King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 151
Monday 28/12/2015
Net Time Allowed: 180 minutes

MASTER VERSION

1. $\displaystyle\int_1^4 \frac{1}{\sqrt{w}(1+\sqrt{w})^2}\,dw =$

$u = 1+\sqrt{w} \Rightarrow du = \frac{1}{2\sqrt{w}}\,dw$

$, w=1 \Rightarrow u = 1+\sqrt{1} = 2$

$. w=4 \Rightarrow u = 1+\sqrt{4} = 3$

(a) $\frac{1}{3}$

(b) $-\frac{2}{3}$

(c) $\frac{1}{9}$

(d) $\frac{3}{5}$

(e) $\frac{1}{4}$

$= 2\displaystyle\int_2^3 \frac{1}{u^2}\,du = 2\cdot -\frac{1}{u}\Big]_{u=2}^{3}$

$= 2\cdot\left(-\frac{1}{3}+\frac{1}{2}\right)$

$= 2\cdot\frac{-2+3}{6}$

$= 2\cdot\frac{1}{6} = \frac{1}{3}$

2. $\displaystyle\int (3-\tan x)^2\,dx =$ $\displaystyle\int 9-6\tan x+\tan^2 x\;dx$

$= \displaystyle\int 9-6\tan x+\sec^2 x-1\;dx$

$= \displaystyle\int 8-6\tan x+\sec^2 x\;dx$

$= 8x - 6\ln|\sec x| + \tan x + C$

(a) $8x+\tan x-6\ln|\sec x|+C$

(b) $\dfrac{(3-\tan x)^3}{3}+C$

(c) $(3-\tan x)\sec^2 x+C$

(d) $9x+\frac{1}{3}\tan^3 x-3\tan^2 x+C$

(e) $9x+\tan x-6\sec x+C$

3. If $f(x) = \int_3^{x^3} \sqrt[3]{t - t^2}\, dt$, then $f'(x) =$ $\sqrt[3]{x^3 - (x^3)^2} \cdot \frac{d}{dx}[x^3]$

$= \sqrt[3]{x^3 - x^6} \cdot 3x^2$

$= \sqrt[3]{x^3(1-x^3)} \cdot 3x^2$

$= x\sqrt[3]{1-x^3} \cdot 3x^2$

$= 3x^3 \sqrt[3]{1-x^3}$

(a) $3x^3\sqrt[3]{1-x^3}$

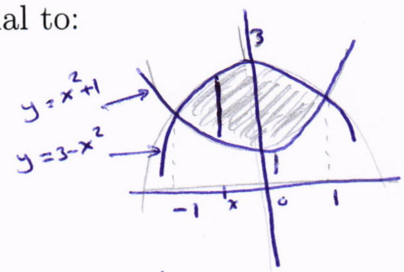
(b) $\sqrt[3]{x^3 - x^6}$

(c) $\sqrt[3]{x^3 - x^3} + \sqrt[3]{3}$

(d) $3x^2\sqrt[3]{x - x^2}$

(e) $3x\sqrt[3]{1-x^3}$

4. The **area** of the region bounded by the curves $y = x^2 + 1$ and $y = 3 - x^2$ is equal to:



pts of intersection:
$x^2 + 1 = 3 - x^2$
$\Rightarrow 2x^2 = 2 \Rightarrow x^2 = 1$
$\Rightarrow x = \pm 1$

(a) $\frac{8}{3}$

(b) $\frac{4}{3}$

(c) $\frac{2}{5}$

(d) $\frac{1}{6}$

(e) $\frac{4}{5}$

$A = \int_{-1}^{1} (3 - x^2) - (x^2+1)\, dx$

$= \int_{-1}^{1} 2 - 2x^2\, dx$

$= 2x - \frac{2}{3}x^3 \Big]_{-1}^{1}$

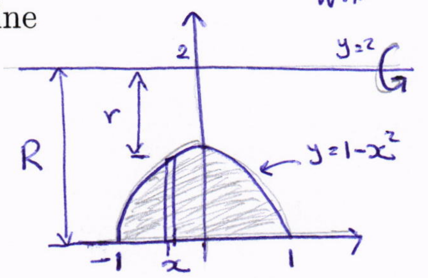
$= \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right)$

