Properties of Graphs

X-intercepts	Inflection points
Y-intercepts	Concavity
Relative extrema	Increasing and decreasing

periodicity	Trig functions
asymptotes	Rational functions

Examples

Polynomials	
Rational functions	
Functions involve radicals or fractional exponents	

Procedure for analyzing graphs

1-Find x-int , y-int
2- symm , period , domain restriction
3- Behavior x → inf (asymptotes)
4- Calculate f' and f'' study the sign (inc , dec , extrem , concave , inflection)

Analysis of Polynomials

1- Domain (-inf ,inf)
2- Range : even , odd
3- Behavior x → inf (asymptotes)
4- Calculate f' and f'' study the sign (inc , dec , extrem , concave , inflection)
5- cont (-inf , inf)
6- no corner , no vertical tangent
7- poly → + - inf as x→ + - inf
8- poly with deg n * at most n x-int at most n-1 rel. Ext at most n-2 reflec. points

Analysis of Polynomials

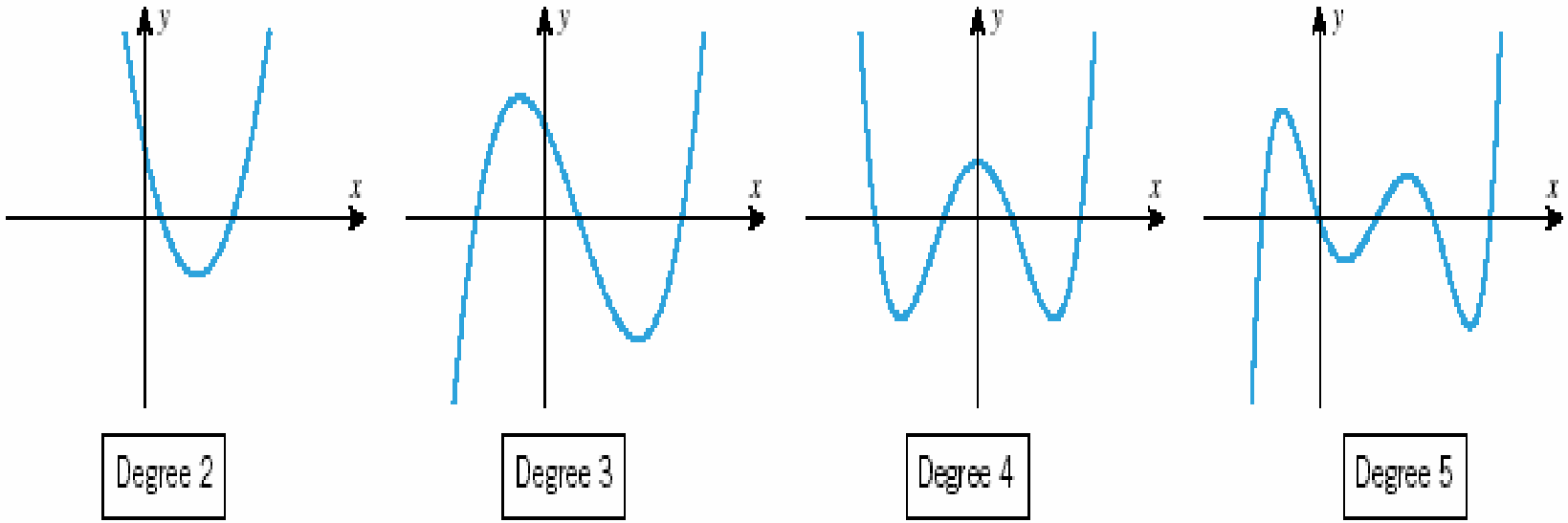
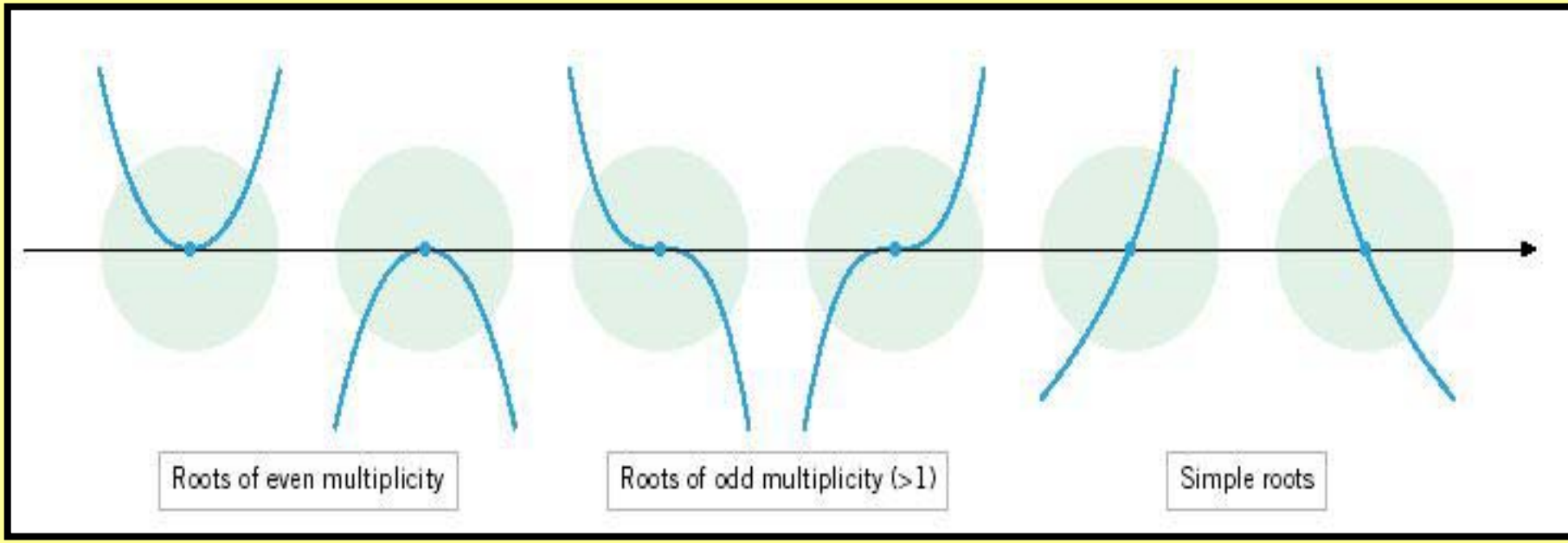


