MATH 102 FINAL (061)

4. The integral $\int_0^1 \frac{x^2-5x+7}{x^2-5x+6}dx$ is equal to

(a) $1-\ln\frac{4}{3}$
(b) $2+\ln\frac{3}{4}$
(c) $1-\ln\frac{2}{3}$
(d) $1+\ln\frac{4}{3}$
(e) $1+\ln\frac{2}{3}$

5. The integral $\int_{-\infty}^{\infty} x^2e^{-x^3}dx$

(a) converges to $\frac{1}{3}$.
(b) converges to $\frac{1}{6}$.
(c) diverges.
(d) converges to 0.
(e) converges to $\frac{2}{3}$.

6. The series $\sum_{n=1}^{\infty}\frac{1}{n\sqrt[n]{n}}$

(a) diverges by limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{n}$
(b) converges by limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{n}$
(c) converges by limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{\sqrt[n]{n}}$
(d) converges by comparison test with $\sum_{n=1}^{\infty}\frac{1}{\sqrt[n]{n}}$
(e) diverges because $\lim_{n\to\infty}\frac{1}{n\sqrt[n]{n}}=\frac{1}{e}$.

1. The integral $\int_1^e 9x^2 \ln x dx$ is equal to

   (a) $2e^3 + 3$
   (b) $2e^3 - 2$
   (c) $3e^3 - 1$
   (d) $2e^3 + 1$
   (e) $3e^3 + 1$

2. The integral $\int \sin 2\theta \cos\theta d\theta$ is equal to

   (a) $-\frac{2}{3}\sin^3\theta + C$
   (b) $-\frac{1}{3}\cos 3\theta + C$
   (c) $\frac{1}{3}\sin 3\theta + C$
   (d) $-\frac{2}{3}\cos^3\theta + C$
   (e) $\frac{1}{4}\sin^2 2\theta + C$

3. The integral $\int_0^1 \frac{dx}{(x^2+1)^{\frac{3}{2}}}$ is equal to

   (a) $\frac{\sqrt{2}}{2} - 1$
   (b) $\frac{\sqrt{2}}{2} + 1$
   (c) $\frac{\sqrt{2}}{2}$
   (d) $\sqrt{2}$
   (e) $\sqrt{2} - 1$

7. The series $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n}{4n^2+1}\right)$

(a) diverges.
(b) converges.
(c) converges absolutely.
(d) diverges by comparison with $\sum_{n=1}^{\infty}\frac{1}{n}$.
(e) converges because $\lim_{n\to\infty}\left(\frac{n}{4n^2+1}\right)=0$.

8. The series $\sum_{n=1}^{\infty}(-1)^n\frac{n^2\,2^n}{n\,!}$

(a) converges conditionally.
(b) diverges by ratio test.
(c) diverges by integral test.
(d) diverges by root test.
(e) converges absolutely.

9. The limit $\lim_{n\to+\infty}\sum_{i=1}^{n}\frac{3}{n}\sqrt{1+\frac{3i}{n}}$ represents the area of the region under the graph of

(a) $y=\sqrt{x}$ on the interval $[0,3]$
(b) $y=\sqrt{x+1}$ on the interval $[1,4]$
(c) $y=\sqrt{x}$ on the interval $[1,4]$
(d) $y=\sqrt{x+1}$ on the interval $[0,4]$
(e) $y=\sqrt{x+1}$ on the interval $[1,3]$

10. Use the properties of the integrals to determine which of the following relations is **correct**

(a) $\int_1^3 \sqrt{x^4+1}dx \geq \frac{26}{3}$

(b) $2 < \int_1^3 \sqrt{x^4+1}dx < 2\sqrt{2}$

(c) $\int_1^3 \sqrt{x^4+1}dx < 0$

(d) $2\sqrt{2} < \int_1^3 \sqrt{x^4+1}dx < \frac{26}{3}$

(e) $0 < \int_1^3 \sqrt{x^4+1}dx \leq 2$

11. The integral $\int_0^2 |x - x^2|\, dx$ is equal to

(a) $-2/3$
(b) 1
(c) -1
(d) $5/6$
(e) $2/3$

12. The integral $\int_0^{\frac{\pi^2}{9}} \sin\sqrt{x}dx$ is equal to

(a) $2\sin\frac{\pi^2}{9} - \frac{2\pi^2}{9}\cos\frac{\pi^2}{9}$
(b) $\frac{\pi\sqrt{3}}{2} - 1$
(c) $1 - \cos\frac{\pi^2}{9}$
(d) $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$
(e) $\sqrt{3} - \frac{\pi}{3}$

13. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^3}$ is

(a) $[0, 1/2]$
(b) $[0, 1]$
(c) $(0, 1)$
(d) $(-\infty, +\infty)$
(e) $\{0\}$

14. If the first three nonzero terms of the Maclaurin series for $\tan^{-1} x$ are used, then the approximation of $\tan^{-1} 1$ is:

(a) 13/15
(b) 11/15
(c) 2/5
(d) 23/5
(e) 14/15

15. The Maclaurin series for the function $f(x) = \frac{1-\cos x}{x^2}$, $x \neq 0$ is

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+2)!}$
(b) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n+2)!}$
(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+2)!}$
(d) $\sum_{n=1}^{\infty} \frac{x^{2n+3}}{(2n)!}$
(e) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

16. For $x \neq 0$, the sum of the series $\sum_{n=0}^{\infty} \frac{x^n}{2^n(n+1)!}$ is equal to

(a) $e^{\frac{x}{2}} - 1 - \frac{x^2}{4}$
(b) $\frac{1}{2}\sin x$
(c) $\frac{x}{2}\cos x$
(d) $\frac{2e^x}{x}$
(e) $\frac{2}{x}(e^{\frac{x}{2}} - 1)$

17. The area of the region enclosed by the graphs of $y = \frac{1}{x}$, $y = \frac{1}{x^2}$ and $x = 2$ is equal to

(a) $\ln 2 + \frac{3}{2}$
(b) $\frac{1}{2} - \ln 2$
(c) $\ln 2 + \frac{1}{2}$
(d) $\ln \frac{1}{2} - \frac{1}{2}$
(e) $\ln 2 - \frac{1}{2}$

18. The volume of the solid obtained by rotating the region bounded by the curves of $y = \sqrt{x-1}$, $x = 2$, $x = 5$ and $y = 0$ about the x-axis is equal to

(a) 21π
(b) 15π
(c) $15\pi/2$
(d) $21\pi/2$
(e) $15\pi/4$

19. The volume generated by rotating the region bounded by the graphs of $y = x^2$, $y = 0$, $x = 1$, and $x = 2$ about $x = 4$ is equal to

(a) $14\pi/3$
(b) $67\pi/2$
(c) 67π
(d) $67\pi/6$
(e) $7\pi/6$

20. The length of the curve $y = \frac{x^2}{2} - \frac{\ln x}{4}$, $2 \leq x \leq 4$ is equal to

(a) $6 + \frac{\ln 2}{4}$
(b) $6 - \ln 2$
(c) $6 - \frac{\ln 2}{4}$
(d) $6 + \ln 2$
(e) $6 - \ln 3$

21. The area of the surface obtained by rotating the curve $x = \frac{1}{3}(y^2 + 2)^{\frac{3}{2}}$, $1 \leq y \leq 2$, about the $x-$axis is equal to

(a) 21π
(b) $21\pi/2$
(c) $21\pi/4$
(d) $20\pi/3$
(e) 20π

22. The sequence $a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$

(a) converges and the limit is 1
(b) diverges
(c) converges and the limit is $\sqrt{2}$
(d) converges and the limit is $\frac{\sqrt{2}}{1+\sqrt{2}}$
(e) converges and the limit is 0

23. The series $\sum_{n=1}^{\infty} \frac{3^n+2^n}{6^n}$ is

(a) convergent and the sum is 7
(b) convergent and the sum is 3
(c) divergent
(d) convergent and the sum is 5/2
(e) convergent and the sum is 3/2

24. The series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is

(a) divergent by the integral test
(b) convergent by the integral test
(c) convergent by the comparaison test
(d) convergent because $\lim_{n\to\infty} \frac{1}{n \ln n} = 0$
(e) convergent by the ratio test

25. The integral $\int_0^1 \frac{dx}{1+\sqrt[3]{x}}$ is equal to

(a) $\ln 2 - \frac{1}{2}$
(b) $\ln 2 + \frac{1}{2}$
(c) $3 \ln 2 - \frac{1}{2}$
(d) $3 \ln 2 - \frac{3}{2}$
(e) $\ln 2$