1] Multiple Choice Questions (each 1 point)

1] What is the average value of $f(x) = xe^{x^2}$ on the interval [2,4]?

a) $\frac{e^{16} - e^4}{4}$

b) $\frac{e^{16} - e^4}{2}$

c) $\frac{2e^{16} - e^4}{2}$

d) $2e^{16} - e^4$

e) $2e^{16} + e^4$

$$f_{av} = \frac{1}{2}\int_2^4 xe^{x^2}dx = \frac{1}{4}\int_4^{16} e^u du$$
$$= \frac{1}{4}[e^{16} - e^4]$$

2] If $\int_a^c f(x)dx = 6$, $\int_a^b f(x)dx = 3$, $\int_b^d g(x)dx = -3$ and $\int_c^d g(x)dx = 7$ then the value of :

$\int_b^c (3f(x) - 4g(x))dx =$

a) -31
b) -19
c) 11
d) 30
e) 49

$$\int_b^c f(x)dx = \int_b^a f(x)dx + \int_a^c f(x)dx = -3 + 6 = 3$$
$$\int_b^c g(x)dx = \int_b^d g(x)dx + \int_d^c g(x)dx = -3 + (-7) = -10$$
$$\int_b^c [3f(x) - 4g(x)]dx = 3\int_b^c f(x)dx - 4\int_b^c g(x)dx = 3(3) - 4(-10) = 49$$

3] $\int_0^3 \frac{x}{\sqrt{x+1}}dx =$

a) $\frac{3}{8}$

b) $\frac{2}{3}$

c) $\frac{3}{2}$

d) $\frac{9}{4}$

e) $\frac{8}{3}$

$$u = x + 1 \Rightarrow du = dx \quad ; x = u - 1$$
$$If \;\; x = 0 \Rightarrow u = 1$$
$$If \;\; x = 3 \Rightarrow u = 4$$
$$\int_0^3 \frac{x}{\sqrt{x+1}}dx = \int_1^4 \frac{u-1}{\sqrt{u}}du = \int_1^4 (u^{1/2} - u^{-1/2})du$$
$$= \frac{2}{3}u^{3/2} - 2u^{1/2}]_1^4 = (\frac{16}{3} - 4) - (\frac{2}{3} - 2)$$
$$\frac{14}{3} - 2 = \frac{8}{3}$$

4] If $F(x)=\int_{\sin x}^{\cos x}\frac{1}{1-t^2}dt,\ 0<x<\pi/2$, then $F'(x)$ is equal to:

a) $-(\sec x+\csc x)$
b) $\sec x+\csc x$
c) $\sec^2 x-\csc^2 x$
d) $\csc^2 x-\sec^2 x$
e) $\sec^2 x+\csc^2 x$

$$F'(x)=\frac{1}{1-\cos^2 x}(-\sin x)-\frac{1}{1-\sin^2 x}(\cos x)$$
$$F'(x)=\frac{1}{\sin^2 x}(-\sin x)-\frac{1}{\cos^2 x}(\cos x)$$
$$F'(x)=-\csc x-\sec x$$

5] If $\int_0^x f(t)dt=\sqrt{x^2+1}-1$; then $f(x)=$

a) $\frac{1}{\sqrt{x^2+1}}$
b) $\frac{1}{2\sqrt{x^2+1}}$
c) $\frac{x}{\sqrt{x^2+1}}$
d) $x\sqrt{x^2+1}$
e) $\frac{x}{2\sqrt{x^2+1}}$

$$\frac{d}{dx}\int_0^x f(t)dt=\frac{d}{dx}(\sqrt{x^2+1}-1)$$
$$f(x)=\frac{\not{2}x}{\not{2}\sqrt{x^2+1}}$$

6] If $\int_{-1}^{4} f(x)dx=3$, then $\int_0^1 xf(5x^2-1)dx=$

a) 3
b) 3/10
c) 10/3
d) 30
e) 15

$$u=5x^2-1\Rightarrow du=10xdx$$
$$x=0\Rightarrow u=-1;x=1\Rightarrow u=4$$
$$\int_0^1 xf(5x^2-1)dx=\int_{-1}^{4} f(u)\frac{du}{10}=\frac{3}{10}$$

2] Do the following integrations. (each 1.5 points)

1] $\int \frac{\sin x + \cos x}{e^{-x} + \sin x} dx$

$$\int \frac{\sin x + \cos x}{e^{-x} + \sin x} dx \bullet \frac{e^x}{e^x} = \int \frac{e^x \sin x + e^x \cos x}{1 + e^x \sin x} dx$$

$$Let \;\; u = 1 + e^x \sin x \Rightarrow du = e^x \sin x + e^x \cos x$$

$$\int \frac{e^x \sin x + e^x \cos x}{1 + e^x \sin x} dx = \int \frac{du}{u} = \ln|u| + c$$

$$= \ln\left|1 + e^x \sin x\right| + c$$

2] $\int e^{-x} \cdot 3^{2x} dx$

$$\int e^{-x} \cdot 3^{2x} dx = \int \frac{3^{2x}}{e^x} dx = \int \left(\frac{9}{e}\right)^x dx$$

$$= \frac{\left(\frac{9}{e}\right)^x}{\ln \frac{9}{e}} = \frac{\left(\frac{9}{e}\right)^x}{\ln 9 - \ln e}$$