Figure 5.3.1

Multiplicity

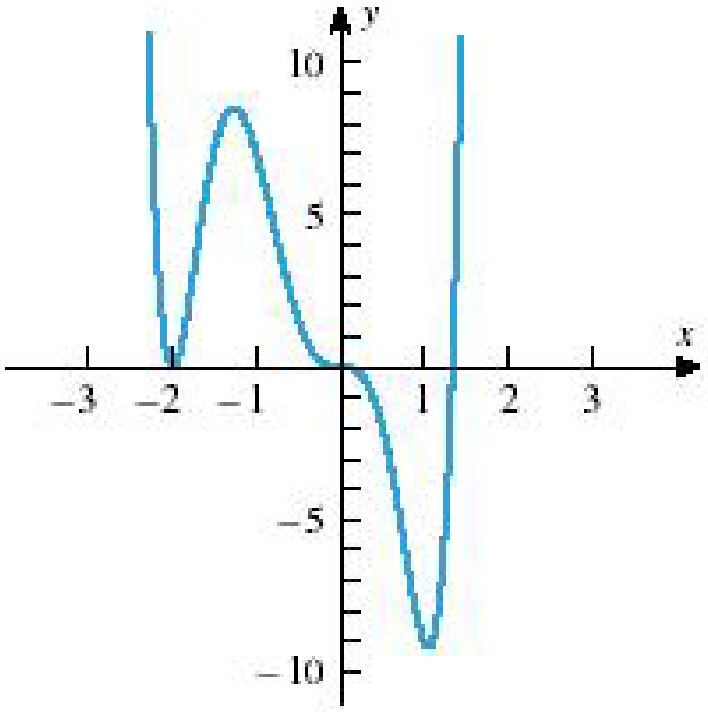
$$f(x) = x^3(3x - 4)(x + 2)^2$$

X= 0 multi 3 X=-2 multi 2 x= 4/3 multi 1 (simple)