$= 4 - \frac{4}{3}$

$= \frac{12-4}{3} = \frac{8}{3}$

5. The **volume** of the solid generated by rotating the region enclosed by the curves $y = 1 - x^2$ and $y = 0$ about the line $y = 2$ is equal to

Washer



$r = 2-(1-x^2) = 1+x^2$

$R = 2-0 = 2$

Washer Method

$V = \pi \int_{-1}^{1} 2^2 - (1+x^2)^2 \, dx$

$= \pi \int_{-1}^{1} 4 - (1+2x^2+x^4) \, dx$

$= \pi \int_{-1}^{1} 3 - 2x^2 - x^4 \, dx$

$= \pi \cdot 3x - \frac{2}{3}x^3 - \frac{1}{5}x^5 \Big]_{-1}^{1}$

$= \pi \cdot (3 - \frac{2}{3} - \frac{1}{5}) - (-3 + \frac{2}{3} + \frac{1}{5})$

$= \pi \cdot (6 - \frac{4}{3} - \frac{2}{5})$

$= \pi \cdot \frac{90-20-6}{15} = \pi \cdot \frac{64}{15}$

(a) $\frac{64}{15}\pi$

(b) $\frac{8}{3}\pi$

(c) $\frac{4}{5}\pi$

(d) $\frac{32}{3}\pi$

(e) $\frac{2}{7}\pi$

6. $\int_0^{\pi/6} \cos^3(3t)\, dt = \int_0^{\pi/6} \cos(3t) \cdot \cos^2(3t) \, dt$

$= \int_0^{\pi/6} \cos(3t) \cdot (1 - \sin^2(3t)) \, dt$

$u = \sin(3t) \Rightarrow du = 3\cos(3t)\, dt$

$t = 0 \Rightarrow u = 0$ & $t = \frac{\pi}{6} \Rightarrow u = 1$

$= \frac{1}{3} \int_0^1 1 - u^2 \, du$

$= \frac{1}{3} \cdot u - \frac{1}{3}u^3 \Big]_0^1$

$= \frac{1}{3} \left(1 - \frac{1}{3}\right)$

$= \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$

(a) $\frac{2}{9}$

(b) $-\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{9}$

(e) $\frac{5}{3}$

7. The **length** of the curve

$$y = \cosh x, \ 0 \le x \le 1$$

is equal to:

(a) sinh 1

(b) cosh 1

(c) $1 - \sinh 1$

(d) $1 + \cosh 1$

(e) tanh 1

$$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$
$$= \int_0^1 \sqrt{1+\sinh^2 x}\, dx$$
$$= \int_0^1 \sqrt{\cosh^2 x}\, dx$$
$$= \int_0^1 |\cosh x|\, dx$$
$$= \int_0^1 \cosh x\, dx \qquad (\cosh x > 0 \quad \forall x)$$
$$= \sinh x \Big]_0^1$$
$$= \sinh 1 - \sinh 0 = \sinh 1 - 0$$
$$= \sinh 1.$$

8. The **area of the surface** obtained by rotating the line segment

$$y = 2x + 1, \ 0 \le x \le 2$$

about the x-axis is equal to:

(a) $12\pi\sqrt{5}$

(b) $\pi\sqrt{5}$

(c) $6\pi + 1$

(d) $\pi + \sqrt{5}$

(e) $2\pi\sqrt{5}$

$$S = \int_0^2 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$
$$= \int_0^2 2\pi (2x+1) \sqrt{1+(2)^2}\, dx$$
$$= 2\pi\sqrt{5} \int_0^2 2x+1\, dx$$
$$= 2\pi\sqrt{5} \cdot x^2+x \Big]_0^2$$
$$= 2\pi\sqrt{5} \cdot (4+2-0)$$
$$= 2\pi\sqrt{5} \cdot 6$$
$$= 12\pi\sqrt{5}$$

9. $\int_1^2 \frac{x-1}{x^2+x}\,dx =$

$\frac{x-1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$\Rightarrow x-1 = A(x+1) + Bx$

- $x=-1 \Rightarrow -2 = -B \Rightarrow B=2$
- $x=0 \Rightarrow -1 = A$

$\int \frac{x-1}{x^2+x}\,dx = \int \frac{-1}{x} + \frac{2}{x+1}\,dx$

$= -\ln|x| + 2\ln|x+1| + C$

$= \ln\left|\frac{(x+1)^2}{x}\right| + C$

$\int_1^2 \frac{x-1}{x^2+x}\,dx = \ln\left|\frac{(x+1)^2}{x}\right|\Big]_1^2$

$= \ln\left(\frac{9}{2}\right) - \ln 4$

$= \ln\left(\frac{9}{8}\right)$.

