Q	MM	V1	V2	V3	V4
1	a	a	a	b	a
2	a	d	c	a	c
3	a	c	c	a	e
4	a	a	a	a	d
5	a	e	b	c	d
6	a	b	e	c	e
7	a	d	a	d	b
8	a	d	c	e	c
9	a	c	c	a	c
10	a	b	e	a	c
11	a	c	a	e	e
12	a	e	e	e	a
13	a	e	a	e	b
14	a	c	b	b	e
15	a	a	d	d	c
16	a	c	d	b	c
17	a	c	d	b	c
18	a	e	e	e	d
19	a	b	c	b	b
20	a	d	d	a	d

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 141
Tuesday 21/10/2014
Net Time Allowed: 120 minutes

MASTER VERSION

1. If P is a partition of $[1,3]$, then

$$\lim_{\|p\|\to 0} \sum_{k=1}^{n} (c_k \tan c_k) \triangle x_k =$$

(a) $\int_1^3 x \tan x \, dx$

(b) $\int_1^3 \tan x \, dx$

(c) $\int_1^3 x \, dx$

(d) $\int_0^2 x \tan x \, dx$

(e) $\int_0^2 \tan x \, dx$

2. Suppose f and g are integrable functions and that

$$\int_1^3 f(x)dx = 2, \ \int_1^{10} f(x)dx = 3, \ \int_1^3 g(x)dx = -4.$$

Which one of the following statements is **FALSE:**

(a) $\int_1^3 f(x)g(x) \, dx = -8$

(b) $\int_{10}^1 f(x) \, dx = -3$

(c) $\int_1^3 2f(x) \, dx = 4$

(d) $\int_1^3 [f(x) - g(x)] \, dx = 6$

(e) $\int_3^{10} f(x) \, dx = 1$

3. $\int_1^4 \left(\frac{1}{2\sqrt{x}} - e^{-x} \right) dx =$

$\sqrt{x} + e^{-x} \Big|_1^4$

$= (2 + e^{-4}) - (1 + e^{-1})$

$= 1 + \frac{1}{e^4} - \frac{1}{e}$

(a) $1 + \frac{1}{e^4} - \frac{1}{e}$

(b) $\frac{1}{2} - \frac{1}{e^2}$

(c) $2 + \frac{1}{e^4}$

(d) $\frac{1}{e} - \frac{1}{e^4}$

(e) $1 - \frac{1}{e}$

4. The **area** of the region enclosed by the curves $y = \frac{1}{x}, y = 0, x = -3, x = -2$, is equal to

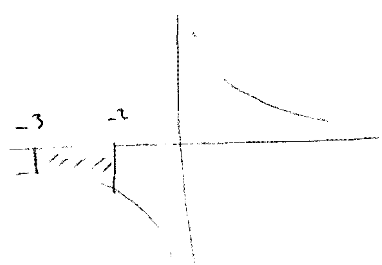
(a) $\ln(1.5)$

(b) $\ln 2$

(c) 1

(d) 2.5

(e) $-\ln 3$



$A = \int_{-3}^{-2} \left(0 - \frac{1}{x}\right) dx$

$= -\ln|x| \Big|_{-3}^{-2}$

$= -(\ln 2 - \ln 3)$

$= \ln 3 - \ln 2$

$= \ln\left(\frac{3}{2}\right)$

$= \ln(1.5)$

5. If $g(x) = \int_{-x}^{x} (t \tan^{-1} t)\, dt$, then $g'(1) =$

$g'(x) = (x \tan^{-1} x) \cdot 1 - [(-x \tan^{-1}(-x)) \cdot (-1)]$

$= x \tan^{-1} x + x \tan^{-1} x$

$= 2x \tan^{-1} x$

$g'(1) = 2(1) \tan^{-1}(1) = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$