Examples

$$f(x) = x^3(3x-4)(x+2)^2$$

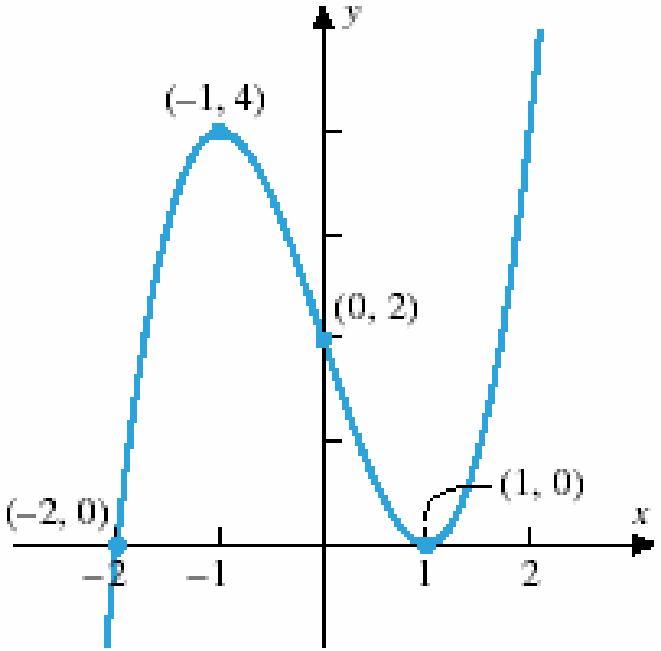


$y = x^3(3x-4)(x+2)^2$

Figure 5.3.4

$$f(x) = x^3 - 3x + 2$$

$$= (x+2)(x-1)^2$$



$y = x^3 - 3x + 2$

Figure 5.3.7

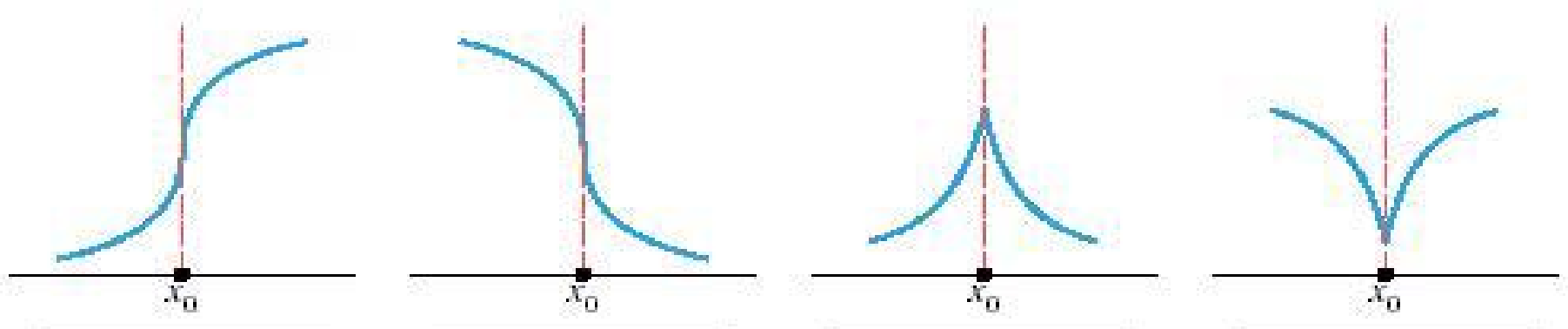
Graphs with vertical Tangents and CUSPS

. f has a **vertical tangent** line at c

* f cont at c
* Lim f ' (x) = + inf or - inf as x → c+ and x→ c-

. f has a **cuspt** at c

* f cont at c
* Lim f ' (x) = + inf as x → c+ and x→ c- (-inf)

