KFUPM SEM II (Term 062) Name: KEY Serial #: ________

MATH 102 Quiz # 4 ID: #: ________ Section #: ________

1. (4-points) Evaluate $\int \frac{dx}{(4-x^2)^{3/2}}$. $= I$

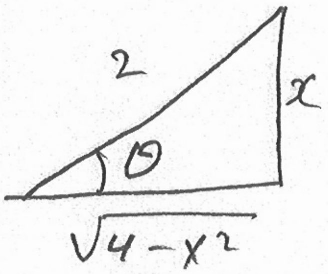
Let $x = 2\sin\theta \Rightarrow (4-x^2)^{3/2} = (4\cos^2\theta)^{3/2} = 8\cos^3\theta$,

and $dx = 2\cos\theta \, d\theta$

$$\Rightarrow I = \int \frac{2\cos\theta \, d\theta}{8\cos^3\theta} = \frac{1}{4}\int \frac{1}{\cos^2\theta} \, d\theta$$

$$= \frac{1}{4}\int \sec^2\theta \, d\theta = \frac{1}{4}\tan\theta + C$$

$$= \frac{1}{4} \frac{x}{\sqrt{4-x^2}} + C$$



2. (4-points) Test for convergence or divergence the improper integral $\int_0^{\sqrt{2}} \frac{dx}{\sqrt{x}(x+2)}$.

$$I = \lim_{t\to 0^+} \int_t^{\sqrt{2}} \frac{dx}{\sqrt{x}\,(x+2)},$$

to find $\int \frac{dx}{\sqrt{x}\,(x+2)}$, let $u^2 = x \Rightarrow 2u\,du = dx$

$$\Rightarrow \int \frac{dx}{\sqrt{x}\,(x+2)} = \int \frac{2u\,du}{u\,(u^2+2)} = 2\int \frac{du}{u^2+2} = \frac{2}{\sqrt{2}}\tan^{-1}\frac{\sqrt{x}}{\sqrt{2}} + C$$

$$I = \lim_{t\to 0^+} \left[\tan^{-1}\frac{\sqrt{x}}{\sqrt{2}}\right]_t^{\sqrt{2}} = \lim_{t\to 0^+}\left[\tan^{-1}\left(\frac{\sqrt{\sqrt{2}}}{\sqrt{2}}\right) - \tan^{-1}\frac{\sqrt{t}}{\sqrt{2}}\right]$$

$$= \tan^{-1}\left(\sqrt{\frac{\sqrt{2}}{2}}\right),$$

$\Rightarrow$ the integral converges and has the value $\tan^{-1}\left(\sqrt{\frac{\sqrt{2}}{2}}\right)$.

3. (7-points) Evaluate $\int \frac{2x-3}{x^2(x^2+1)}\,dx. = I$

$$\frac{2x-3}{x^2(x^2+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{x^2+1}$$

$$2x-3 = A(x^2+1) + Bx(x^2+1) + (Cx+D)x^2$$

$x=0 \Longrightarrow \boxed{-3 = A}$

$\underline{\text{Coef } x^2}\,|$ $\quad 0 = A + D \Longrightarrow \boxed{D=3}$

$\underline{\text{Coef } x}\,|$ $\quad \boxed{2 = B}$

$\underline{\text{Coef } x^3}\,|$ $\quad 0 = B + C \Longrightarrow \boxed{C=-2}$

$$\Longrightarrow I = \int \left[\frac{-3}{x^2} + \frac{2}{x} + \frac{-2x+3}{x^2+1} \right] dx$$

$$= -3\int \frac{1}{x^2}\,dx + 2\int \frac{1}{x}\,dx - \int \frac{2x}{x^2+1}\,dx + 3\int \frac{1}{x^2+1}\,dx$$

$$= \frac{3}{x} + 2\ln|x| - \ln(x^2+1) + 3\tan^{-1}x + C.$$