(a) $\frac{\pi}{2}$

(b) 0

(c) $\frac{\pi}{4}$

(d) $-\frac{\pi}{2}$

(e) $-\frac{\pi}{4}$

6. The **average value** of $f(x) = \sqrt{16\pi^2 - x^2}$ on $[0, 4\pi]$ is (**Hint:** you may use a known area).

$f_{ave} = \frac{1}{4\pi} \int_0^{4\pi} \sqrt{16\pi^2 - x^2}\, dx$

$= \frac{1}{4\pi} \cdot \frac{1}{4} \pi (16\pi^2)$

$= \pi^2$

(a) π^2

(b) 4π

(c) $\frac{\pi}{4}$

(d) π^3

(e) $2\pi^2$

7. $\displaystyle\int \frac{1+\sin^3\theta}{\sec\theta}\, d\theta =$ $\displaystyle\int \left(\cos\theta + \sin^3\theta \cos\theta\right) d\theta$

$= \sin\theta + \frac{1}{4}\sin^4\theta + C$

(a) $\sin\theta + \dfrac{1}{4}\sin^4\theta + C$

(b) $\ln|\sec\theta + \tan\theta| + C$

(c) $\cos\theta - \dfrac{1}{4}\sin^4\theta + C$

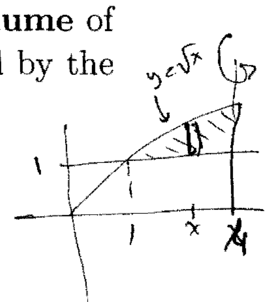
(d) $1 + \cos^2\theta + C$

(e) $-\sin\theta - \dfrac{1}{3}\sin^3\theta + C$

8. Using the **method of cylindrical shells**, the **volume** of the solid generated by rotating the region enclosed by the curves

$y = \sqrt{x}$, $y = 1$, $x = 4$

about the line $x = 4$ is given by



$\displaystyle\int_1^4 2\pi\,(4-x)(\sqrt{x}-1)\,dx$

(a) $2\pi \int_1^4 (4-x)(\sqrt{x}-1)\, dx$

(b) $2\pi \int_1^4 x(\sqrt{x}-1)\, dx$

(c) $2\pi \int_1^4 (4-x)\sqrt{x}\, dx$

(d) $2\pi \int_1^4 x\sqrt{x}\, dx$

(e) $2\pi \int_1^4 (x-1)(\sqrt{x}-4)\, dx$

9. $\int_{\pi/2}^{2\pi} |5\cos t|\, dt =$ $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -5\cos t\, dt + \int_{\frac{3\pi}{2}}^{2\pi} 5\cos t\, dt$

   (a) 15

   (b) -3

   (c) -7

   (d) 9

   (e) 12

$= 5\left[-\sin t \Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \sin t \Big]_{\frac{3\pi}{2}}^{2\pi} \right]$

$5\left[-(-1-1) + (0-(-1)) \right]$

$5\left[2 + 1 \right] = 15$

10. The **volume** of the solid generated by rotating the region bounded by the curves $y = x^2$ and $y = 1$ about the x-axis is

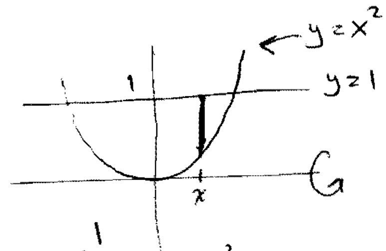
   (a) $\dfrac{8\pi}{5}$

   (b) $\dfrac{4\pi}{3}$

   (c) $\dfrac{8\pi}{3}$

   (d) $\dfrac{4\pi}{5}$

   (e) 2π



$V = \pi \int_{-1}^{1} \left(1^2 - (x^2)^2\right) dx$

$= \pi \int_{-1}^{1} (1 - x^4)\, dx$

$= \pi \cdot x - \frac{1}{5}x^5 \Big]_{-1}^{1}$

$= \pi \left[\left(1 - \frac{1}{5}\right) - \left(-1 + \frac{1}{5}\right) \right]$