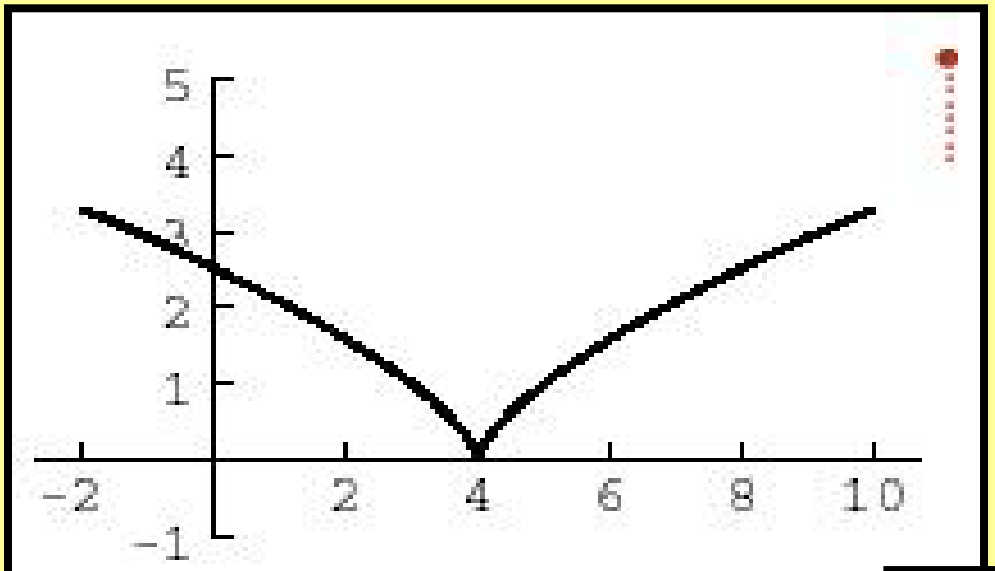


(a) $\lim_{x \to x_0^+} f'(x) = +\infty$, $\lim_{x \to x_0^-} f'(x) = +\infty$

(b) $\lim_{x \to x_0^+} f'(x) = -\infty$, $\lim_{x \to x_0^-} f'(x) = -\infty$

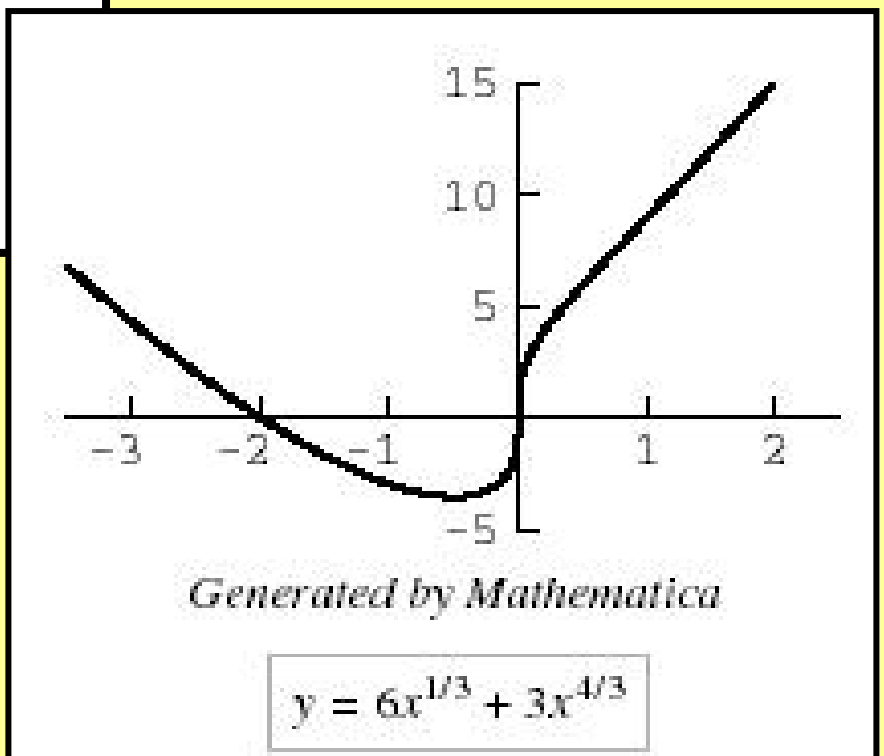
(c) $\lim_{x \to x_0^+} f'(x) = -\infty$, $\lim_{x \to x_0^-} f'(x) = +\infty$