(a) $\ln\left(\frac{9}{8}\right)$

(b) $\ln\left(\frac{9}{7}\right)$

(c) $\ln\left(\frac{9}{10}\right)$

(d) $\ln\left(\frac{9}{5}\right)$

(e) $\ln\left(\frac{9}{11}\right)$

10. $\int \frac{x^4}{x^2-4}\,dx =$ $\int x^2+4 + \frac{16}{x^2-4}\,dx$ $= \frac{1}{3}x^3 + 4x + 16\cdot\frac{1}{2\cdot 2}\ln\left|\frac{x-2}{x+2}\right| + C$

Long divisim

$= \frac{1}{3}x^3 + 4x + 4\ln\left|\frac{x-2}{x+2}\right| + C$

(a) $\frac{1}{3}x^3 + 4x + 4\ln\left|\frac{x-2}{x+2}\right| + C$

(b) $\frac{1}{3}x^3 + 4x + 16\ln|x^2-4| + C$

(c) $x^2 + 4 - 8\ln\left|\frac{x-2}{x+2}\right| + C$

(d) $\frac{1}{3}x^3 + 4x - 2\ln\left|\frac{x-2}{x+2}\right| + C$

(e) $2x + 4x - 8\ln|x^2-4| + C$

11. $\int (t \ln t)^2 \, dt = \int t^2 (\ln t)^2 \, dt$

: $u = (\ln t)^2$, $dv = t^2$

$du = 2\frac{\ln t}{t}$, $v = \frac{1}{3}t^3$

by Parts

$= \frac{1}{3}t^3 (\ln t)^2 - \frac{2}{3}\int t^2 \ln t \, dt$

$u = \ln t$, $dv = t^2$

$du = \frac{1}{t}$, $v = \frac{1}{3}t^3$

$= \frac{1}{3}t^3 (\ln t)^2 - \frac{2}{3}\left[\frac{1}{3}t^3 \ln t - \frac{1}{3}\int t^2 dt\right]$

$= \frac{1}{3}t^3 (\ln t)^2 - \frac{2}{9} t^3 \ln t - \frac{2}{9} \cdot \frac{1}{3}t^3 + C$

$= \frac{1}{3}t^3 (\ln t)^2 - \frac{2}{9} t^3 \ln t - \frac{2}{27}t^3 + C$

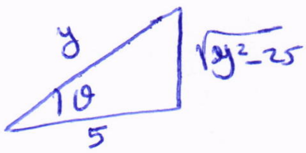
(a) $\frac{1}{3}t^3(\ln t)^2 - \frac{2}{9}t^3 \ln t + \frac{2}{27}t^3 + C$

(b) $\frac{1}{3}t^3 \ln t - \frac{2}{9}t^3 + C$

(c) $\frac{1}{3}t^3(\ln t)^2 - \frac{2}{3}t^3 \ln t + \frac{2}{9}t^3 + C$

(d) $\frac{1}{3}(t \ln t)^3 + \frac{1}{2}(t \ln t)^2 + t \ln t + C$

(e) $3t^3 \ln t - \frac{2}{9}t^2 \ln t + \frac{2}{27}t \ln t + C$



let $y = 5\sec\theta$, $0 < \theta < \frac{\pi}{2}$

$dy = 5 \sec\theta \tan\theta \, d\theta$

$\sqrt{y^2 - 25} = \sqrt{25\sec^2\theta - 25} = \sqrt{25\tan^2\theta} = |5\tan\theta| = 5\tan\theta$

12. For $y > 5$, $\int \frac{\sqrt{y^2 - 25}}{y^3} dy = \int \frac{5\tan\theta}{5^3 \sec^3\theta} \cdot 5\sec\theta\tan\theta \, d\theta$