$= \pi \left(\frac{4}{5} - \left(-\frac{4}{5}\right) \right)$

$= \pi \cdot \frac{8}{5}$

11. The **length** of the curve

$$y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt, \quad -2 \le x \le -1$$

is

(a) $\frac{7\sqrt{3}}{3}$

(b) $\frac{\sqrt{3}}{3}$

(c) 3

(d) $2\sqrt{3}$

(e) $\frac{3\sqrt{3}}{4}$

$$y' = \sqrt{3x^4 - 1}$$

$$1 + (y')^2 = 1 + (3x^4 - 1) = 3x^4$$

$$L = \int_{-2}^{-1} \sqrt{1 + (y')^2}\, dx = \int_{-2}^{-1} \sqrt{3}\sqrt{x^4}\, dx$$

$$= \sqrt{3} \int_{-2}^{-1} |x^2|\, dx = \sqrt{3} \int_{-2}^{-1} x^2\, dx$$

$$= \sqrt{3} \cdot \frac{1}{3} x^3 \Big]_{-2}^{-1}$$

$$= \frac{\sqrt{3}}{3} \left(-1 - (-8)\right) = \frac{7\sqrt{3}}{3}$$

12. The **area** of the region bounded by the curves $y = \sqrt{x+1}$ and $y = \frac{x+1}{2}$ is equal to

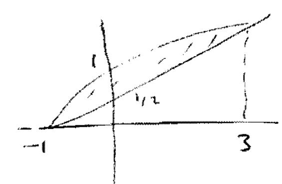
(a) $\frac{4}{3}$

(b) $\frac{9}{5}$

(c) $2 - \sqrt{3}$

(d) $\frac{5}{6}$

(e) $\frac{8}{5}$



$$A = \int_{-1}^{3} \sqrt{x+1} - \frac{1}{2}(x+1)\, dx$$

$$= \frac{2}{3}(x+1)^{3/2} - \frac{1}{4}x^2 - \frac{1}{2}x \Big]_{-1}^{3}$$

$$= \left(\frac{2}{3} 4^{3/2} - \frac{9}{4} - \frac{3}{2}\right) - \left(0 - \frac{1}{4} + \frac{1}{2}\right)$$

$$= \frac{2}{3} \cdot 8 - \frac{9}{4} - \frac{6}{4} - \frac{1}{4}$$

$$= \frac{16}{3} - \frac{16}{4} = 16\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= 16\left(\frac{1}{12}\right)$$

$$= \frac{4}{3}$$

13. $\displaystyle\int \sqrt{\frac{x^3-1}{x^{11}}}\,dx =$

$$\int \sqrt{\frac{x^3-1}{x^3\cdot x^8}}\,dx = \int \frac{1}{x^4}\sqrt{1-\frac{1}{x^3}}\,dx$$

$u = 1 - x^{-3} \Rightarrow du = 3x^{-4}\,dx$

$= \frac{1}{3}\int \sqrt{u}\,du$

$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{9}\left(1-\frac{1}{x^3}\right)^{3/2} + C$