(d) $\lim_{x \to x_0^+} f'(x) = +\infty$, $\lim_{x \to x_0^-} f'(x) = -\infty$



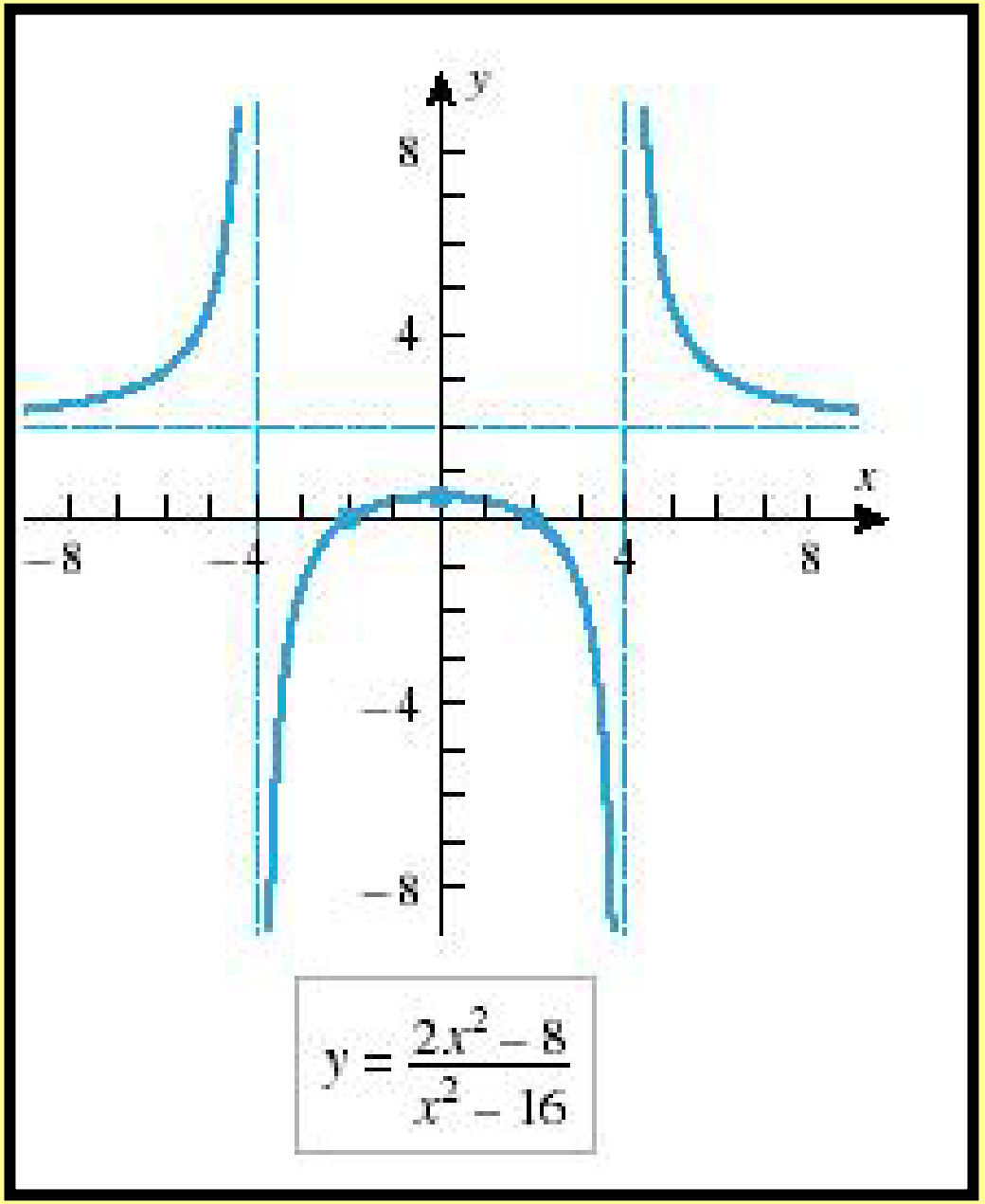
Generated by Mathematica

