College of Sciences	King Fahd University of Petroleum & Minerals	Department of Mathematical Sciences	
Name: KEY	**Quiz-5** **MATH 101**	**ID:** KEY	SEC: **11** **28**

(Success is a Journey). Say a prayer and START

1) (15)

Find $\lim_{x\to+\infty} \frac{\ln(x)}{x}$ of type ∞/∞ (3)

(4) Apply L'H

$$\lim_{x\to+\infty} \frac{\ln(x)}{x} = \lim_{x\to+\infty} \frac{1/x}{1} = \lim_{x\to+\infty} \frac{1}{x} = 0$$ (8)

2) (15)

Find $\lim_{x\to 0}(e^x + x)^{\frac{1}{x}}$ of type 1^{∞}

Let $y = \lim_{x\to 0} (e^x + x)^{1/x}$ (2)

take ln $\ln y = \lim_{x\to 0} \left[\frac{1}{x} \ln(e^x + x)\right]$ (2)

$= \lim_{x\to 0} \left[\frac{\ln(e^x+x)}{x}\right]$ of type $\frac{0}{0}$ Apply L'H

$= \lim_{x\to 0} \left[\frac{\frac{e^x+1}{e^x+x}}{1}\right] = \lim_{x\to 0} \frac{e^x+1}{e^x+x}$ (2)

$= \frac{2}{1} = 2 \Rightarrow \boxed{y = e^2}$

Hence, $\lim_{x\to 0} (e^x+x)^{1/x} = e^2$ (4)

20

3) Find all relative extrema and all inflection points .

$f(x)=\dfrac{x^2}{x^2+1}$, $f'(x)=\dfrac{2x(x^2+1)-2x(x^2)}{(x^2+1)^2}=\dfrac{2x}{(x^2+1)^2}$

$f''(x)=\dfrac{2(x^2+1)^2-2(x^2+1)(2x)(2x)}{(x^2+1)^4}$

$=\dfrac{2(x^2+1)[x^2+1-4x^2]}{(x^2+1)^4}=\dfrac{2(x^2+1)(1-3x^2)}{(x^2+1)^4}$

4 $\boxed{f''(x)=\dfrac{2(1-\sqrt{3}x)(1+\sqrt{3}x)}{(x^2+1)^3}}$ $\boxed{f'(x)=\dfrac{2x}{(x^2+1)^2}}$ 4

$f'(x)=0 \Rightarrow x=0$ is critical number.

$f''(0)=\dfrac{2(1)(1)}{1}=2$

$\Rightarrow$ f has relative min at $x=0$ 4

$f''(x)=0 \Rightarrow x=\frac{1}{\sqrt{3}}$, $x=-\frac{1}{\sqrt{3}}$

	$-\frac{1}{\sqrt{3}}$		0		$\frac{1}{\sqrt{3}}$	
$2(1-\sqrt{3}x)$	+		+		+	
$(1+\sqrt{3}x)$	−		+		+	
$(x^2+1)^3$	+		+		+	
$f''(x)$	−		+		−	4

Hence $\boxed{x=\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \text{ are inflection points}}$ 4