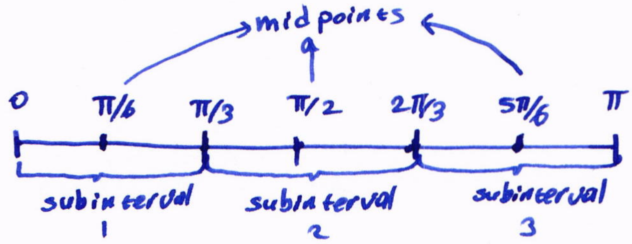
King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 152
Tuesday 23/02/2016
Net Time Allowed: 120 minutes

MASTER VERSION

1. Using three rectangles and midpoints, the estimate of the area under the graph of $f(x) = x + \cos^2(x)$ from $x = 0$ to $x = \pi$ is



(a) $\frac{\pi}{2}(\pi + 1)$

(b) $\frac{\pi}{2}(2\pi/3 + 1)$

(c) $\frac{\pi}{2}(4\pi/3 + 1)$

(d) $\frac{\pi}{2}(\pi + 1/3)$

(e) $\frac{\pi}{2}(\pi + 1/2)$

Estimate of area $= \frac{\pi}{3}\left[\frac{\pi}{6} + \cos^2(\pi/6) + \frac{\pi}{2} + \cos^2(\pi/2) + \frac{5\pi}{6} + \cos^2(5\pi/6)\right]$

$= \frac{\pi}{3}\left[\frac{3\pi}{2} + \frac{3}{4} + \frac{3}{4}\right]$

$= \frac{\pi}{2}(\pi + 1)$

2. If the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{2i}{n}\right)^{10}$ is expressed as a definite integral of a function on the interval $[1, 3]$, then its value is

$b = 3 \quad a = 1$

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$

$x_i = a + i\Delta x = 1 + \frac{2}{n} i$

Then $f(x) = x^{10}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{2i}{n}\right)^{10} = \int_1^3 x^{10}\,dx = \frac{1}{11} x^{11}\Big|_1^3 = \frac{1}{11}(3^{11} - 1)$

(a) $\frac{1}{11}(3^{11} - 1)$

(b) $\frac{2^{11}}{11}(2^{11} - 1)$

(c) $\frac{1}{11}(2^{11} - 1)$

(d) $\frac{3^{11}}{11}(3^{11} - 1)$

(e) $\frac{1}{11}(3^{11} - 2^{11})$

3. If $A = \int_{11}^{-20} f(x)dx$, $B = \int_{4}^{-20} f(x)dx$, and $C = \int_{11}^{-4} f(x)\,dx$,

then $A - B - C$ is equal to

(a) $\int_{-4}^{4} f(x)\,dx$

(b) $\int_{4}^{11} f(x)\,dx$

(c) $\int_{-20}^{4} f(x)\,dx$

(d) $\int_{-20}^{-4} f(x)\,dx$

(e) $\int_{-4}^{11} f(x)\,dx$

$$A-B-C = \int_{11}^{-20} f(x)dx - \int_{4}^{-20} f(x)dx - \int_{11}^{-4} f(x)dx$$

$$= \underbrace{\int_{11}^{-20} f(x)dx + \int_{-20}^{4} f(x)dx}_{} + \int_{-4}^{11} f(x)dx$$

$$= \int_{11}^{4} f(x)\,dx + \int_{-4}^{11} f(x)dx$$

$$= \int_{-4}^{4} f(x)\,dx$$

4. $\int_{0}^{1} 5^x - \frac{1}{\sqrt{1-x^2}}\,dx = \int_0^1 5^x\,dx - \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

