

- 7. For which values of p, the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$  is convergent
  - (a) diverges for all p
  - (b) p > e
  - (c) converges for all p
  - (d) p > 1
  - (e) p < 1

- 8. A power series representing the function  $f(x) = \frac{3x^3}{(x-3)^2}$  is [Hint: You may consider the function:  $\frac{d}{dx}\left(\frac{1}{3-x}\right) = \frac{1}{(3-x)^2}$ ]
  - (a)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^{n+2}}$
  - (b)  $\sum_{n=1}^{+\infty} \frac{x^{n+3}}{3^n}$
  - (c)  $\sum_{n=1}^{+\infty} \frac{x^n}{3^n}$
  - (d)  $\sum_{n=1}^{+\infty} \frac{nx^{n+2}}{3^n}$
  - (e)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^n}$

- 9. The sequence  $\left\{\frac{\ln n}{n^2}\right\}_{n>3}$  is
  - (a) Increasing and convergent
  - (b) Decreasing and convergent
  - (c) Decreasing and divergent
  - (d) Neither increasing, nor decreasing and convergent
  - (e) Increasing and divergent

- 10. The series  $\sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}}$  is
  - (a) Divergent
  - (b) Convergent and its sum is  $\frac{e^2}{e^2-2}$
  - (c) Convergent and its sum is  $\frac{e}{2}$
  - (d) Convergent and its sum is  $\frac{1}{e^3}$
  - (e) Convergent and its sum is  $\frac{e^3}{2e^2-4}$

- 11. The volume of the solid obtained by rotating the region bounded by the curves  $y = e^{x^2}$ , y = e and y = 0 (in the first quadrant) about y-axis is
  - (a)  $\pi$
  - (b)  $2\pi$
  - (c)  $\pi e$
  - (d)  $e^2\pi$
  - (e)  $\pi e^2$

- 12. The value of the integral  $\int_0^1 x\sqrt{1-x^4} dx$  is
  - (a)  $\frac{\pi}{3}$
  - (b)  $\frac{\pi}{16}$
  - (c)  $\frac{\pi}{2}$
  - (d)  $\frac{\pi}{8}$
  - (e)  $\frac{\pi}{6}$

- 13. The value of the integral  $\int_{-1}^{0} x^2 \sqrt{1+x} \ dx$  is
  - (a)  $\frac{102}{105}$
  - (b)  $\frac{16}{105}$
  - (c)  $\frac{8}{105}$
  - (d)  $\frac{22}{105}$
  - (e)  $\frac{19}{105}$

- 14. The series  $\sum_{n=0}^{+\infty} \frac{2^n \sin\left(n\frac{\pi}{2}\right)}{n!}$  is
  - (a) Convergent by Alternating Series Test
  - (b) Divergent by Divergence Test
  - (c) Divergent by Ratio Test
  - (d) Convergent by Limit Comparison Test
  - (e) Divergent by Comparison Test

- 15. The area of the region bounded by the curves  $4x + y^2 = 12$  and x = y is
  - (a)  $\frac{64}{3}$
  - (b)  $\frac{44}{3}$
  - (c) 17
  - (d) 21
  - (e)  $\frac{52}{3}$

- 16. The length of the curve  $y = \ln(\sec x)$ ,  $0 \le x \le \frac{\pi}{4}$  is
  - (a)  $\ln(1+\sqrt{2})$
  - (b)  $1 + \sqrt{2}$
  - (c)  $\ln(\sqrt{2})$
  - (d)  $\ln(\sqrt{2} + \sqrt{3})$
  - (e)  $\ln(2 + \sqrt{2})$

- 17. The series  $\sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n$  is
  - (a) Divergent by Ratio Test
  - (b) Conditionally convergent
  - (c) Divergent by Divergence Test
  - (d) Convergent
  - (e) Divergent by Comparison Test

- 18. The value of the integral  $\int_0^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) dx$  is
  - (a) 1/2
  - (b) -e/2
  - (c) -1/2
  - (d) e/2
  - (e)  $-\sqrt{2}/2$

- 19. The improper integral  $\int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx$  is
  - (a) Convergent and its value is  $-e^{-2}$
  - (b) Convergent and its value is  $e^{+2}$
  - (c) Convergent and its value is  $e^{-2}$
  - (d) Divergent
  - (e) Convergent and its value is 0

- 20. The value of the integral  $\int_0^1 \frac{3x+2}{x^2-4} dx$  is
  - (a)  $2 \ln 3 \ln 2$
  - (b)  $\ln 3 \ln 2$
  - (c)  $3 \ln 2 \ln 3$
  - (d)  $\ln 3 3 \ln 2$
  - (e)  $\ln 2 2 \ln 3$

- 21. The minimum number of terms needed to estimate the sum of the series  $\sum_{n=1}^{+\infty} \frac{(-1)^n}{(n+1)^4}$  within 0.0001 (error) is
  - (a) n = 10
  - (b) n = 8
  - (c) n = 13
  - (d) n = 12
  - (e) n = 11

- 22. If  $F(x) = \int_x^{x^2} \frac{dt}{\sqrt{1-t^2}}$ , then  $F'\left(\frac{1}{2}\right)$  is
  - (a)  $\frac{4+2\sqrt{5}}{\sqrt{15}}$
  - (b)  $\frac{4}{\sqrt{15}} \frac{1}{\sqrt{3}}$
  - (c)  $\frac{2+2\sqrt{5}}{\sqrt{15}}$
  - (d)  $\frac{4-2\sqrt{5}}{\sqrt{15}}$
  - (e)  $\frac{2 2\sqrt{5}}{\sqrt{15}}$

- 23. The radius of convergence of the series  $\sum_{n=0}^{+\infty} \frac{(n!)^2 x^n}{(2n)!}$  is
  - (a)  $e^2$
  - (b) k
  - (c) 1
  - (d) 0
  - (e) 4

- 24. The series  $\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n}$  is
  - (a) Absolutely convergent
  - (b) Divergent
  - (c) Divergent by Comparison Test
  - (d) Conditionally convergent
  - (e) Divergent by Ratio Test

- 25. The value of the integral  $\int_0^1 x^3 \sqrt{1+x^2} dx$  is
  - (a)  $\frac{2}{15}\sqrt{2} + \frac{2}{15}$
  - (b)  $\frac{1}{6} \frac{\sqrt{2}}{2}$
  - (c)  $\sqrt{2} 1$
  - (d)  $\frac{1}{3} \sqrt{2}$
  - (e)  $\frac{\sqrt{2}}{3} \frac{1}{15}$