(a) $\frac{2}{9}\left(1-\frac{1}{x^3}\right)^{3/2} + C$

(b) $\frac{1}{9}\left(1-\frac{1}{x^4}\right)^{1/2} + C$

(c) $\frac{9}{2}\left(1-\frac{1}{x^3}\right)^{3/2} + C$

(d) $\frac{2}{9}\left(1-\frac{1}{x^3}\right)^{1/2} + C$

(e) $\frac{4}{9}\left(1-\frac{1}{x^3}\right)^{3/2} + C$

14. $\displaystyle\int \frac{2\tan^2 x\sec^2 x}{(3+\tan^3 x)^3}\,dx =$

$u = 3 + \tan^3 x \Rightarrow du = 3\tan^2 x \cdot \sec^2 x\,dx$

$\frac{2}{3}\int \frac{1}{u^3}\,du$

$\frac{2}{3}\int u^{-3}\,du$

$\frac{2}{3}\;\frac{u^{-2}}{-2} + C$

$-\frac{1}{3}\;\frac{1}{u^2} + C$

$-\frac{1}{3}\cdot\frac{1}{(3+\tan^3 x)^2} + C$

(a) $-\frac{1}{3}\cdot\frac{1}{(3+\tan^3 x)^2} + C$

(b) $\frac{2}{3}\cdot\frac{1}{(3+\tan^3 x)^2} + C$

(c) $-\frac{3}{4}\cdot\frac{1}{3+\tan^3 x} + C$

(d) $\frac{1}{(3+\tan^3 x)^3} + C$

(e) $\frac{1}{3}\cdot\frac{\sec x}{(3+\tan^3 x)^3} + C$

15. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\sin x}{10x^2 + \cos x}\, dx =$

~~an odd~~ an odd function on a symmetric interval. $\to 0$

(a) 0

(b) 2π

(c) $2 \ln 10$

(d) 1

(e) $\pi + \ln 2$

16. The **area** of the region enclosed by the curve $y = x^3 - 5x^2 + 4x$ and the x-axis between $x = 0$ and $x = 2$ is

$x(x^2 - 5x + 4) = x(x-1)(x-4) = 0$

$x = 0, 1, 4$

+ −

0 1 4

(a) $\frac{5}{2}$

(b) $\frac{3}{4}$

(c) $-\frac{1}{6}$

(d) $\frac{2}{7}$

(e) $\frac{13}{8}$

$$\int_0^1 x^3 - 5x^2 + 4x \, dx - \int_1^2 x^3 - 5x^2 + 4x \, dx$$

$$\left.\frac{1}{4}x^4 - \frac{5}{3}x^3 + 2x^2\right]_0^1 - \left.\frac{x^4}{4} - \frac{5}{3}x^3 + 2x^2\right]_1^2$$

$$\left(\frac{1}{4} - \frac{5}{3} + 2\right) - \left[\left(4 - \frac{40}{3} + 8\right) - \left(\frac{1}{4} - \frac{5}{3} + 2\right)\right]$$

$$\frac{3 - 20 + 24}{12} - \left[\left(12 - \frac{40}{3}\right) - \left(\frac{-17}{12} + 2\right)\right]$$

$$\frac{7}{12} - \left[10 - \frac{160}{12} + \frac{17}{12}\right]$$

$$\frac{7}{12} \underbrace{- \frac{120}{12} + \frac{160}{12}}_{\cancel{\frac{40}{12} = \frac{10}{3}}} - \frac{17}{12} = \frac{40}{12} - \frac{10}{12} = \frac{30}{12} = \frac{15}{6} = \frac{5}{2}$$

17. $\int \frac{x^2+4x+3}{(2+x)^3-(2+x)^2}\,dx =$ $\int \frac{(x+1)(x+3)}{(2+x)^2(2+x-1)}\,dx$ Factor

$= \int \frac{\cancel{(x+1)}(x+3)}{(2+x)^2\cancel{(1+x)}}\,dx$

$= \int \frac{x+3}{(2+x)^2}\,dx$

$= \int \frac{1}{x+2} + \frac{1}{(2+x)^2}\,dx$

$= \ln|x+2| - \frac{1}{2+x} + C$

(a) $\ln|x+2| - \dfrac{1}{x+2} + C$

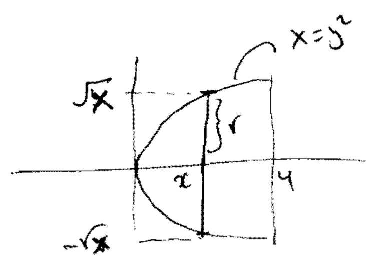
(b) $\dfrac{x^2+3x}{x+2} + C$

(c) $\dfrac{x+1}{(x+2)^3} + C$

(d) $2\ln|x+2| - (x+2)^2 + C$

(e) $\dfrac{x}{x+3} - \ln|x| + C$

18. The base of a solid is bounded by the curves $x = y^2$ and $x = 4$. If the **cross sections** of the solid, perpendicular to the x-axis, are **semicircles**, then the volume of the solid is