(a) $\frac{4}{\ln 5} - \frac{\pi}{2}$

(b) $\frac{4}{\ln 5}$

(c) $5\ln 5 + \pi$

(d) $4\ln 5 - \pi$

(e) $\frac{5}{\ln 5} - \frac{\pi}{2}$

$$= \frac{5^x}{\ln 5}\Big|_0^1 - \sin^{-1}(x)\Big|_0^1$$

$$= \left(\frac{5}{\ln 5} - \frac{1}{\ln 5}\right) - \left(\sin^{-1}(1) - \sin^{-1}(0)\right)$$

$$= \frac{4}{\ln 5} - \frac{\pi}{2}$$

5. $\int (3\tan x - 2\sin 2x)\sec x\,dx =$ $3\int \tan x \sec x\,dx - 2\int \sin 2x \sec x\,dx$

(a) $3\sec x + 4\cos x + C$

(b) $3\sec x - 4\cos x + C$

(c) $2\cos x - 3\sec x + C$

(d) $3\sec x + 2\cos x + C$

(e) $-3\sec x - 4\cos x + C$

$= 3\int \tan x \sec x\,dx - 2\int 2\sin x\cos x \frac{1}{\cos x}\,dx$

$= 3\int \tan x \sec x\,dx - 4\int \sin x\,dx$

$= 3\sec x + 4\cos x + C$

6. $\int_0^{\pi/2} \sin x\,\tan(\cos x)\,dx =$ $-\int_1^0 \tan(u)\,du$

substitution

$u = \cos x$

$du = -\sin x\,dx$

$x = 0 \Rightarrow u = 1$

$x = \pi/2 \Rightarrow u = 0$

(a) $\ln(\sec(1))$

(b) $\ln(\cos(1))$

(c) $\ln(\csc(1))$

(d) $\ln(\sin(1))$

(e) 0

$= +\ln|\cos(u)|\Big|_1^0$

$= \ln|\cos(0)| - \ln|\cos(1)|$

$= -\ln(\cos(1))$

$= \ln(\sec(1))$

7. $\int x(2x-1)^5\,dx = I$

(a) $\frac{1}{28}(2x-1)^7 + \frac{1}{24}(2x-1)^6 + C$

(b) $\frac{1}{14}(2x-1)^7 + \frac{1}{12}(2x-1)^6 + C$

(c) $\frac{1}{7}(2x-1)^7 + \frac{1}{6}(2x-1)^6 + C$

(d) $\frac{1}{28}(2x-1)^7 + C$

(e) $\frac{1}{12}(2x-1)^6 + C$

substitution

$u = 2x-1 \rightsquigarrow x = \frac{u+1}{2}$

$du = 2dx$

$I = \int \frac{u+1}{2} \cdot u^5 \frac{du}{2}$

$= \frac{1}{4} \int u^6 + u^5\,du$

$= \frac{1}{4}\left(\frac{1}{7}u^7 + \frac{1}{6}u^6\right) + C$

$= \frac{1}{28}(2x-1)^7 + \frac{1}{24}(2x-1)^6 + C$

8. If $\int_{-3}^{a} \frac{[\ln(x+4)]^2}{x+4}\,dx = \frac{1}{3}$, then a is equal to

(a) $e-4$

(b) 1

(c) $4-e$

(d) e

(e) $1/e$

substitution

$u = \ln(x+4)$

$du = \frac{1}{x+4}dx$

$x=-3 \Rightarrow u=0$

$x=a \Rightarrow u=\ln(a+4)$

$\int_{-3}^{a} \frac{[\ln(x+4)]^2}{x+4}dx = \int_{0}^{\ln(a+4)} u^2\,du = \frac{1}{3}$

$\Rightarrow \frac{1}{3}u^3\Big|_0^{\ln(a+4)} = \frac{1}{3} \Rightarrow [\ln(a+4)]^3 = 1 \Rightarrow \ln(a+4) = 1$

$\Rightarrow a+4 = e$

$\Rightarrow a = e-4$

9. If f is an **EVEN** continuous function and $\int_0^4 f(x)dx = 5$, then $\int_{-2}^{2}[xf(x^2) + f(2x)]\,dx$ is equal to