- 26. For x > 0 (but close to 1), the sum of the series  $\sum_{n=0}^{+\infty} \frac{(-2)^n (\ln x)^n}{n!}$  is
  - (a)  $e^x$
  - (b)  $e^{x^2}$
  - (c)  $\frac{1}{x^2}$
  - (d)  $e^{-2x}$
  - (e)  $x^2$

- 27. The value of the integral  $\int_0^{\pi/4} \tan^4 x \sec^4 x \ dx$  is
  - (a)  $\frac{12}{35}$
  - (b)  $\frac{2}{35}$
  - (c)  $-\frac{2}{35}$
  - (d)  $\frac{5}{12}$
  - (e)  $\frac{1}{5}$

28. The first four terms of the Taylor series of the function  $f(x) = \ln(3+x)$  at a = 1 are:

(a) 
$$\ln 4 - \frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192}$$

(b) 
$$\ln 4 + \frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

(c) 
$$\frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192} - \frac{(x-1)^4}{256}$$

(d) 
$$\ln 4 + \frac{(x-1)}{4} + \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

(e) 
$$-\frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192} + \frac{(x-1)^4}{256}$$

Name		
ID	 Sec	

1	a	b	c	d	е	f
2	a	b	c	d	е	f
3	a	b	$\mathbf{c}$	d	е	f
4	a	b	$\mathbf{c}$	d	е	f
5	a	b	c	d	е	f
6	a	b	c	d	е	f
7	a	b	c	d	e	f
8	a	b	c	d	e	f
9	a	b	c	d	e	f
10	a	b	c	d	e	f
11	a	b	$\mathbf{c}$	d	е	f
12	a	b	$\mathbf{c}$	d	е	f
13	a	b	$\mathbf{c}$	d	е	f
14	a	b	c	d	е	f
15	a	b	c	d	е	f
16	a	b	С	d	е	f
17	a	b	c	d	е	f
18	a	b	С	d	е	f
19	a	b	c	d	е	f
20	a	b	c	d	е	f
21	a	b	c	d	е	f
22	a	b	c	d	е	f
23	a	b	С	d	е	f
24	a	b	c	d	е	f
25	a	b	c	d	е	f
26	a	b	$\mathbf{c}$	d	е	f
27	a	b	c	d	е	f
28	a	b	С	d	е	f
29	a	b	c	d	е	f
30	a	b	c	d	e	f
31	a	b	c	d	е	f
32	a	b	С	d	е	f
33	a	b	c	d	е	f
34	a	b	c	d	е	f
35	a	b	$\mathbf{c}$	d	е	f

36	a	b	c	d	е	f
37	a	b	c	d	е	f
38	a	b	c	d	е	f
39	a	b	С	d	е	f
40	a	b	c	d	е	f
41	a	b	c	d	е	f
42	a	b	С	d	е	f
43	a	b	c	d	е	f
44	a	b	c	d	е	f
45	a	b	c	d	е	f
46	a	b	С	d	е	f
47	a	b	С	d	е	f
48	a	b	С	d	е	f
49	a	b	c	d	е	f
50	a	b	С	d	е	f
51	a	b	С	d	е	f
52	a	b	С	d	е	f
53	a	b	c	d	е	f
54	a	b	c	d	е	f
55	a	b	c	d	е	f
56	a	b	c	d	е	f
57	a	b	С	d	е	f
58	a	b	С	d	е	f
59	a	b	С	d	е	f
60	a	b	c	d	е	f
61	a	b	С	d	е	f
62	a	b	$^{\mathrm{c}}$	d	е	f
63	a	b	c	d	е	f
64	a	b	С	d	е	f
65	a	b	c	d	е	f
66	a	b	С	d	е	f
67	a	b	c	d	е	f
68	a	b	С	d d	е	f
69 70	a	b	$^{\mathrm{c}}$	d	е	f
70	a	b	С	d	е	f

#### CODE 002

# Math 102 Final Exam Second

#### CODE 002

Saturday, June 11, 2011 Net Time Allowed: 180 minutes

Name:		
ID:	Sec:	 

Check that this exam has 28 questions.

#### **Important Instructions:**

- 1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
- 2. Use HB 2.5 pencils only.
- 3. Use a good eraser. DO NOT use the erasers attached to the pencil.
- 4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
- 5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
- 6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
- 7. When bubbling, make sure that the bubbled space is fully covered.
- 8. When erasing a bubble, make sure that you do not leave any trace of penciling.

- 1. The interval of convergence of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n4^n}$  is
  - (a) (-6,2)
  - (b) [-6, 2]
  - (c) [-2, 2]
  - (d) [-6,2)
  - (e) (-6, 2]

- 2. The value of the integral  $\int_0^1 \frac{3x+2}{x^2-4} dx$  is
  - (a)  $3 \ln 2 \ln 3$
  - (b)  $2 \ln 3 \ln 2$
  - (c)  $\ln 3 3 \ln 2$
  - (d)  $\ln 2 2 \ln 3$
  - (e)  $\ln 3 \ln 2$

- 3. The series  $\sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}}$  is
  - (a) Convergent and its sum is  $\frac{e^2}{e^2-2}$
  - (b) Divergent
  - (c) Convergent and its sum is  $\frac{e}{2}$
  - (d) Convergent and its sum is  $\frac{1}{e^3}$
  - (e) Convergent and its sum is  $\frac{e^3}{2e^2-4}$

- 4. The value of the integral  $\int_{-1}^{0} x^2 \sqrt{1+x} \ dx$  is
  - (a)  $\frac{22}{105}$
  - (b)  $\frac{8}{105}$
  - (c)  $\frac{19}{105}$
  - (d)  $\frac{16}{105}$
  - (e)  $\frac{102}{105}$

- 5. The series  $\sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n$  is
  - (a) Convergent
  - (b) Divergent by Ratio Test
  - (c) Divergent by Divergence Test
  - (d) Conditionally convergent
  - (e) Divergent by Comparison Test

- 6. The sequence  $\left\{\frac{\ln n}{n^2}\right\}_{n\geq 3}$  is
  - (a) Neither increasing, nor decreasing and convergent
  - (b) Increasing and divergent
  - (c) Increasing and convergent
  - (d) Decreasing and divergent
  - (e) Decreasing and convergent