$= \frac{1}{5}\int \frac{\tan^2\theta}{\sec^2\theta} d\theta = \frac{1}{5}\int \sin^2\theta \, d\theta$

$= \frac{1}{10}\int 1 - \cos(2\theta) \, d\theta$

$= \frac{1}{10}\left[\theta - \frac{1}{2}\sin(2\theta)\right] + C$

$= \frac{1}{10}\left[\theta - \sin\theta\cos\theta\right] + C$

$= \frac{1}{10}\left[\sec^{-1}\left(\frac{y}{5}\right) - \frac{\sqrt{y^2-25}}{y} \cdot \frac{5}{y}\right] + C$

$= \frac{1}{10}\sec^{-1}\left(\frac{y}{5}\right) - \frac{1}{2} \cdot \frac{\sqrt{y^2-25}}{y^2} + C$

(a) $\frac{1}{10}\sec^{-1}\left(\frac{y}{5}\right) - \frac{\sqrt{y^2-25}}{2y^2} + C$

(b) $\frac{1}{5}\sec^{-1}\left(\frac{y}{5}\right) - \frac{\sqrt{y^2-25}}{2y^2} + C$

(c) $\sec^{-1}\left(\frac{y}{5}\right) + y^3\sqrt{y^2-25} + C$

(d) $\frac{1}{2}\sec^{-1}\left(\frac{y}{5}\right) - \frac{2y}{\sqrt{25-y^2}} + C$

(e) $y \sec^{-1}(y) + \frac{y^3}{\sqrt{y^2-25}} + C$

13. The improper integral $\int_{-\infty}^{1} \frac{1}{e^{-x}+1}\,dx$ is

$= \int_{-\infty}^{1} \frac{e^x}{e^x+1}\,dx$

$= \lim_{t\to-\infty} \int_t^1 \frac{e^x}{e^x+1}\,dx$

$= \lim_{t\to-\infty} \ln(e^x+1)\Big]_t^1$

$= \lim_{t\to-\infty} \ln(e+1) - \ln(e^t+1)$

$= \ln(e+1) - \ln(0+1)$

$= \ln(e+1)$

Convergent

(a) convergent and its value is $\ln(1+e)$

(b) convergent and its value is $\ln 2$

(c) convergent and its value is $e \ln 2$

(d) convergent and its value is $\ln(2+e)$

(e) divergent.

14. $\int_0^1 \frac{\sinh(2x)}{\sinh x + \cosh x}\,dx = \int_0^1 \frac{\frac{e^{2x}-e^{-2x}}{2}}{e^x}\,dx = \int_0^1 e^{-x} \cdot \frac{e^{2x}-e^{-2x}}{2}\,dx$

$= \frac{1}{2}\int_0^1 e^x - e^{-3x}\,dx$

$= \frac{1}{2} \cdot e^x + \frac{1}{3}e^{-3x}\Big]_0^1$

$= \frac{1}{2} \cdot \left[\left(e+\frac{1}{3}e^{-3}\right) - \left(1+\frac{1}{3}\right)\right]$

$= \frac{1}{2} \cdot \left[e + \frac{1}{3e^3} - \frac{4}{3}\right]$

$= \frac{1}{2} \cdot \left[\frac{3e^4+1}{3e^3} - \frac{4}{3}\right]$

$= \frac{3e^4+1}{6e^3} - \frac{2}{3}$

(a) $\frac{3e^4+1}{6e^3} - \frac{2}{3}$

(b) $\frac{e^4+1}{2e^3}$

(c) $\frac{e^4+1}{2e^3} - \frac{2}{3}$

(d) $e^4 - \frac{4}{3}$

(e) $\frac{e+1}{2e} - \frac{1}{3}$

15. Let $a_n = \ln(2n^2+1) - \ln(n^2+2)$. Then the sequence $\{a_n\}_{n=1}^{\infty}$

$= \ln\left(\frac{2n^2+1}{n^2+2}\right) \longrightarrow \ln 2$

(a) converges to $\ln 2$

(b) converges to 0

(c) converges to 2

(d) converges to 1

(e) diverges

16. Which one of the following series is **Convergent?**

a p-series is convergent if $p > 1$
divergent if $p \leq 1$

(a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.01}}$ $p = 1.01 > 1$ ✓

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ $p = \frac{1}{2} < 1$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^{\pi-3}}$ $p = \pi - 3 < 1$