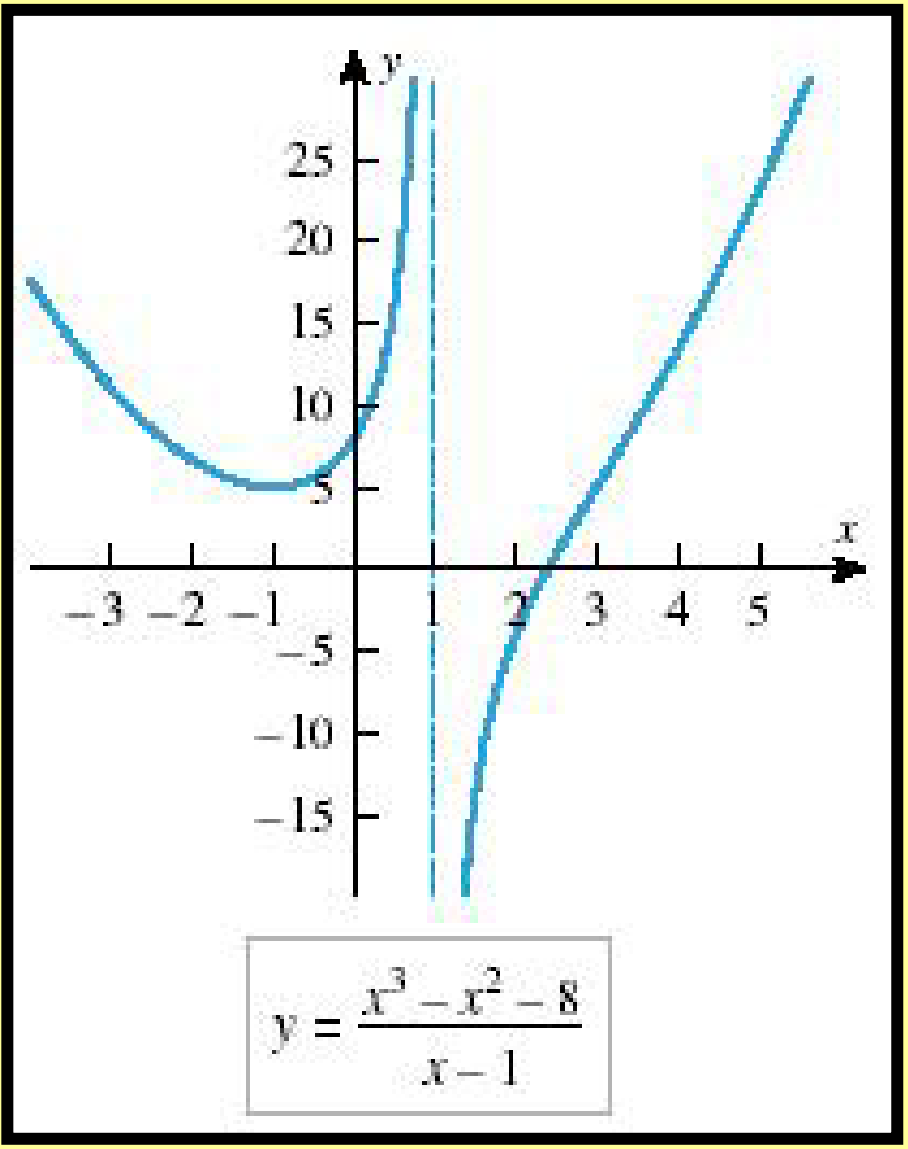
$$y = (x-4)^{2/3}$$

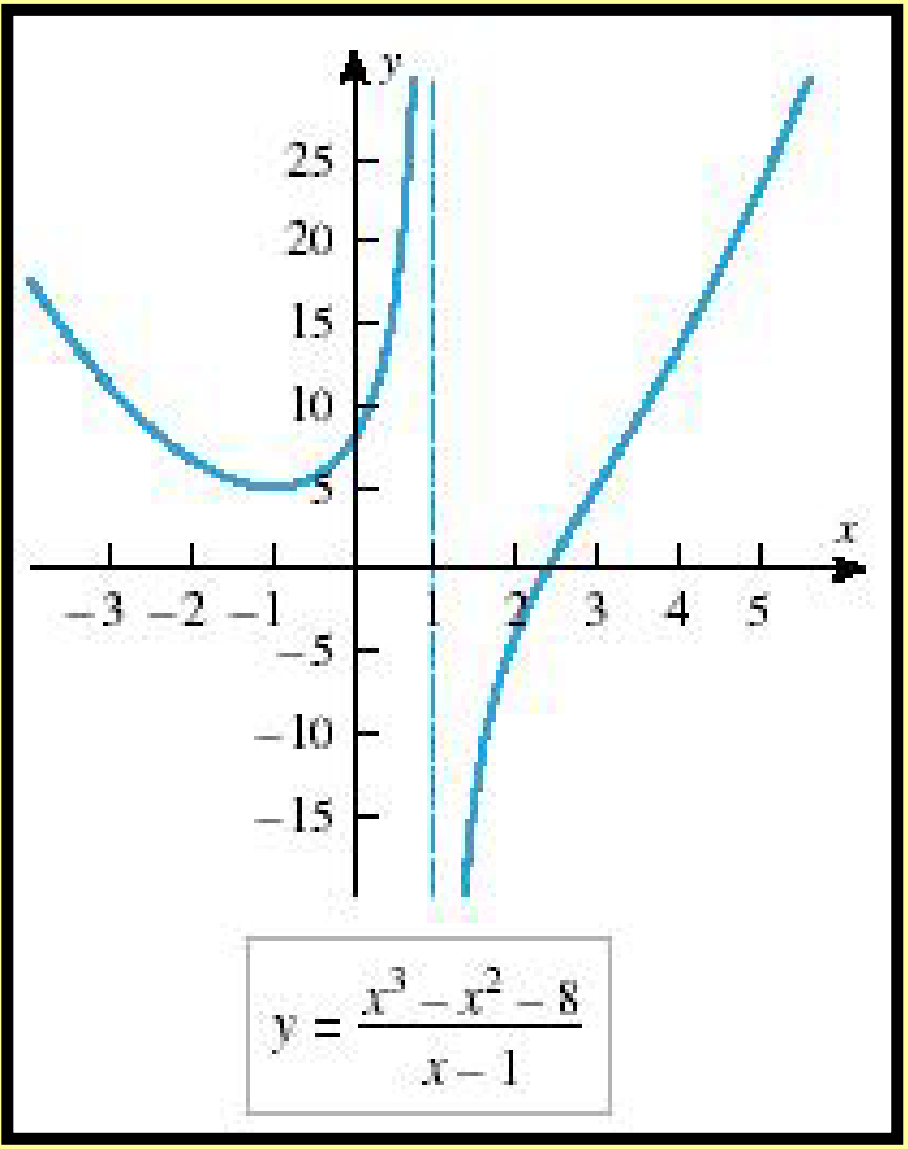


Generated by Mathematica