- 7. The value of the integral  $\int_0^1 x \sqrt{1-x^4} \, dx$  is
  - (a)  $\frac{\pi}{2}$
  - (b)  $\frac{\pi}{3}$
  - (c)  $\frac{\pi}{16}$
  - (d)  $\frac{\pi}{8}$
  - (e)  $\frac{\pi}{6}$

- 8. The volume of the solid obtained by rotating the region bounded by the curves  $y = e^{x^2}$ , y = e and y = 0 (in the first quadrant) about y-axis is
  - (a)  $\pi e^2$
  - (b)  $\pi$
  - (c)  $2\pi$
  - (d)  $\pi e$
  - (e)  $e^2\pi$

9. If 
$$F(x) = \int_x^{x^2} \frac{dt}{\sqrt{1-t^2}}$$
, then  $F'\left(\frac{1}{2}\right)$  is

- (a)  $\frac{4}{\sqrt{15}} \frac{1}{\sqrt{3}}$
- (b)  $\frac{4-2\sqrt{5}}{\sqrt{15}}$
- (c)  $\frac{2-2\sqrt{5}}{\sqrt{15}}$
- (d)  $\frac{2+2\sqrt{5}}{\sqrt{15}}$
- (e)  $\frac{4+2\sqrt{5}}{\sqrt{15}}$

- 10. If f(x) = f'(x) + 3, f(0) = 1 and f(1) = 4, then the value of the integral  $\int_0^1 e^x f'(x) dx$  is
  - (a)  $\frac{e+1}{2}$
  - (b)  $\frac{e-2}{2}$
  - (c)  $\frac{2e-1}{2}$
  - (d)  $\frac{e+2}{2}$
  - (e)  $\frac{e-1}{2}$

- 11. The series  $\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n}$  is
  - (a) Divergent
  - (b) Divergent by Comparison Test
  - (c) Absolutely convergent
  - (d) Divergent by Ratio Test
  - (e) Conditionally convergent

- 12. A power series representing the function  $f(x) = \frac{3x^3}{(x-3)^2}$  is [Hint: You may consider the function:  $\frac{d}{dx} \left( \frac{1}{3-x} \right) = \frac{1}{(3-x)^2}$ ]
  - (a)  $\sum_{n=1}^{+\infty} \frac{x^{n+3}}{3^n}$
  - (b)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^{n+2}}$
  - (c)  $\sum_{n=1}^{+\infty} \frac{x^n}{3^n}$
  - (d)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^n}$
  - (e)  $\sum_{n=1}^{+\infty} \frac{nx^{n+2}}{3^n}$

- 13. The area of the region bounded by the curves  $4x + y^2 = 12$  and x = y is
  - (a)  $\frac{52}{3}$
  - (b)  $\frac{64}{3}$
  - (c) 21
  - (d)  $\frac{44}{3}$
  - (e) 17

- 14. For which values of p, the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$  is convergent
  - (a) p > e
  - (b) p > 1
  - (c) converges for all p
  - (d) p < 1
  - (e) diverges for all p

- 15. The length of the curve  $y = \ln(\sec x)$ ,  $0 \le x \le \frac{\pi}{4}$  is
  - (a)  $\ln(\sqrt{2} + \sqrt{3})$
  - (b)  $\ln(2+\sqrt{2})$
  - (c)  $\ln(\sqrt{2})$
  - (d)  $1 + \sqrt{2}$
  - (e)  $\ln(1+\sqrt{2})$

16. The first four terms of the Taylor series of the function  $f(x) = \ln(3+x)$  at a=1 are:

(a) 
$$\ln 4 + \frac{(x-1)}{4} + \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

(b) 
$$\ln 4 - \frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192}$$

(c) 
$$-\frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192} + \frac{(x-1)^4}{256}$$

(d) 
$$\ln 4 + \frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

(e) 
$$\frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192} - \frac{(x-1)^4}{256}$$

17. The value of the integral  $\int_0^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) dx$  is

- (a) 1/2
- (b)  $-\sqrt{2}/2$
- (c) -e/2
- (d) -1/2
- (e) e/2

18. The value of the integral  $\int_0^1 x^3 \sqrt{1+x^2} dx$  is

- (a)  $\frac{2}{15}\sqrt{2} + \frac{2}{15}$
- (b)  $\frac{\sqrt{2}}{3} \frac{1}{15}$
- (c)  $\frac{1}{3} \sqrt{2}$
- (d)  $\frac{1}{6} \frac{\sqrt{2}}{2}$
- (e)  $\sqrt{2} 1$

19. The value of the integral  $\int_0^{1/3} \frac{x^2 dx}{1 + x^7}$  is

(a) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+2) \cdot 3^{7n+2}}$$

(b) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+2}}$$

(c) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+3) \cdot 3^{7n+3}}$$

(d) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}}$$

(e) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+1}}$$

- 20. The improper integral  $\int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx$  is
  - (a) Convergent and its value is  $e^{+2}$
  - (b) Convergent and its value is  $-e^{-2}$
  - (c) Convergent and its value is 0
  - (d) Convergent and its value is  $e^{-2}$
  - (e) Divergent

- 21. The series  $\sum_{n=0}^{+\infty} \frac{2^n \sin\left(n\frac{\pi}{2}\right)}{n!}$  is
  - (a) Convergent by Alternating Series Test
  - (b) Divergent by Divergence Test
  - (c) Convergent by Limit Comparison Test
  - (d) Divergent by Ratio Test
  - (e) Divergent by Comparison Test

- 22. The radius of convergence of the series  $\sum_{n=0}^{+\infty} \frac{(n!)^2 x^n}{(2n)!}$  is
  - (a) 4
  - (b) 1
  - (c) 0
  - (d) k
  - (e)  $e^2$

- 23. The minimum number of terms needed to estimate the sum of the series  $\sum_{n=1}^{+\infty} \frac{(-1)^n}{(n+1)^4}$  within 0.0001 (error) is
  - (a) n = 11
  - (b) n = 13
  - (c) n = 10
  - (d) n = 12
  - (e) n = 8