(a) 5

(b) 10

(c) 15

(d) 20

(e) 0

If f is even, then $xf(x^2)$ is odd

Therefore $\int_{-2}^{2} x f(x^2)dx = 0$

For $\int_{-2}^{2} f(2x)\,dx$, do the substitution $u=2x, du=2dx$

$x=-2 \Rightarrow u=-4$

$x=2 \Rightarrow u=4.$

$$\int_{-2}^{2} f(2x)dx = \frac{1}{2}\int_{-4}^{4} f(u)\,du \underset{\text{b/c } f \text{ is even}}{=} \frac{1}{2}\cdot 2\cdot\int_0^4 f(u)\,du = 5.$$

Then $\int_{-2}^{2}\left[x f(x^2) + f(2x)\right]dx = \int_{-2}^{2} x f(x^2)\,dx + \int_{-2}^{2} f(2x)dx = 5$

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10. $\int_1^{16} \frac{2\sqrt{y} - y}{y^2}\,dy = \int_1^{16} \frac{2\sqrt{y}}{y^2} - \frac{y}{y^2}\,dy = \int_1^{16} 2y^{-3/2} - \frac{1}{y}\,dy$

(a) $3 - 4\ln 2$

(b) $-6 - 4\ln 2$

(c) $-4 + 4\ln 2$

(d) $\frac{63}{16}$

(e) $\frac{79}{16}$

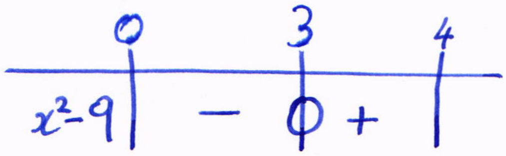
$= 2\cdot(-2)\cdot y^{-1/2}\Big|_1^{16} - \ln|y|\Big|_1^{16}$

$= -4\left(\frac{1}{4} - 1\right) - \left(\ln(16) - \ln(1)\right)$

$= -1 + 4 - \ln(16)$

$= 3 - 4\ln 2$

11. $\int_0^4 |x^2 - 9|\, dx = I$



(a) $\frac{64}{3}$

(b) $\frac{280}{3}$

(c) $-\frac{44}{3}$

(d) $\frac{32}{3}$

(e) $\frac{140}{3}$

From the above table $|x^2-9| = \begin{cases} 9-x^2, & 0 \le x < 3 \\ x^2-9, & 3 \le x \le 4 \end{cases}$

$$I = \int_0^3 9-x^2\, dx + \int_3^4 x^2 - 9\, dx$$

$$= \left(9x - \frac{x^3}{3}\right)\Big|_0^3 + \left(\frac{x^3}{3} - 9x\right)\Big|_3^4$$

$$= (27-9) + \left(\frac{64}{3} - 36\right) - (9-27)$$

$$= 18 + \frac{64}{3} - 36 + 18$$

$$= \frac{64}{3}$$

12. If $f(x) = \begin{cases} \sqrt{4-x^2}, & -2 \le x \le 0 \\ e^{2x}+1, & 0 \le x \le 2 \end{cases}$, then $\int_{-2}^{2} f(x)\, dx =$

$$I = \int_{-2}^{2} f(x)\, dx = \underbrace{\int_{-2}^{0} \sqrt{4-x^2}\, dx}_{I_1} + \underbrace{\int_0^2 e^{2x}+1\, dx}_{I_2.}$$

(a) $\frac{1}{2}(2\pi + e^4 + 3)$

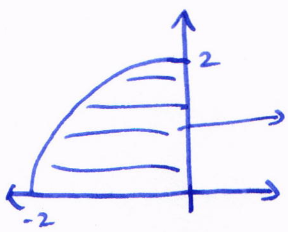
(b) $\frac{1}{2}(2\pi + e^4 + 2)$

(c) $\frac{1}{2}(2\pi + e^4 + 1)$

(d) $\frac{1}{2}(\pi + e^4 + 3)$

(e) $\frac{1}{2}(\pi + e^4 + 1)$

For I_1, we use area below curve.