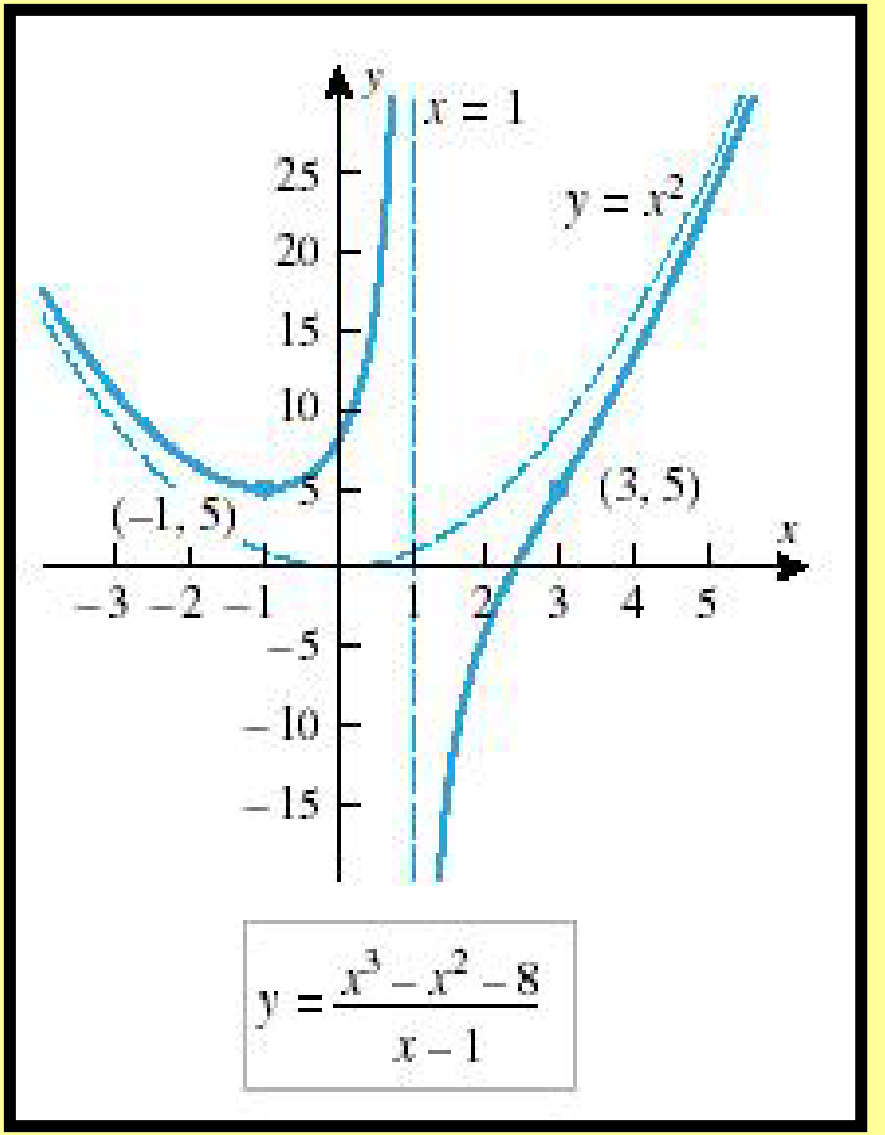
$$y = 6x^{1/3} + 3x^{4/3}$$


y
8
4
x
−8
−4
4
8
−4
−8
$y = \frac{2x^2 - 8}{x^2 - 16}$


y
x
25
20
15
10
5
−5
−10
−15
−3
−2
−1
1
2
3
4
5
$y = \frac{x^3 - x^2 - 8}{x - 1}$



$$y = \frac{x^3 - x^2 - 8}{x - 1}$$



$$y = \frac{x^3 - x^2 - 8}{x - 1}$$