- 24. The areas of the surface of the solid obtained by rotating the curve  $y = 2\sqrt{x}$ ,  $8 \le x \le 15$  about x-axis is
  - (a)  $\frac{74\pi}{3}$
  - (b)  $\frac{148\pi}{3}$
  - (c)  $\frac{96\pi}{3}$
  - (d)  $\frac{48\pi}{3}$
  - (e)  $\frac{296\pi}{3}$

- 25. For x > 0 (but close to 1), the sum of the series  $\sum_{n=0}^{+\infty} \frac{(-2)^n (\ln x)^n}{n!}$  is
  - (a)  $x^2$
  - (b)  $\frac{1}{x^2}$
  - (c)  $e^x$
  - (d)  $e^{x^2}$
  - (e)  $e^{-2x}$

- 26. The value of the integral  $\int_0^{\pi/4} \tan^4 x \sec^4 x \ dx$  is
  - (a)  $\frac{1}{5}$
  - (b)  $\frac{5}{12}$
  - (c)  $-\frac{2}{35}$
  - (d)  $\frac{2}{35}$
  - (e)  $\frac{12}{35}$

- 27. The volume of the solid obtained by rotating the region bounded by the curves  $x^2 y^2 = 1$  and x = 3 about the line x = -2 is given by
  - (a)  $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 (1 + y^2)] dy$
  - (b)  $\int_0^{2\sqrt{2}} \pi [25 (2+y)^2] dy$
  - (c)  $\int_{-3}^{3} \pi [9 (x^2 1)] dx$
  - (d)  $\int_{-3}^{3} \pi [25 (2 + \sqrt{x^2 1})^2] dx$
  - (e)  $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 (2 + \sqrt{1 + y^2})^2] dy$

- 28. If the sum of the first n terms of a series  $\sum_{n=1}^{+\infty} a_n$  is given by  $S_n = \frac{2n}{n+1}$ , then
  - $(a) \quad a_n = \frac{2}{n^2 + 2n}$
  - (b)  $a_n = \frac{2}{n^2 + 1}$
  - (c)  $a_n = \frac{4}{n^2 + n}$
  - $(d) \quad a_n = \frac{1}{n^2 + n}$
  - $(e) \quad a_n = \frac{2}{n^2 + n}$

2 3 4 5 6 7 8 9 10	aa aa aa aa aa aa	b b b b b b b b b b b b b b b b	C C C C C C C C C C C C C C C C C C C	d d d d d d d d d d d	e e e e e e e e e e e e e e e e e e e	f f f f f f f
4 5 6 7 8 9 10 11	a a a a a a a a a a a a a a a a a a a	b b b b b b b b b b b b	C C C C C C C	d d d d d d d d	e e e e e e e	f f f f
5 6 7 8 9 10	a a a a a a a a a a	b b b b b b b b b	c c c c c c	d d d d d d d d	e e e e e e	f f f f
6 7 8 9 10	a a a a a a a a	b b b b b b b b	c c c c c c	d d d d d d d	e e e e e	f f f f
7 8 9 10 11	a a a a a a	b b b b b b b	c c c c c c	d d d d d	e e e e e	f f f
8 9 10 11	a a a a a	b b b c b b b	c c c	d d d	e e e	f f
9 10 11	a a a a	b b b	c c	d d d	e e e	f f
10 11	a a a	b b b	c c	d d	e e	f
11	a a a	b b	c	d	е	
	a a	b				f
12	a		С	_1		
		L		d	е	f
	~		c	d	e	f
	a	b	c	d	e	f
	a	b	c	d	e	f
	a	b	c	d	е	f
	a	b	c	d	e	f
	a	b	c	d	e	f
	a	b	c	d	е	f
	a	b	c	d	e	f
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	a	b	c	d	e	f
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	a	b	c	d	е	f
	a	b	c	d	e	f
	a	b	c	d	e	f
	a	b	c	d	e	f
	a	b	c	d	e	f
	a	b	c	d	e	f
	a	b	c	d	e	f
	a	b	$\mathbf{c}$	d	e	f
	a	b	c	d	е	f
	a	b	c	d	е	f
35	a	b	c	d	е	f

36	a	b	c	d	е	f
37	a	b	c	d	е	f
38	a	b	c	d	е	f
39	a	b	$^{\mathrm{c}}$	d	е	f
40	a	b	c	d	e	f
41	a	b	c	d	е	f
42	a	b	$\mathbf{c}$	d	e	f
43	a	b	$\mathbf{c}$	d	e	f
44	a	b	c	d	е	f
45	a	b	$\mathbf{c}$	d	e	f
46	a	b	c	d	е	f
47	a	b	c	d	е	f
48	a	b	c	d	е	f
49	a	b	c	d	е	f
50	a	b	c	d	е	f
51	a	b	С	d	е	f
52	a	b	С	d	е	f
53	a	b	С	d	е	f
54	a	b	С	d	е	f
55	a	b	С	d	е	f
56	a	b	c	d	е	f
57	a	b	С	d	е	f
58	a	b	С	d	е	f
59	a	b	c	d	е	f
60	a	b	c	d	е	f
61	a	b	c	d	е	f
62	a	b	c	d	е	f
63	a	b	c	d	е	f
64	a	b	c	d	е	f
65	a	b	c	d	е	f
66	a	b	c	d	е	f
67	a	b	c	d	е	f
68	a	b	c	d	е	f
69	a	b	c	d	е	f
70	a	b	С	d	е	f

## King Fahd University of Petroleum & Minerals Department of Mathematics and Statistics

#### CODE 003

## Math 102 Final Exam Second

#### CODE 003

Saturday, June 11, 2011 Net Time Allowed: 180 minutes

Name:			
ID:	Sec	:	

Check that this exam has 28 questions.