$A(x) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(\sqrt{x})^2 = \frac{1}{2}\pi x$

$V = \int_0^4 A(x)\,dx = \frac{\pi}{2}\int_0^4 x\,dx$

$= \frac{\pi}{2}\cdot \frac{1}{2}x^2\Big]_0^4$

$= \frac{\pi}{4}\cdot 4^2 = 4\pi$

(a) 4π

(b) π

(c) 8π

(d) $\dfrac{\pi}{2}$

(e) $\dfrac{3\pi}{13}$

19. The **area of the surface** generated by revolving the curve $x = \dfrac{e^y + e^{-y}}{2}$, $0 \le y \le \ln 2$, about the y-axis is

$S = \int_0^{\ln 2} 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^y + e^{-y}) \cdot \sqrt{1 + \left(\frac{e^y - e^{-y}}{2}\right)^2}\, dy$

$\sqrt{1 + \frac{e^{2y} - 2 + e^{-2y}}{4}}$

$\sqrt{\frac{e^{2y} + 2 + e^{-2y}}{4}}$

$\sqrt{\left(\frac{e^y + e^{-y}}{2}\right)^2}$

(a) $\pi\left(\dfrac{15}{16} + \ln 2\right)$

(b) $\dfrac{\pi}{2}(15 + 2\ln 2)$

(c) $2\pi(1 + \ln 2)$

(d) $\pi\left(\dfrac{7}{8} + \ln 4\right)$

(e) $\dfrac{9}{4}(\pi + \ln 2)$

$= 2\pi \int_0^{\ln 2} \frac{e^y + e^{-y}}{2} \cdot \frac{e^y + e^{-y}}{2}$

$= \frac{\pi}{2} \int_0^{\ln 2} e^{2y} + 2 + e^{-2y}\, dy$

$= \frac{\pi}{2}\left[\frac{1}{2}e^{2y} + 2y - \frac{1}{2}e^{-2y}\right]_0^{\ln 2}$

$= \frac{\pi}{2}\left[\left(\frac{4}{2} + 2\ln 2 - \frac{1}{8}\right) - \left(\cancel{\frac{1}{2} + 0 - \frac{1}{2}}\right)\right]$

$= \frac{\pi}{2}\left[\frac{15}{8} + 2\ln 2\right] = \pi\left(\frac{15}{16} + \ln 2\right)$

20. $\displaystyle\int \frac{e^{\sin^2(\ln x)} \cdot \sin(2\ln x)}{x e^{\cos^2(\ln x)}}\, dx =$

(a) $\dfrac{1}{2e^{\cos(2\ln x)}} + C$

(b) $\dfrac{-1}{2e^{\cos(2\ln x)}} + C$

(c) $\dfrac{1}{2e^{\cos(\ln x)}} + C$

(d) $\dfrac{1}{e^{\sin(2\ln x)}} + C$

(e) $\dfrac{-\sin x}{e^{\cos(\ln x)}} + C$

$\int \frac{1}{x} e^{\sin^2(\ln x) - \cos^2(\ln x)} \cdot \sin(2\ln x)\, dx$, $\sin^2(\ln x) - \cos^2(\ln x) = -\cos(2\ln x)$

$u = -\cos(2\ln x) \Rightarrow du = \sin(2\ln x) \cdot \frac{2}{x}$

$\frac{1}{2}\int e^u\, du$

$\frac{1}{2} e^u + C$

$\frac{1}{2} e^{-\cos(2\ln x)} + C$ ✓