King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 101
Final Exam
Semester I, 2001–2002 (011)
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Serial No:	

Form (4)

Question #		Points
1		10
2		27
3		34
4		27
5		27
6		9
7-23		8 each
24		3 each
Total:		300

1. Let $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2$
   (a) Find the interval on which f is decreasing

   (b) Find the open interval on which f is concave up.

2. The equation $x^3 - x^2 - 2x + 1 = 0$ has one real solution for $0 < x < 1$. Approximate it by Newton's Method. If $x_1 = 1$, then find x_6.

3. Find the extreme values for $f(x) = x^{4/3} - 3x^{1/3}$ on the interval $[-1, 8]$ and determine where those values occur.

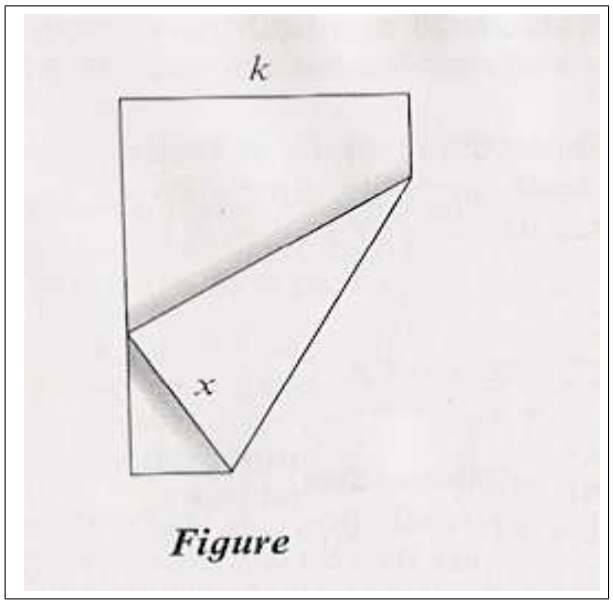
4. Let $f(x) = x^{2/3}(x+5)$
   (a) Find all relative max. and all relative min.

   (b) Find all inflection points

   (c) Find all cusp which may or may not exist.

5. A rectangular field is to be bounded by a fence on three sides and by a wall on the fourth one. Find the dimensions of the field with maximum area that can be enclosed with 1000 meters of fence ?

6. The lower corner of a page of width k is folded over so as just to reach the inner edge of the page (see Figure). Find the width of the part folded over (the value of x) when the area of the triangle floded over is a minimum.



Figure

7. Use dy to approximate $\sqrt{4.04}$ starting at $x = 4$

   (a) 2.01
   (b) 1.99
   (c) 4.01
   (d) 3.99
   (e) 1.59

8. Find $\dfrac{dy}{dx}$ if $x^2 + y^2 = 49$.

   (a) $\dfrac{49x}{y}$
   (b) $\dfrac{x}{y}$
   (c) $-\dfrac{x}{y}$
   (d) $-\dfrac{49x}{y}$
   (e) $\dfrac{y}{x}$

9. The number of critical points for $f(x) = \quad | \, (x-1)(x-2)(x-3) \, | \quad$ is

   (a) 5
   (b) 2
   (c) 3
   (d) 4
   (e) 1

10. $y = \cot^{-1}\sqrt{x}$. Find dy/dx.

    (a) $-\dfrac{x}{1+x}$
    (b) $-\dfrac{1}{2\sqrt{x}(1+x)}$
    (c) $-\dfrac{x}{(1+x^2)}$
    (d) $-\sqrt{\dfrac{1}{1+x^2}}$
    (e) $\dfrac{x}{1+x}$