#### **Important Instructions:**

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- 6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
- 7. When bubbling, make sure that the bubbled space is fully covered.
- 8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The value of the integral  $\int_0^{1/3} \frac{x^2 dx}{1+x^7}$  is

(a) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+3) \cdot 3^{7n+3}}$$

(b) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+1}}$$

(c) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+2}}$$

(d) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+2) \cdot 3^{7n+2}}$$

(e) 
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}}$$

2. The length of the curve  $y = \ln(\sec x)$ ,  $0 \le x \le \frac{\pi}{4}$  is

(a) 
$$\ln(\sqrt{2} + \sqrt{3})$$

(b) 
$$\ln(\sqrt{2})$$

(c) 
$$\ln(1+\sqrt{2})$$

(d) 
$$1 + \sqrt{2}$$

(e) 
$$\ln(2 + \sqrt{2})$$

- 3. The first four terms of the Taylor series of the function  $f(x) = \ln(3+x)$  at a=1 are:
  - (a)  $\ln 4 + \frac{(x-1)}{4} + \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$
  - (b)  $\ln 4 \frac{(x-1)}{4} + \frac{(x-1)^2}{32} \frac{(x-1)^3}{192}$
  - (c)  $\ln 4 + \frac{(x-1)}{4} \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$
  - (d)  $\frac{(x-1)}{4} \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192} \frac{(x-1)^4}{256}$
  - (e)  $-\frac{(x-1)}{4} + \frac{(x-1)^2}{32} \frac{(x-1)^3}{192} + \frac{(x-1)^4}{256}$

- 4. The value of the integral  $\int_{-1}^{0} x^2 \sqrt{1+x} \ dx$  is
  - (a)  $\frac{19}{105}$
  - (b)  $\frac{102}{105}$
  - (c)  $\frac{22}{105}$
  - (d)  $\frac{16}{105}$
  - (e)  $\frac{8}{105}$

- 5. For which values of p, the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$  is convergent
  - (a) p > 1
  - (b) p < 1
  - (c) p > e
  - (d) converges for all p
  - (e) diverges for all p

- 6. The minimum number of terms needed to estimate the sum of the series  $\sum_{n=1}^{+\infty} \frac{(-1)^n}{(n+1)^4}$  within 0.0001 (error) is
  - (a) n = 10
  - (b) n = 8
  - (c) n = 13
  - (d) n = 12
  - (e) n = 11

- 7. The radius of convergence of the series  $\sum_{n=0}^{+\infty} \frac{(n!)^2 x^n}{(2n)!}$  is
  - (a) 1
  - (b) 0
  - (c) 4
  - (d)  $e^2$
  - (e) k

- 8. The area of the region bounded by the curves  $4x + y^2 = 12$  and x = y is
  - (a)  $\frac{52}{3}$
  - (b)  $\frac{44}{3}$
  - (c) 17
  - (d)  $\frac{64}{3}$
  - (e) 21

- 9. A power series representing the function  $f(x) = \frac{3x^3}{(x-3)^2}$  is [Hint: You may consider the function:  $\frac{d}{dx} \left( \frac{1}{3-x} \right) = \frac{1}{(3-x)^2}$ ]
  - (a)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^{n+2}}$
  - (b)  $\sum_{n=1}^{+\infty} \frac{x^{n+3}}{3^n}$
  - (c)  $\sum_{n=1}^{+\infty} \frac{nx^{n+2}}{3^n}$
  - (d)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^n}$
  - (e)  $\sum_{n=1}^{+\infty} \frac{x^n}{3^n}$

- 10. The sequence  $\left\{\frac{\ln n}{n^2}\right\}_{n\geq 3}$  is
  - (a) Decreasing and convergent
  - (b) Increasing and divergent
  - (c) Decreasing and divergent
  - (d) Increasing and convergent
  - (e) Neither increasing, nor decreasing and convergent

- 11. The value of the integral  $\int_0^1 \frac{3x+2}{x^2-4} dx$  is
  - (a)  $\ln 3 \ln 2$
  - (b)  $\ln 3 3 \ln 2$
  - (c)  $\ln 2 2 \ln 3$
  - (d)  $3 \ln 2 \ln 3$
  - (e)  $2 \ln 3 \ln 2$

- 12. The value of the integral  $\int_0^1 x^3 \sqrt{1+x^2} dx$  is
  - (a)  $\sqrt{2} 1$
  - (b)  $\frac{\sqrt{2}}{3} \frac{1}{15}$
  - (c)  $\frac{1}{3} \sqrt{2}$
  - (d)  $\frac{2}{15}\sqrt{2} + \frac{2}{15}$
  - (e)  $\frac{1}{6} \frac{\sqrt{2}}{2}$

- 13. If f(x) = f'(x) + 3, f(0) = 1 and f(1) = 4, then the value of the integral  $\int_0^1 e^x f'(x) dx$  is
  - (a)  $\frac{e+1}{2}$
  - (b)  $\frac{e-2}{2}$
  - (c)  $\frac{2e-1}{2}$
  - (d)  $\frac{e-1}{2}$
  - (e)  $\frac{e+2}{2}$

- 14. The areas of the surface of the solid obtained by rotating the curve  $y=2\sqrt{x}, \quad 8 \leq x \leq 15$  about x-axis is
  - (a)  $\frac{296\pi}{3}$
  - (b)  $\frac{48\pi}{3}$
  - (c)  $\frac{148\pi}{3}$
  - (d)  $\frac{96\pi}{3}$
  - (e)  $\frac{74\pi}{3}$

- 15. The interval of convergence of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n4^n}$  is
  - (a) [-6, 2]
  - (b) [-2, 2]
  - (c) (-6,2)
  - (d) (-6, 2]
  - (e) [-6, 2)

- 16. The value of the integral  $\int_0^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) dx$  is
  - (a) e/2
  - (b) -1/2
  - (c) 1/2
  - (d)  $-\sqrt{2}/2$
  - (e) -e/2

- 17. The series  $\sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}}$  is
  - (a) Divergent
  - (b) Convergent and its sum is  $\frac{e}{2}$
  - (c) Convergent and its sum is  $\frac{e^2}{e^2-2}$
  - (d) Convergent and its sum is  $\frac{e^3}{2e^2-4}$
  - (e) Convergent and its sum is  $\frac{1}{e^3}$

- 18. The volume of the solid obtained by rotating the region bounded by the curves  $y = e^{x^2}$ , y = e and y = 0 (in the first quadrant) about y-axis is
  - (a)  $e^2\pi$
  - (b)  $\pi e$
  - (c)  $\pi e^2$
  - (d)  $\pi$
  - (e)  $2\pi$

- 19. The series  $\sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n$  is
  - (a) Conditionally convergent
  - (b) Divergent by Divergence Test
  - (c) Divergent by Comparison Test
  - (d) Divergent by Ratio Test
  - (e) Convergent