(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^3}}$ $p = \frac{3}{5} < 1$

(e) $\sum_{n=1}^{\infty} \frac{1}{n^{e-2}}$ $p = e - 2 < 1$

17. The series $\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$ is $\sum_{n=0}^{\infty} 5\left(-\frac{1}{4}\right)^n = 5 - \frac{5}{4} + \cdots$

a geometric series with

$a = 5$ & $r = -\frac{1}{4}$

$. |r| = \frac{1}{4} < 1 \Rightarrow$ Conv.

$\text{Sum} = \frac{a}{1-r} = \frac{5}{1+\frac{1}{4}} = \frac{5}{\frac{5}{4}}$

$= 5 \cdot \frac{4}{5} = 4$

(a) convergent and its sum is 4

(b) convergent and its sum is $\frac{20}{3}$

(c) convergent and its sum is $\frac{5}{4}$

(d) convergent and its sum is $\frac{2}{3}$

(e) divergent

18. The series $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$ is

✓ (a) divergent by the n^{th} term test for divergence :

$\lim_{n\to\infty} \cos\left(\frac{1}{n^2}\right) = \cos 0 = 1 \neq 0$

(b) convergent by the integral test

(c) convergent by the comparison test

(d) convergent by the limit comparison test

(e) a convergent $p-$ series.

19. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{21+n}$ is

$\cdot \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{21+n}$ Conv. by the alternating series (check)

$\cdot \sum_{n=1}^{\infty}\left|(-1)^{n+1}\frac{1}{21+n}\right| = \sum_{n=1}^{\infty}\frac{1}{21+n}$ div. by the integral test (check)

Then $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{21+n}$ is conditionally convergent.

(a) conditionally convergent

(b) absolutely convergent

(c) divergent

(d) convergent by the ratio test

(e) divergent by the integral test

20. The **Taylor polynomial** of order 2 generated by $f(x) = \ln(\cos x)$ at $a = 0$ is:

$P_2(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2$

$= f(0) + f'(0)x + \frac{f''(0)}{2}x^2$

$\cdot f(0) = \ln(\cos 0) = \ln 1 = 0$

$\cdot f'(x) = \frac{-\sin x}{\cos x} = -\tan x \Rightarrow f'(0) = 0$

$\cdot f''(x) = -\sec^2 x \Rightarrow f''(0) = -1$

$P_2(x) = 0 + 0 - \frac{1}{2}x^2$

$= -\frac{1}{2}x^2$

(a) $P_2(x) = -\frac{1}{2}x^2$

(b) $P_2(x) = 1 + x + \frac{1}{2}x^2$

(c) $P_2(x) = x - x^2$

(d) $P_2(x) = \frac{1}{3}x^2$

(e) $P_2(x) = -\frac{1}{4}x^2$

21. The series $\sum\limits_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ is:

Use the Integral Test

. Conditions are satisfied (check)

. $\int_2^\infty \frac{1}{x\sqrt{\ln x}}\,dx = \lim\limits_{t\to\infty} \int_2^t \frac{1}{x\sqrt{\ln x}}\,du$

$= \lim\limits_{t\to\infty} 2\sqrt{\ln x}\Big]_2^t$

$= \lim\limits_{t\to\infty} 2\sqrt{\ln t} - 2\sqrt{\ln 2} = \infty$

Since $\int_2^\infty \frac{1}{x\sqrt{\ln x}}\,dx$ diverges,

then $\sum\limits_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ diverges.

(a) divergent

(b) convergent

(c) neither convergent nor divergent

(d) divergent by the ratio test

(e) convergent by the limit comparison test

22. Applying the **ratio test** to the series $\sum\limits_{n=1}^{\infty} \frac{3\cdot 5\cdot 7\cdot \ldots \cdot (2n+1)}{6^n\cdot n!}$, with $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$, we conclude that:

$a_n = \frac{3\cdot 5\cdot 7 \cdots (2n+1)}{6^n \cdot n!}$

$a_{n+1} = \frac{3\cdot 5\cdot 7\cdots (2n+1)\cdot(2n+3)}{6^{n+1}\cdot (n+1)!}$

$\frac{a_{n+1}}{a_n} = \frac{(2n+3)}{6\cdot(n+1)} = \frac{2n+3}{6n+6}$

$\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{2}{6} = \frac{1}{3} = \rho < 1$

$\Longrightarrow$ Conv.