3] $\int \frac{dx}{x\sqrt{x^6 - 4}}$

$$\int \frac{dx}{x\sqrt{x^6 - 4}} = \int \frac{dx}{x\sqrt{(x^3)^2 - 4}}$$

$$u = x^3 \Rightarrow du = 3x^2 dx \Rightarrow dx = \frac{du}{3x^2}$$

$$\int \frac{dx}{x\sqrt{(x^3)^2 - 4}} = \int \frac{\frac{du}{3x^2}}{x\sqrt{(u)^2 - 4}} = \int \frac{du}{3x^3\sqrt{(u)^2 - 4}}$$

$$\frac{1}{3}\int \frac{du}{u\sqrt{(u)^2 - 4}} = \frac{1}{6}\sec^{-1}\left|\frac{u}{2}\right| + c = \frac{1}{6}\sec^{-1}\left|\frac{x^2}{2}\right| + c$$

4] $\int_0^1 \sqrt{1-\sqrt{x}}\,dx$

$$u = 1-\sqrt{x} \Rightarrow du = \frac{-dx}{2\sqrt{x}} \;;\sqrt{x} = 1-u$$

$$dx = -2(1-u)du$$

$$x = 0 \Rightarrow u = 1$$

$$x = 1 \Rightarrow u = 0$$

$$\int_0^1 \sqrt{1-\sqrt{x}}\,dx = \int_1^0 \sqrt{u}[-2(1-u)du]$$

$$2\int_0^1 (u^{1/2} - u^{3/2})du = 2[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}]_0^1$$

$$2[\frac{2}{3} - \frac{2}{5}] = \frac{8}{15}$$

5] $\int \frac{\sin x}{1+\sin x}dx$

$$\int \frac{\sin x}{1+\sin x}dx = \int \frac{\sin x}{1+\sin x}\frac{1-\sin x}{1-\sin x}dx$$

$$\int \frac{\sin x - \sin^2 x}{1-\sin^2 x}dx$$

$$\int \frac{\sin x - \sin^2 x}{\cos^2 x}dx = \int \frac{\sin x}{\cos^2 x}dx - \int \frac{\sin^2 x}{\cos^2 x}dx$$

$$= \int \tan x \sec x dx - \int \tan^2 x dx$$

$$= \sec x - \tan x + x + c$$

6] $\int \frac{dx}{x\sqrt{2+\ln x}}$

$$u = 2+\ln x \Rightarrow du = \frac{dx}{x}$$

$$\int \frac{dx}{x\sqrt{2+\ln x}} = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + c$$

$$= 2\sqrt{2+\ln x} + c$$

3] Solve the differential equation $\frac{d^2y}{dx^2} = \frac{3\sqrt{x}}{2} + \frac{2}{x^3} + 1, \quad y'(1) = 2, \quad y(1) = \frac{19}{10}$ (1.5 points)

$$\frac{d^2y}{dx^2} = \frac{3\sqrt{x}}{2} + \frac{2}{x^3} + 1, \quad y'(1) = 2, \quad y(1) = \frac{19}{10}$$

$$\frac{dy}{dx} = \int (\frac{3}{2}x^{1/2} + 2x^{-3} + 1)dx = x^{3/2} - x^{-2} + x + c$$

$$2 = 1 - 1 + 1 + c \to c = 1$$

$$y = \int (x^{3/2} - x^{-2} + x + 1)dx = \frac{2}{5}x^{5/2} + x^{-1} + \frac{1}{2}x^2 + x + c$$

$$\frac{19}{10} = \frac{2}{5} + \frac{1}{2} + 1 + 1 + c \Rightarrow c = \frac{19}{10} - \frac{29}{10} = -1$$

$$y = \frac{2}{5}x^{5/2} + x^{-1} + \frac{1}{2}x^2 + x - 1$$

4] (1.5 points) If P denote any partition of [-3,3] determined by the points $-3 < x_1 < x_2 < ... < x_{n-1} < 3$, $\Delta x_k = x_k - x_{k+1}$ and $x_k^* \in [x_{k-1}, x_k]$. Then find

$\lim_{Max.\Delta x \to 0} \sum_{k=1}^{n} [\sqrt{9 - (x_k^*)^2} \Delta x_k$ (Hint use a formula from geometry)

$$\lim_{Max.\Delta x \to 0} \sum_{k=1}^{n} [\sqrt{9 - (x_k^*)^2} \Delta x_k = \int_{-3}^{3} \sqrt{9 - x^2} dx = \frac{9\pi}{2}$$

(Bonus Question 1 point)

Evaluate $\int_0^{\pi/2} |\sin x - \cos x| dx$