5.1.9 $f(x) = x(x+4)^3$

(a) Find the largest intervals where f is increasing and where f is decreasing.

(b) Find the largest intervals where f is concave up and where f is concave down.

(c) Find the location of any inflection points.

5.1.10 $f(x) = (x-4)^4 + 4$

(a) Find the largest intervals where f is increasing and where f is decreasing.

(b) Find the largest intervals where f is concave up and where f is concave down.

(c) Find the location of any inflection points.

5.1.11 $f(x) = x(x-3)^5$

(a) Find the largest intervals where f is increasing and where f is decreasing.

(b) Find the largest intervals where f is concave up and where f is concave down.

(c) Find the location of any inflection points.

5.1.12 $f(x) = \sin 2x (0, \pi)$

(a) Find the largest intervals where f is increasing and where f is decreasing.

(b) Find the largest intervals where f is concave up and where f is concave down.

(c) Find the location of any inflection points.

5.1.13 Are the following true or false?

(a) If $f''(x) > 0$ on the open interval (a, b) then $f'(x)$ is increasing on (a, b).

(b) If $f''(x) > 0$ on the open interval (a, b) then $f(x)$ is increasing on (a, b).

(c) If $f''(x) = 0$, then x is a point of inflection.

(d) If x_0 is a point of inflection, then $f''(x_0) = 0$.

(e) If $f'(x)$ is decreasing on (a, b), then $f(x)$ is concave down on (a, b).

5.1.14 Which of the following is correct if $f'(x) < 0$ and $f''(x) > 0$ on (a, b):

(a) $f(x)$ is increasing and concave up.

(b) $f(x)$ is decreasing and concave up.

(c) $f(x)$ is increasing and concave down.

(d) $f(x)$ is decreasing and concave down.

5.1.15 Sketch a continuous curve having the following properties:

$f(-3) = 27$, $f(0) = 27/2$, $f(3) = 0$, $f'(x) > 0$ for $|x| > 3$

$f'(-3) = f'(3) = 0$, $f''(x) < 0$ for $x < 0$, $f''x) > 0$ for $x > 0$.

5.1.16 Sketch a continuous curve $y = f(x)$ for $x > 0$ if $f(1) = 0$, and $f'(x) = 1/x$ for all $x > 0$. Is the curve concave up or concave down?

5.1.17 Sketch a continuous curve having the following properties:

$f(0) = 4$, $f(-2) = f(2) = 0$; $f'(x) > 0$ for $(-\infty, 0)$ and

$f'(x) < 0$ for $(0, +\infty)$, $f''(x) < 0$ for $(-\infty, +\infty)$.

SOLUTIONS

SECTION 5.1

5.1.1 $f'(x) = 4x^3 - 48x$, $f''(x) = 12x^2 - 48$

(a) Increasing $[-2\sqrt{3}, 0]$, $[2\sqrt{3}, +\infty)$ decreasing $(-\infty, -2\sqrt{3}]$, $[0, 2\sqrt{3}]$

(b) Concave up $(-\infty, -2)$, $(2, +\infty)$; concave down $(-2, 2)$

(c) $(-2, -80)$ and $(2, -80)$

5.1.2 $f'(x) = 4x^3 - 12x^2$, $f''(x) = 12x^2 - 24x$

(a) Increasing $[3, +\infty)$; decreasing $(-\infty, 3]$

(b) Concave up $(-\infty, 0)$, $(2, +\infty)$; concave down $(0, 2)$

(c) $(0, 0)$ and $(2, -16)$

5.1.3 $f'(x) = 4x^3 + 24x^2$, $f''(x) = 12x^2 + 48x$

(a) Increasing $[-6, +\infty)$; decreasing $(-\infty, -6]$

(b) Concave up $(-\infty, -4)$, $(0, +\infty)$; concave down $(-4, 0)$

(c) $(-4, -232)$ and $(0, 24)$

5.1.4 $f'(x) = 20x^3 - 5x^4$, $f''(x) = 60x^2 - 20x^3$

(a) Increasing $[0, 4]$; decreasing $(-\infty, 0]$, $[4, +\infty)$

(b) Concave up $(-\infty, 0)$, $(0, 3)$; concave down $(3, +\infty)$

(c) $(3, 162)$

5.1.5 $f'(x) = 12x^2 - 30x - 18$, $f''(x) = 24x - 30$

(a) Increasing $(-\infty, -1/2]$, $[3, +\infty)$; decreasing $[-1/2, 3]$

(b) Concave up $(5/4, +\infty)$; concave down $(-\infty, 5/4)$

(c) $\left(\frac{5}{4}, -\frac{225}{16}\right)$

5.1.6 $f'(x) = (x - 6)(3x - 6)$, $f''(x) = 6x - 24$

(a) Increasing $(-\infty, 2]$, $[6, +\infty)$; decreasing $[2, 6]$

(b) Concave up $(4, +\infty)$; concave down $(-\infty, 4)$

(c) $(4, 16)$