(a) $\rho = \frac{1}{3}$ and the series is convergent

(b) $\rho = 0$ and the series is convergent

(c) $\rho = \frac{1}{6}$ and the sereis is convergent

(d) $\rho = 1$ and the test is inconclusive

(e) $\rho = 2$ and the sereis is divergent.

23. For some suitable values of x, the **Maclaurin series** for $f(x) = \dfrac{8x}{2-x^2}$ is given by

(a) $\sum_{n=0}^{\infty} \dfrac{x^{2n+1}}{2^{n-2}}$

(b) $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n-1}}{2^{n-1}}$

(c) $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{2^n}$

(d) $\sum_{n=0}^{\infty} \dfrac{8\,x^n}{2^n}$

(e) $\sum_{n=0}^{\infty} \dfrac{x^{2n+1}}{2^{n+1}}$

$$f(x) = 8x \cdot \frac{1}{2(1-\frac{x^2}{2})}$$
$$= 4x \cdot \frac{1}{1-\frac{x^2}{2}}$$
$$= 4x \sum_{n=0}^{\infty} \left(\frac{x^2}{2}\right)^n \quad , \quad \left|\frac{x^2}{2}\right| < 1$$
$$= 4x \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n} \quad , \quad |x|^2 < 2$$
$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2^{n-2}} \quad , \quad |x| < \sqrt{2}$$

24. $\int_0^2 e^{-x^3}\,dx =$

(a) $\sum_{n=0}^{\infty} \dfrac{(-1)^n}{n!} \cdot \dfrac{2^{3n+1}}{3n+1}$

(b) $\sum_{n=0}^{\infty} \dfrac{(-1)^n}{n!} \cdot \dfrac{2^{3n}}{3n}$

(c) $\sum_{n=0}^{\infty} \dfrac{1}{n!} \cdot \dfrac{2^{3n+1}}{3n+1}$

(d) $\sum_{n=0}^{\infty} \dfrac{1}{n!} \cdot \dfrac{8^n}{2n+1}$

(e) $\sum_{n=0}^{\infty} (-1)^n \dfrac{2^{3n}}{(3n)!}$

$$\int_0^2 \sum_{n=0}^{\infty} \frac{(-x^3)^n}{n!}\,dx = \int_0^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{3n}\,dx$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^2 x^{3n}\,dx$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \left.\frac{x^{3n+1}}{3n+1}\right]_0^2$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \frac{2^{3n+1}}{3n+1}$$

25. Using the **binomial series**, if

$$\sqrt{1-2x} = a_0 + a_1\,x + a_2\,x^2 + a_3\,x^3 + \ldots$$

then $a_0 + a_1 + a_2 + a_3 =$

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots$$

$$\sqrt{1-2x} = 1 + \tfrac{1}{2}\cdot(-2x) + \frac{\frac{1}{2}\cdot(\frac{1}{2}-1)}{2}(-2x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{6}(-2x)^3 + \cdots$$

$$= 1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + \cdots$$

$$a_0 + a_1 + a_2 + a_3 = 1 - 1 - \frac{1}{2} - \frac{1}{2} = -1$$

(a) -1

(b) 0

(c) $-\dfrac{3}{2}$

(d) $\dfrac{1}{2}$

(e) 3

26. The **sum** of the series $\sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{n\,3^{n+1}}$ is equal to

(**Hint:** You may use the power series $\ln(1+x) = \sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}\,x^n}{n}$, $-1 < x \le 1$)

$$\ln(1+x) = x + \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

Sub $x = \frac{1}{3} \in (-1, 1]$

$$\ln(1+\tfrac{1}{3}) = \frac{1}{3} - \sum_{n=2}^{\infty} \frac{(-1)^n}{n}\cdot\frac{1}{3^n}$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n}\cdot\frac{1}{3^n} = \frac{1}{3} - \ln(\tfrac{4}{3})$$

$$\frac{1}{3}\cdot\sum_{n=2}^{\infty} \frac{(-1)^n}{n}\cdot\frac{1}{3^n} = \frac{1}{3}\left(\frac{1}{3} - \ln(\tfrac{4}{3})\right)$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n}\cdot\frac{1}{3^{n+1}} = \frac{1}{9} - \frac{1}{3}\ln(\tfrac{4}{3})$$