$I_1 = \frac{1}{4} 4\pi = \pi$

$$I_2 = \frac{e^{2x}}{2} + x\Big|_0^2 = \left(\frac{e^4}{2} + 2\right) - \frac{1}{2} = \frac{e^4}{2} + \frac{3}{2}$$

$$I = \pi + \frac{e^4}{2} + \frac{3}{2} = \frac{1}{2}(2\pi + e^4 + 3)$$

13. If $f(x) = \int_{\sin(x)}^{1} \sqrt{2+t^2}\, dt$, then $\frac{df}{dx}|_{x=\pi/3}$ is equal to

(a) $-\frac{\sqrt{11}}{4}$

(b) $\frac{\sqrt{7}}{4}$

(c) $\frac{3\sqrt{11}}{2}$

(d) $\frac{3\sqrt{3}}{4}$

(e) $-\frac{\sqrt{3}}{4}$

Using F.T.C $\frac{df}{dx} = -\cos x \sqrt{2+\sin^2 x}$

Then $\frac{df}{dx}\Big|_{x=\pi/3} = -\cos(\pi/3)\sqrt{2+\sin^2(\pi/3)}$

$= -\frac{1}{2}\sqrt{2+\frac{3}{4}}$

$= -\frac{\sqrt{11}}{4}$

14. Let f and g be continuous functions on the interval $[a, b]$ and $c \in [a, b]$. If $0 \le m \le g(x) < f(x) \le M$, for $a \le x \le b$, then which one of the following statements is **FALSE**.

(a) $(M-m)(b-a) < \int_a^b [f(x) - g(x)]\, dx$

(b) $\int_a^b [f(x) + m]\, dx \le \int_a^b [g(x) + M]\, dx$

(c) $\int_a^b [M - f(x)]dx < \int_a^b [M - g(x)]\, dx$

(d) $\int_a^c [f(x) - g(x)]dx \le \int_a^b [f(x) - g(x)]\, dx$

(e) $\int_a^b [g(x) - f(x)]dx \le \int_a^c [g(x) - f(x)]\, dx$

Observe that

$f(x) - g(x) < M - m$, $a \le x \le b$.

Therefore,

$\int_a^b f(x) - g(x)\,dx < \int_a^b M - m\,dx$

$\int_a^b f(x) - g(x)\,dx < (M-m)(b-a)$

15. $\displaystyle\int_0^{\ln\sqrt{3}} \frac{1}{e^x + e^{-x}}\, dx = \int_0^{\ln\sqrt{3}} \frac{e^x}{e^{2x}+1}\, dx$

substitution
$u = e^x$
$du = e^x dx$
$x = 0 \Rightarrow u = 1$
$x = \ln\sqrt{3} \Rightarrow u = \sqrt{3}$

$$= \int_1^{\sqrt{3}} \frac{1}{u^2+1}\, du$$

$$= \tan^{-1}(u)\Big|_1^{\sqrt{3}}$$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

