SECTION 3.6

3.6.1 Let $y = x^3 - 1$.

    (a) Find Δy if $\Delta x = 1$ and the initial value of x is $x = 1$.

    (b) Find dy if $dx = 1$ and the initial value of x is $x = 1$.

    (c) Make a sketch of $y = x^3 - 1$ and show Δy and dy in the picture.

3.6.2 Let $y = \frac{1}{2}x^2 + 1$.

    (a) Find Δy if $\Delta x = 1$ and the initial value of x is $x = 1$.

    (b) Find dy if $dx = 1$ and the initial value of x is $x = 1$.

    (c) Make a sketch of $y = \frac{1}{2}x^2 + 1$ and show Δy and dy in the picture.

3.6.3 Use a differential to approximate $\sqrt{14}$.

3.6.4 Use a differential to approximate $\sqrt[3]{9}$.

3.6.5 Use a differential to approximate $\sqrt[5]{29}$.

3.6.6 Use a differential to approximate $\sqrt[3]{10}$.

3.6.7 Use a differential to approximate $(1.98)^4$.

3.6.8 Use a differential to approximate $\cos 58°$.

3.6.9 Use a differential to approximate $\sin 31°$.

3.6.10 Use a differential to approximate $\tan 43°$.

3.6.11 The surface area of a sphere is given by $S = 4\pi r^2$ where r is the radius of the sphere. The radius is measured to be 3 cm with an error of ± 0.1 cm.

    (a) Use differentials to estimate the error in the calculated surface area.

    (b) Estimate the percentage error in the radius and surface area.

3.6.12 The surface area S of a cube is to be computed from a measured value of its side x. Estimate the maximum permissible percentage error in the side measurement if the percentage error in the surface area must be kept to within $\pm 4\%$.

3.6.13 A circular hole 6 inches in diameter and 10 feet deep is to be drilled out of a glacier. The diameter of the hole is exact but the depth of the hole is measured with an error of $\pm 1\%$. Estimate the percentage error in the volume of ice removed. ($V = \frac{\pi}{4}d^2h$ is the volume of a cylinder of diameter d and height h.)

3.6.14 The pressure P, the volume V, and the temperature T of an enclosed gas are related by the Ideal Gas Law, $PV = kT$ where k is a constant. With the temperature held constant, the volume of the gas is calculated from a measured value of its pressure. Estimate the maximum permissible error in the pressure measurement if the percentage error in the volume must be kept to within $\pm 2\%$.

3.6.15 The magnetic force F acting on a particle is given by $F = \dfrac{k}{r^2}$, where r is the distance from the magnetic source and k is a constant. r is measured to be 3 cm with a possible error of $\pm 6\%$.

(a) Use differentials to estimate the error in the calculated value of F.

(b) Estimate the percentage error in F and r.

3.6.16 When a cubical block of metal is heated, each edge increases by 0.1% per degree increase in temperature. Use differentials to estimate the percentage increase in the surface area and volume of the block per degree increase in temperature.

3.6.17 When a spherical ball of metal is heated, the radius of the sphere increases by 0.1% per degree increase in temperature. Use differentials to estimate the percentage increase in the surface area and volume of the ball per degree increase in temperature.

$$\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3. \right)$$

3.6.18 The area of a circle is to be computed from a measured value of its diameter. Estimate the maximum permissible percentage error in the measurement if the percentage error in the area must be kept within 0.5%.