SECTION 6.5

6.5.1 Verify that $f(x) = x^3 - x$ satisfies the hypothesis of Rolle's Theorem on the interval $[-1, 1]$ and find all values of C in $(-1, 1)$ such that $f'(C) = 0$.

6.5.2 Verify that $f(x) = x^3 - 3x + 2$ satisfies the hypothesis of the Mean-Value Theorem over the interval $[-2, 3]$ and find all values of C that satisfy the conclusion of the theorem.

6.5.3 Verify that $f(x) = x^2 + 2x - 1$ satisfies the hypothesis of the Mean-Value Theorem over the interval $[0, 1]$ and find all values of C that satisfy the conclusion of the theorem.

6.5.4 Verify that $f(x) = x^3 - 4x$ satisfies the hypothesis of Rolle's Theorem on the interval $[-2, 2]$ and find all values of C that satisfy the conclusion of the theorem.

6.5.5 Does $f(x) = \dfrac{1}{x^2}$ satisfy the hypothesis of the Mean-Value Theorem over the interval $[-1, 1]$? If so, find all values of C that satisfy the conclusion.

6.5.6 Verify that $f(x) = x^2 + 4$ satisfies the hypothesis of the Mean-Value Theorem on the interval $[0, 2]$ and find all values of C that satisfy the conclusion of the theorem.

6.5.7 Verify that $f(x) = x^3 - 3x + 1$ satisfies the hypothesis of the Mean-Value Theorem on the interval $[-2, 2]$ and find all values of C that satisfy the conclusion of the theorem.

6.5.8 Verify that $f(x) = \dfrac{4x}{4 - x}$ satisfies the hypothesis of the Mean-Value Theorem over the interval $[1, 3]$ and find all values of C that satisfy the conclusion of the theorem.

6.5.9 Use Rolle's Theorem to prove that the equation $7x^6 - 9x^2 + 2 = 0$ has at least one solution in the interval $(0, 1)$.

6.5.10 Verify that $f(x) = x^3 - 3x^2 - 3x + 1$ satisfies the hypothesis of the Mean-Value Theorem over the interval $[0, 2]$ and find all values of C that satisfy the conclusion of the theorem.

6.5.11 Use Rolle's Theorem to show that $f(x) = x^3 + x - 2$ does not have more than one real root.

6.5.12 Does $f(x) = \sqrt{x}$ satisfy the hypothesis of the Mean-Value Theorem over the interval $[0, 4]$? If so, find all values of C that satisfy the conclusion of the theorem.

6.5.13 Does $f(x) = \sqrt[3]{x}$ satisfy the hypothesis of the Mean-Value Theorem over the interval $[-1, 1]$? If so, find all values of C that satisfy the conclusion of the theorem.

6.5.14 An automobile starts from rest and travels 3 miles along a straight road in 4 minutes. Use the Mean-Value Theorem to show that at some instant during the trip its velocity was exactly 45 miles per hour.

6.5.15 Does $f(x) = \dfrac{x}{x - 1}$ satisfy the hypothesis of the Mean-Value Theorem over the interval $[0, 2]$? If so, find all values of C that satisfy the conclusion of the theorem.

6.5.16 Does $f(x) = \sqrt[3]{x}$ satisfy the hypothesis of the Mean-Value Theorem over the interval $[0, 1]$? If so, find all values of C that satisfy the conclusion.

6.5.17 Use Rolle's Theorem to show that $f(x) = x^3 + ax + b$, where $a > 0$, cannot have more than one real root.

6.5.18 A cyclist starts from rest and travels 4 miles along a straight road in 20 minutes. Use the Mean-Value Theorem to show that at some instant during the trip his velocity was exactly 12 miles per hour.