20. If the sum of the first n terms of a series  $\sum_{n=1}^{+\infty} a_n$  is given by  $S_n = \frac{2n}{n+1}$ , then

(a) 
$$a_n = \frac{2}{n^2 + 2n}$$

(b) 
$$a_n = \frac{1}{n^2 + n}$$

(c) 
$$a_n = \frac{2}{n^2 + n}$$

(d) 
$$a_n = \frac{2}{n^2 + 1}$$

(e) 
$$a_n = \frac{4}{n^2 + n}$$

- 21. The value of the integral  $\int_0^1 x\sqrt{1-x^4} dx$  is
  - (a)  $\frac{\pi}{16}$
  - (b)  $\frac{\pi}{2}$
  - (c)  $\frac{\pi}{3}$
  - (d)  $\frac{\pi}{6}$
  - (e)  $\frac{\pi}{8}$

- 22. The improper integral  $\int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx$  is
  - (a) Convergent and its value is  $e^{+2}$
  - (b) Divergent
  - (c) Convergent and its value is  $e^{-2}$
  - (d) Convergent and its value is  $-e^{-2}$
  - (e) Convergent and its value is 0

- 23. For x > 0 (but close to 1), the sum of the series  $\sum_{n=0}^{+\infty} \frac{(-2)^n (\ln x)^n}{n!}$  is
  - (a)  $e^{x^2}$
  - (b)  $e^x$
  - (c)  $\frac{1}{x^2}$
  - (d)  $e^{-2x}$
  - (e)  $x^2$

- 24. If  $F(x) = \int_x^{x^2} \frac{dt}{\sqrt{1-t^2}}$ , then  $F'\left(\frac{1}{2}\right)$  is
  - (a)  $\frac{2+2\sqrt{5}}{\sqrt{15}}$
  - (b)  $\frac{4}{\sqrt{15}} \frac{1}{\sqrt{3}}$
  - (c)  $\frac{4-2\sqrt{5}}{\sqrt{15}}$
  - (d)  $\frac{2 2\sqrt{5}}{\sqrt{15}}$
  - (e)  $\frac{4+2\sqrt{5}}{\sqrt{15}}$

25. The volume of the solid obtained by rotating the region bounded by the curves  $x^2 - y^2 = 1$  and x = 3 about the line x = -2 is given by

(a) 
$$\int_0^{2\sqrt{2}} \pi [25 - (2+y)^2] dy$$

(b) 
$$\int_{-3}^{3} \pi [25 - (2 + \sqrt{x^2 - 1})^2] dx$$

(c) 
$$\int_{-3}^{3} \pi [9 - (x^2 - 1)] dx$$

(d) 
$$\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 - (2 + \sqrt{1 + y^2})^2] dy$$

(e) 
$$\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 - (1 + y^2)] dy$$

- 26. The series  $\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n}$  is
  - (a) Divergent by Ratio Test
  - (b) Divergent
  - (c) Divergent by Comparison Test
  - (d) Conditionally convergent
  - (e) Absolutely convergent

- 27. The value of the integral  $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$  is
  - (a)  $\frac{5}{12}$
  - (b)  $\frac{12}{35}$
  - (c)  $\frac{1}{5}$
  - (d)  $\frac{2}{35}$
  - (e)  $-\frac{2}{35}$

- 28. The series  $\sum_{n=0}^{+\infty} \frac{2^n \sin\left(n\frac{\pi}{2}\right)}{n!}$  is
  - (a) Divergent by Ratio Test
  - (b) Convergent by Alternating Series Test
  - (c) Convergent by Limit Comparison Test
  - (d) Divergent by Comparison Test
  - (e) Divergent by Divergence Test

Name		
ID	 Sec	

1	a	b	c	d	е	f
2	a	b	c	d	е	f
3	a	b	$\mathbf{c}$	d	е	f
4	a	b	$\mathbf{c}$	d	е	f
5	a	b	c	d	е	f
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13	a	b	$\mathbf{c}$	d	е	f
14	a	b	c	d	е	f
15	a	b	c	d	е	f
16	a	b	С	d	е	f
17	a	b	c	d	е	f
18	a	b	С	d	е	f
19	a	b	$^{\mathrm{c}}$	d	e	f
20	a	b	c	d	е	f
21	a	b	С	d	е	f
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25	a	b	c	d	е	f
26	a	b	$\mathbf{c}$	d	е	f
27	a	b	c	d	е	f
28	a	b	С	d	е	f
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35	a	b	c	d	е	f

36	a	b	c	d	е	f
37	a	b	c	d	е	f
38	a	b	c	d	е	f
39	a	b	$^{\mathrm{c}}$	d	е	f
40	a	b	c	d	e	f
41	a	b	c	d	е	f
42	a	b	$\mathbf{c}$	d	e	f
43	a	b	$\mathbf{c}$	d	e	f
44	a	b	c	d	е	f
45	a	b	$\mathbf{c}$	d	e	f
46	a	b	c	d	е	f
47	a	b	c	d	е	f
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49	a	b	c	d	е	f
50	a	b	c	d	е	f
51	a	b	С	d	е	f
52	a	b	С	d	е	f
53	a	b	С	d	е	f
54	a	b	С	d	е	f
55	a	b	С	d	е	f
56	a	b	c	d	е	f
57	a	b	С	d	е	f
58	a	b	С	d	е	f
59	a	b	c	d	е	f
60	a	b	c	d	е	f
61	a	b	С	d	е	f
62	a	b	c	d	е	f
63	a	b	c	d	е	f
64	a	b	c	d	е	f
65	a	b	c	d	е	f
66	a	b	c	d	е	f
67	a	b	c	d	е	f
68	a	b	c	d	е	f
69	a	b	c	d	е	f
70	a	b	С	d	е	f

## King Fahd University of Petroleum & Minerals Department of Mathematics and Statistics

#### CODE 004

# Math 102 Final Exam Second

#### CODE 004

Saturday, June 11, 2011 Net Time Allowed: 180 minutes

Name:		
ID:	Sec:	 

Check that this exam has 28 questions.

#### **Important Instructions:**

- 1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
- 2. Use HB 2.5 pencils only.
- 3. Use a good eraser. DO NOT use the erasers attached to the pencil.
- 4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
- 5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
- 6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
- 7. When bubbling, make sure that the bubbled space is fully covered.
- 8. When erasing a bubble, make sure that you do not leave any trace of penciling.