(a) $\pi/12$

(b) $-\pi/12$

(c) $\pi/3$

(d) $\pi/6$

(e) $-\pi/3$

16. The area of the region enclosed by the curves $y = (1+x)^2$ and $y = \sqrt{1-x}$, and the x-axis is

(a) 1

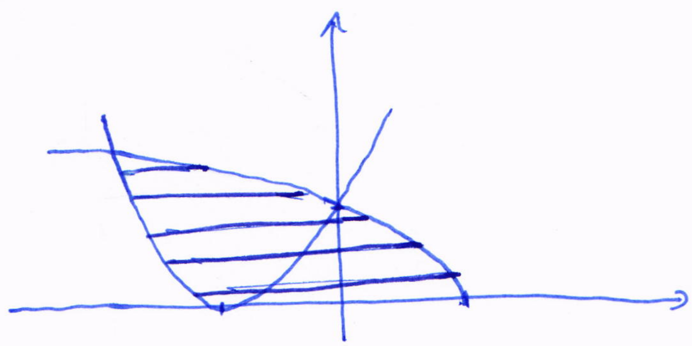
(b) 2

(c) 1/3

(d) 2/3

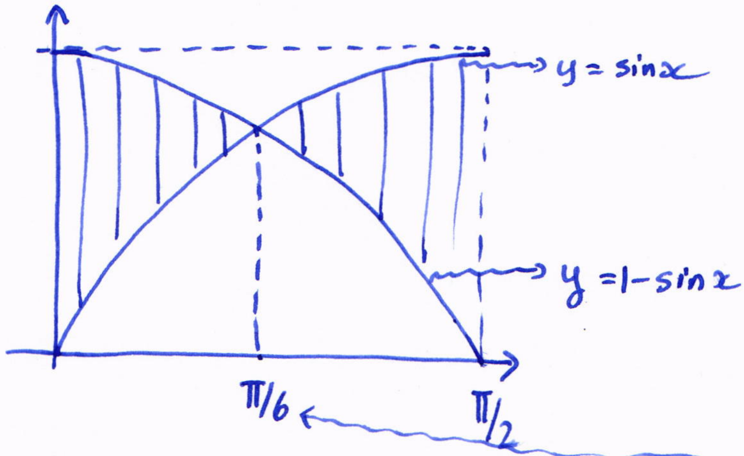
(e) 7/3

This question is cancelled because the area described in the question is



and the correct answer is not among the choices.

17. The area of the region enclosed by the curves $y = \sin x$, $y = 1 - \sin x$, $x = 0$, and $x = \dfrac{\pi}{2}$ is



(a) $2\sqrt{3} - 2 - \pi/6$

(b) $2\sqrt{3} - \pi/6$

(c) $\pi/2 - 1$

(d) $2 + \pi/6$

(e) $2\sqrt{3} + 2 + \pi/6$

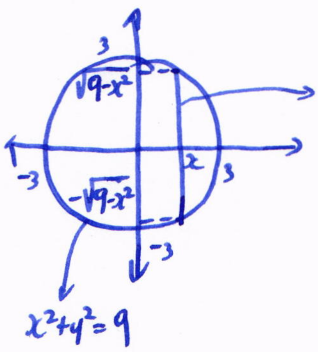
$1 - \sin x = \sin x \Rightarrow 2\sin x = 1 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \pi/6.$

$$\text{Area} = \int_0^{\pi/6} (1-\sin x) - \sin x \, dx + \int_{\pi/6}^{\pi/2} \sin x - (1 - \sin x)\, dx$$

$$= \int_0^{\pi/6} 1 - 2\sin x \, dx + \int_{\pi/6}^{\pi/2} 2\sin x - 1 \, dx = (x + 2\cos x)\Big|_0^{\pi/6} + (-2\cos x - x)\Big|_{\pi/6}^{\pi/2}$$

$$= 2\sqrt{3} - 2 - \pi/6$$

18. The volume of the solid whose base is a circular disk with radius 3 and whose parallel cross-sections perpendicular to the base are squares is