(a) $\dfrac{1}{9} - \dfrac{1}{3}\ln\left(\dfrac{4}{3}\right)$

(b) $\ln\left(\dfrac{4}{3}\right)$

(c) $-\ln\left(\dfrac{4}{3}\right)$

(d) $\dfrac{1}{3} + \ln\left(\dfrac{4}{3}\right)$

(e) $\dfrac{1}{9} - \dfrac{1}{9}\ln\left(\dfrac{4}{3}\right)$

27. The **interval of convergence** of the power series

$$\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n})\,(x-3)^n = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}(x-3)^n$$

is

$$\frac{a_{n+1}}{a_n} = \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+2}+\sqrt{n+1}}\cdot\frac{(x-3)^{n+1}}{(x-3)^n} = \frac{\sqrt{n+1}}{\sqrt{n+2}}\cdot\frac{1+\frac{\sqrt{n}}{\sqrt{n+1}}}{1+\frac{\sqrt{n+1}}{\sqrt{n+2}}}\cdot(x-3)$$

$$= \sqrt{\frac{n+1}{n+2}}\cdot\frac{1+\sqrt{\frac{n}{n+1}}}{1+\sqrt{\frac{n+1}{n+2}}}\cdot(x-3)$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1\cdot\frac{1+1}{1+1}\cdot|x-3| = |x-3|$$

it conv. if $|x-3|<1 \Rightarrow -1<x-3<1 \Rightarrow \boxed{2<x<4}$

endpts

$\cdot\ x=2$: $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+1}+\sqrt{n}}$ Conv. by alternating Series Test (check)

$\cdot\ x=4$: $\displaystyle\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n})$ Div as $\lim_{n\to\infty} S_n = \infty$

So, Interval of Conv. is $[2,4)$

(a) $[2,4)$

(b) $(2,4)$

(c) $(2,4]$

(d) $(1,5)$

(e) $(1,5]$

28. Which one the following statements is **TRUE?**

✓ (a) If $\sum_{n=1}^{\infty}|a_n|$ is convergent, then $\sum_{n=1}^{\infty}a_n$ is convergent

(b) If $\sum_{n=1}^{\infty}|a_n|$ is divergent, then $\sum_{n=1}^{\infty}a_n$ is divergent $\quad a_n=\frac{(-1)^n}{n}$

(c) If $\sum_{n=1}^{\infty}a_n$ is divergent, then $\sum_{n=1}^{\infty}2a_n$ is convergent $\quad a_n=\frac{1}{n}$

(d) If $\sum_{n=1}^{\infty}a_n$ is convergent, then $\sum_{n=1}^{\infty}(-1)^n a_n$ is convergent $\quad a_n=\frac{(-1)^n}{n}$

(e) If $\sum_{n=1}^{\infty}a_n$ is divergent, then $\sum_{n=1}^{\infty}a_n^2$ is convergent $\quad a_n=\frac{1}{\sqrt{n}}$

Q	MM	V1	V2	V3	V4
1	a	c	e	c	e
2	a	c	d	a	e
3	a	a	a	b	d
4	a	b	b	b	c
5	a	d	e	a	a
6	a	d	b	d	a
7	a	d	a	d	c
8	a	b	a	a	a
9	a	c	d	d	c
10	a	b	d	b	b
11	a	d	d	c	b
12	a	a	c	b	e
13	a	c	b	c	d
14	a	b	c	b	a
15	a	d	a	d	d
16	a	d	d	a	b
17	a	a	d	c	a
18	a	a	d	c	c
19	a	a	c	a	e
20	a	d	d	a	b
21	a	d	d	a	a
22	a	e	a	c	c
23	a	b	a	e	c
24	a	c	a	e	d
25	a	a	d	c	c
26	a	e	c	d	c
27	a	a	b	b	e
28	a	b	a	a	e