- 1. The volume of the solid obtained by rotating the region bounded by the curves  $x^2 y^2 = 1$  and x = 3 about the line x = -2 is given by
  - (a)  $\int_{-3}^{3} \pi [25 (2 + \sqrt{x^2 1})^2] dx$
  - (b)  $\int_{-3}^{3} \pi [9 (x^2 1)] dx$
  - (c)  $\int_0^{2\sqrt{2}} \pi [25 (2+y)^2] dy$
  - (d)  $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 (2 + \sqrt{1 + y^2})^2] dy$
  - (e)  $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 (1 + y^2)] dy$

- 2. The value of the integral  $\int_0^1 x^3 \sqrt{1+x^2} dx$  is
  - (a)  $\frac{\sqrt{2}}{3} \frac{1}{15}$
  - (b)  $\sqrt{2} 1$
  - (c)  $\frac{1}{6} \frac{\sqrt{2}}{2}$
  - (d)  $\frac{1}{3} \sqrt{2}$
  - (e)  $\frac{2}{15}\sqrt{2} + \frac{2}{15}$

- 3. The areas of the surface of the solid obtained by rotating the curve  $y=2\sqrt{x}, \quad 8 \leq x \leq 15$  about x-axis is
  - (a)  $\frac{48\pi}{3}$
  - (b)  $\frac{296\pi}{3}$
  - (c)  $\frac{74\pi}{3}$
  - (d)  $\frac{148\pi}{3}$
  - (e)  $\frac{96\pi}{3}$

- 4. The radius of convergence of the series  $\sum_{n=0}^{+\infty} \frac{(n!)^2 x^n}{(2n)!}$  is
  - (a)  $e^2$
  - (b) 4
  - (c) k
  - (d) 0
  - (e) 1

- 5. The volume of the solid obtained by rotating the region bounded by the curves  $y=e^{x^2}$ , y=e and y=0 (in the first quadrant) about y-axis is
  - (a)  $\pi e$
  - (b)  $2\pi$
  - (c)  $\pi$
  - (d)  $\pi e^2$
  - (e)  $e^2\pi$

- 6. For which values of p, the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$  is convergent
  - (a) p > e
  - (b) diverges for all p
  - (c) p < 1
  - (d) converges for all p
  - (e) p > 1

- 7. For x > 0 (but close to 1), the sum of the series  $\sum_{n=0}^{+\infty} \frac{(-2)^n (\ln x)^n}{n!}$  is
  - (a)  $x^2$
  - (b)  $\frac{1}{x^2}$
  - (c)  $e^{x^2}$
  - (d)  $e^{x}$
  - (e)  $e^{-2x}$

- 8. The sequence  $\left\{\frac{\ln n}{n^2}\right\}_{n\geq 3}$  is
  - (a) Increasing and divergent
  - (b) Decreasing and divergent
  - (c) Decreasing and convergent
  - (d) Increasing and convergent
  - (e) Neither increasing, nor decreasing and convergent

- 9. The series  $\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n}$  is
  - (a) Divergent by Ratio Test
  - (b) Divergent by Comparison Test
  - (c) Divergent
  - (d) Conditionally convergent
  - (e) Absolutely convergent

- 10. The series  $\sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}}$  is
  - (a) Divergent
  - (b) Convergent and its sum is  $\frac{e^3}{2e^2-4}$
  - (c) Convergent and its sum is  $\frac{1}{e^3}$
  - (d) Convergent and its sum is  $\frac{e}{2}$
  - (e) Convergent and its sum is  $\frac{e^2}{e^2-2}$

- 11. The length of the curve  $y = \ln(\sec x)$ ,  $0 \le x \le \frac{\pi}{4}$  is
  - (a)  $1 + \sqrt{2}$
  - (b)  $\ln(\sqrt{2})$
  - (c)  $\ln(2 + \sqrt{2})$
  - (d)  $\ln(1+\sqrt{2})$
  - (e)  $\ln(\sqrt{2} + \sqrt{3})$

- 12. The value of the integral  $\int_{-1}^{0} x^2 \sqrt{1+x} \, dx$  is
  - (a)  $\frac{16}{105}$
  - (b)  $\frac{102}{105}$
  - (c)  $\frac{19}{105}$
  - (d)  $\frac{22}{105}$
  - (e)  $\frac{8}{105}$

- 13. The minimum number of terms needed to estimate the sum of the series  $\sum_{n=1}^{+\infty} \frac{(-1)^n}{(n+1)^4}$  within 0.0001 (error) is
  - (a) n = 8
  - (b) n = 10
  - (c) n = 11
  - (d) n = 13
  - (e) n = 12

- 14. The value of the integral  $\int_0^1 \frac{3x+2}{x^2-4} dx$  is
  - (a)  $3 \ln 2 \ln 3$
  - (b)  $\ln 3 3 \ln 2$
  - (c)  $\ln 3 \ln 2$
  - (d)  $2 \ln 3 \ln 2$
  - (e)  $\ln 2 2 \ln 3$

- 15. The interval of convergence of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n4^n}$  is
  - (a) [-6, 2]
  - (b) (-6, 2]
  - (c) [-2, 2]
  - (d) [-6, 2)
  - (e) (-6,2)

- 16. If  $F(x) = \int_x^{x^2} \frac{dt}{\sqrt{1-t^2}}$ , then  $F'\left(\frac{1}{2}\right)$  is
  - (a)  $\frac{4}{\sqrt{15}} \frac{1}{\sqrt{3}}$
  - (b)  $\frac{4-2\sqrt{5}}{\sqrt{15}}$
  - (c)  $\frac{4+2\sqrt{5}}{\sqrt{15}}$
  - (d)  $\frac{2-2\sqrt{5}}{\sqrt{15}}$
  - (e)  $\frac{2+2\sqrt{5}}{\sqrt{15}}$

- 17. The series  $\sum_{n=0}^{+\infty} \frac{2^n \sin\left(n\frac{\pi}{2}\right)}{n!}$  is
  - (a) Convergent by Limit Comparison Test
  - (b) Convergent by Alternating Series Test
  - (c) Divergent by Divergence Test
  - (d) Divergent by Comparison Test
  - (e) Divergent by Ratio Test

- 18. If the sum of the first n terms of a series  $\sum_{n=1}^{+\infty} a_n$  is given by  $S_n = \frac{2n}{n+1}$ , then
  - (a)  $a_n = \frac{2}{n^2 + n}$
  - (b)  $a_n = \frac{2}{n^2 + 1}$
  - (c)  $a_n = \frac{2}{n^2 + 2n}$
  - (d)  $a_n = \frac{1}{n^2 + n}$
  - (e)  $a_n = \frac{4}{n^2 + n}$