11. $\lim_{x\to 0^+} \dfrac{\sin x}{\ln(x^2+1)} =$

(a) $-\infty$
(b) 10
(c) 0
(d) $+\infty$
(e) -10

12. $\lim_{x\to 0^+} (1-\ln(2x))^{2x} =$

(a) 0
(b) 1
(c) $+\infty$
(d) $-\infty$
(e) -1

13. If $f(x) = x^4 - x$ on $[-1, 1]$, find the value c that satisfies the Mean value Theorem.

(a) 1
(b) 0
(c) 2
(d) 3
(e) 4

14. $\lim_{x\to 9} \dfrac{x+5}{\sqrt{x}-3} =$

(a) it does not exist
(b) $+\infty$
(c) $-\infty$
(d) 84
(e) -84

15. $\lim_{x\to-\infty}\sqrt{\dfrac{20x^{10}-2x^5+2}{5x^{10}+x^5-3}}=$

(a) $+\infty$

(b) 2

(c) $-\infty$

(d) -2

(e) it does not exist

16. To prove that $\lim_{x\to5}(x-2)=3$ a reasonable relationship between δ and ϵ would be

(a) $\delta=\epsilon$

(b) $\delta=5\epsilon$

(c) $\delta=\sqrt{\epsilon}$

(d) $\delta=\dfrac{1}{\epsilon}$

(e) $\delta=\dfrac{1}{\sqrt{\epsilon}}$

17. Find the value of k, if possible, that will make the function continuous

$$f(x)=\begin{cases} x+2k & x\le 1 \\ kx^2+x+1 & x>1 \end{cases}$$

(a) -1

(b) 2

(c) 1

(d) -2

(e) none exists

18. find the limit $\lim_{x\to+\infty}\left(\cos\left(\frac{4}{x}\right)\cdot\sin\left(\frac{5}{x}\right)\right)=$

(a) 1
(b) -1
(c) $+\infty$
(d) $-\infty$
(e) 0

19. Find an equation for the tangent line to the curve $y=x^7-5$ at $(1,-4)$.

(a) $y=7x$
(b) $y=7x+5$
(c) $y=7x-3$
(d) $y=7x-11$
(e) $y=7x-5$

20. If $y=\dfrac{2}{x+3}$, then $y'(0)=$

(a) $\dfrac{-2}{9}$
(b) 0
(c) $\dfrac{4}{9}$
(d) $\dfrac{2}{9}$
(e) $\dfrac{-4}{9}$

21. $g(x) = x^3 f(x)$. Find $g'(2)$, given that $f(2) = 6$ and $f'(2) = 3$

    (a) 48
    (b) -60
    (c) 96
    (d) 60
    (e) -48

22. $y = x^{-3} + x$. Find y'''

    (a) -6
    (b) $-60x^{-6} + x^{-2}$
    (c) $-60x^{-6} - x^{-2}$
    (d) $-3x^{-2} + 1$
    (e) $-60x^{-6}$

23. If $y = x^5 \cos x$, find d^2y/dx^2

    (a) $20x^3 \cos x$
    (b) $20x^3 \cos x - x^5 \cos x$
    (c) $20x^3 \cos x + x^5 \cos x$
    (d) $20x^3 \cos x - 10x^4 \sin x - x^5 \cos x$
    (e) $-x^5 \cos x$

24. True (T) or False (F)

(a) $f(x) = x^{3/5}$ has a critical point ( )

(b) $f(x) = |x^2 - 4|$ has points of discontinuity at $x = 2$ and $x = -2$. ( )

(c) The function $f(x) = \frac{x+5}{x-1}$ has a removable discontinuity at $x = 1$ ( )

(d) $\tan x$ has a point of inflection on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ( )

(e) All relative extrema occur at critical points ( )

(f) $f(x) = |\tan^2 x|$ has no relative extrema on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ( )

(g) $f(x) = \dfrac{x^3}{x^5 - 2}$ has no horizontal asymptote ( )

(h) $f(x) = \dfrac{1}{x^3}$ on $[-1, 1]$ satisfies the hypotheses of Rolle's Theorem ( )

(i) The Mean Value Theorem can be used on $f(x) = |x - 1|$ on $[-2, 1]$ ( )

(j) The slope of the tangent line to the graph of $f(x) = x^3 - 5$ at $x_0 = 3$ is 22. ( )