one side of the square $= 2\sqrt{9-x^2}$

Area of cross-section $= 4(9-x^2)$

(a) 144

(b) 18

(c) 36

(d) 288

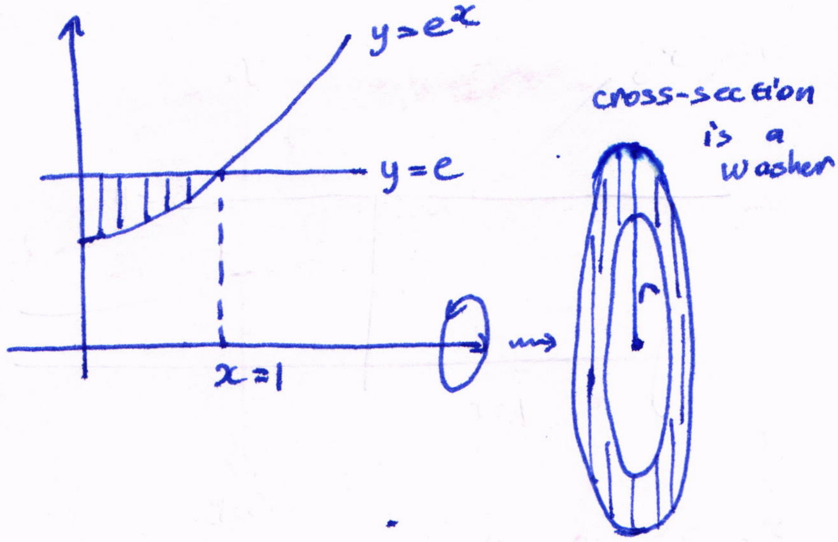
(e) 100

$$\text{Volume} = \int_{-3}^{3} (\text{Area of cross-section})\, dx$$

$$= \int_{-3}^{3} 4(9-x^2)\, dx = 8\int_0^3 9 - x^2 \, dx = 8\left(9x - \frac{x^3}{3}\right)\Big|_0^3$$

$$= 144$$

19. The volume of the solid obtained by rotating the region bounded by $y = e^x$, $y = e$ and $x = 0$ about the x-axis is equal to



(a) $\pi \int_0^1 (e^2 - e^{2x})dx$

(b) $\pi \int_0^1 (e - e^x)^2 dx$

(c) $\pi \int_0^e (e^2 - e^{2x})dx$

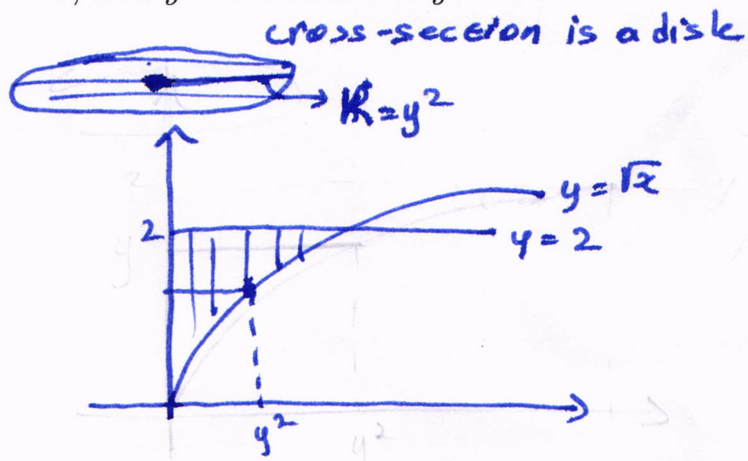
(d) $\pi \int_1^e (\ln x)^2 dx$

(e) $\pi \int_0^e (e - e^x)^2 dx$

Using Washer Method with $R = e$ $r = e^x$

$$\text{Volume} = \pi \int_0^1 (e^2 - e^{2x})\,dx$$

20. The volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $x = 0$, and $y = 2$ about the y-axis is equal to



(a) $\frac{32\pi}{5}$

(b) $\frac{8\pi}{3}$

(c) $\frac{16\pi}{5}$

(d) $\frac{64\pi}{3}$

(e) $\frac{4\pi}{5}$

Using Disk Method with $R = y^2$

$$\text{Volume} = \pi \int_0^2 (y^2)^2\,dy = \frac{\pi}{5} y^5 \Big|_0^2 = \frac{32}{5}\pi$$

Q	MM	V1	V2	V3	V4
1	a	c	b	d	d
2	a	a	c	c	c
3	a	b	d	e	b
4	a	e	b	c	b
5	a	c	e	c	c
6	a	e	e	a	d
7	a	d	c	c	b
8	a	c	a	a	b
9	a	b	e	c	a
10	a	e	d	c	d
11	a	e	a	a	a
12	a	b	e	c	c
13	a	a	a	c	d
14	a	d	c	d	d
15	a	c	e	c	c
16	a	a	a	b	e
17	a	a	e	a	c
18	a	c	c	b	a
19	a	b	d	a	d
20	a	e	b	a	d