- 19. The value of the integral  $\int_0^1 x\sqrt{1-x^4} \, dx$  is
  - (a)  $\frac{\pi}{8}$
  - (b)  $\frac{\pi}{16}$
  - (c)  $\frac{\pi}{3}$
  - (d)  $\frac{\pi}{6}$
  - (e)  $\frac{\pi}{2}$

- 20. The improper integral  $\int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx$  is
  - (a) Convergent and its value is  $e^{+2}$
  - (b) Convergent and its value is  $-e^{-2}$
  - (c) Divergent
  - (d) Convergent and its value is 0
  - (e) Convergent and its value is  $e^{-2}$

- 21. The series  $\sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n$  is
  - (a) Conditionally convergent
  - (b) Divergent by Divergence Test
  - (c) Convergent
  - (d) Divergent by Ratio Test
  - (e) Divergent by Comparison Test

22. The first four terms of the Taylor series of the function  $f(x) = \ln(3+x)$  at a=1 are:

(a) 
$$\ln 4 + \frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

(b) 
$$\frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192} - \frac{(x-1)^4}{256}$$

(c) 
$$-\frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192} + \frac{(x-1)^4}{256}$$

(d) 
$$\ln 4 + \frac{(x-1)}{4} + \frac{(x-1)^2}{32} + \frac{(x-1)^3}{192}$$

(e) 
$$\ln 4 - \frac{(x-1)}{4} + \frac{(x-1)^2}{32} - \frac{(x-1)^3}{192}$$

- 23. The value of the integral  $\int_0^{\pi/4} \tan^4 x \sec^4 x \ dx$  is
  - (a)  $\frac{2}{35}$
  - (b)  $\frac{12}{35}$
  - (c)  $\frac{1}{5}$
  - (d)  $\frac{5}{12}$
  - (e)  $-\frac{2}{35}$

- 24. The value of the integral  $\int_0^{1/3} \frac{x^2 dx}{1+x^7}$  is
  - (a)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+3) \cdot 3^{7n+3}}$
  - (b)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+2) \cdot 3^{7n+2}}$
  - (c)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+2}}$
  - (d)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}}$
  - (e)  $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+1}}$

- 25. A power series representing the function  $f(x) = \frac{3x^3}{(x-3)^2}$  is [Hint: You may consider the function:  $\frac{d}{dx} \left(\frac{1}{3-x}\right) = \frac{1}{(3-x)^2}$ ]
  - (a)  $\sum_{n=1}^{+\infty} \frac{nx^{n+2}}{3^n}$
  - (b)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^n}$
  - (c)  $\sum_{n=1}^{+\infty} \frac{x^{n+3}}{3^n}$
  - (d)  $\sum_{n=1}^{+\infty} \frac{x^n}{3^n}$
  - (e)  $\sum_{n=1}^{+\infty} \frac{nx^n}{3^{n+2}}$

- 26. If f(x) = f'(x) + 3, f(0) = 1 and f(1) = 4, then the value of the integral  $\int_0^1 e^x f'(x) dx$  is
  - (a)  $\frac{e+1}{2}$
  - (b)  $\frac{e-2}{2}$
  - (c)  $\frac{e-1}{2}$
  - $(d) \ \frac{2e-1}{2}$
  - (e)  $\frac{e+2}{2}$

- 27. The value of the integral  $\int_0^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) dx$  is
  - (a) 1/2
  - (b) -1/2
  - (c) e/2
  - (d) -e/2
  - (e)  $-\sqrt{2}/2$

- 28. The area of the region bounded by the curves  $4x + y^2 = 12$  and x = y is
  - (a)  $\frac{64}{3}$
  - (b) 17
  - (c)  $\frac{44}{3}$
  - (d) 21
  - (e)  $\frac{52}{3}$

Name		
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1	a	b	c	d	е	f
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16	a	b	С	d	е	f
17	a	b	c	d	е	f
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29	a	b	c	d	е	f
30	a	b	c	d	е	f
31	a	b	c	d	е	f
32	a	b	С	d	е	f
33	a	b	c	d	е	f
34	a	b	c	d	е	f
35	a	b	c	d	e	f

36	a	b	c	d	е	f
37	a	b	c	d	е	f
38	a	b	С	d	е	f
39	a	b	c	d	е	f
40	a	b	С	d	е	f
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49	a	b	c	d	е	f
50	a	b	c	d	е	f
51	a	b	С	d	е	f
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53	a	b	С	d	е	f
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55	a	b	С	d	е	f
56	a	b	c	d	е	f
57	a	b	С	d	е	f
58	a	b	c	d	е	f
59	a	b	c	d	е	f
60	a	b	$^{\mathrm{c}}$	d	е	f
61	a	b	c	d	е	f
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64	a	b	$^{\mathrm{c}}$	d	е	f
65	a	b	c	d	е	f
66	a	b	$^{\mathrm{c}}$	d	е	f
67	a	b	c	d	е	f
68	a	b	$\mathbf{c}$	d	e	f
69	a	b	c	d	е	f
70	a.	b	С	d	e	f

Q	MM	V1	V2	V3	V4
1	a	d	е	a	d
2	a	е	С	С	е
3	a	е	е	С	b
4	a	d	d	d	b
5	a	d	a	a	С
6	a	е	е	a	е
7	a	d	d	c	b
8	a	d	b	d	c
9	a	b	b	c	d
10	a	е	d	a	b
11	a	a	е	b	d
12	a	d	е	d	a
13	a	b	b	е	b
14	a	a	b	a	b
15	a	a	е	d	b
16	a	a	d	b	b
17	a	d	d	d	b
18	a	c	a	d	a
19	a	c	c	е	a
20	a	d	d	c	е
21	a	a	a	е	$\mathbf{c}$
22	a	d	a	c	a
23	a	е	c	c	b
24	a	d	е	c	a
25	a	a	b	d	a
26	a	c	е	d	е
27	a	a	е	b	b
28	a	b	е	b	a

# Answer Counts

V	a	b	С	d	е
1	3	9	7	5	4
2	5	6	7	3	7
3	4	10	6	3	5
4	11	6	3	2	6