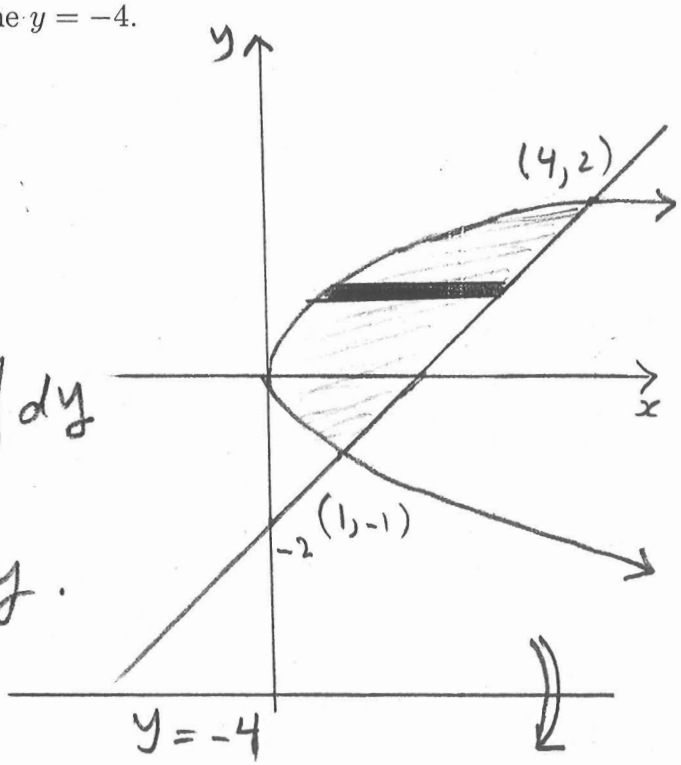
KFUPM SEM II (Term 062) Name: _____ Serial #: _____

MATH 102 Quiz # 3 ID: #: KEY Section #: _____

1. (4-points) Use cylindrical shells to set up, **but do not evaluate**, an integral for the volume of the solid obtained by rotating the region bounded by the parabola $y^2 = x$ and the line $y - x + 2 = 0$ about the lne $y = -4$.



Pts of intersection:

$y^2 = y+2 \Rightarrow y^2 - y - 2 = 0 \Rightarrow$

$(y-2)(y+1) = 0 \Rightarrow y = -1$ or 2

$\Rightarrow (1,-1)$, $(4,2)$ are the int. pts

$$dV \xrightarrow[\text{shell}]{\text{cylindrical}} 2\pi(y+4)\left[(y+2) - y^2\right] dy$$

$$V = \int_{-1}^{2} 2\pi(y+4)\left[2 + y - y^2\right] dy .$$

2. (4-points) Find the average of the function $f(x) = 3x \sin 2x$ on the interval $\left[0, \frac{\pi}{4}\right]$.

$$f_{ave} = \frac{\int_0^{\pi/4} 3x \sin 2x \, dx}{\frac{\pi}{4} - 0}$$

$$= \frac{12}{\pi} \int_0^{\pi/4} x \sin 2x \, dx$$

Integration by Parts

x		$\sin 2x$
	$+$ ↘	
		$-\frac{1}{2}\cos 2x$
1		
	$-$ ↘	
0		$-\frac{1}{4}\sin 2x$

$$= \frac{12}{\pi}\left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi/4}$$

$$= \frac{12}{\pi}\left[\left(0 + \frac{1}{4}\right) - 0\right]$$

$$\Rightarrow f_{ave} = \frac{3}{\pi} .$$

3. (4-points) Evaluate $\int (\ln 5x)^2\,dx$. [Do not use any reduction formula].

Use integration by parts:

$u = (\ln 5x)^2$, $dv = dx$

$du = 2(\ln 5x)\left(\frac{5}{5x}\right)dx = \frac{2}{x}\ln 5x\,dx$, $v = x$

$\Rightarrow I = \int (\ln 5x)^2\,dx = x(\ln 5x)^2 - 2\int x \ln 5x\,dx$

Apply integration by parts again on $J = \int x \ln 5x\,dx$

Put $u = \ln 5x$, $dv = dx$

$\Rightarrow du = \frac{5}{5x}dx = \frac{1}{x}dx$, $v = x$

$\Rightarrow J = x\ln 5x - \int dx = x\ln 5x - x \Rightarrow$

$I = x(\ln 5x)^2 - 2x(\ln 5x) + 2x + C$.

4. (3-points) Evaluate $\int (\sec x)^{5/2}\tan^3 x\,dx$. $= I$

$I = \int (\sec x)^{3/2}\tan^2 x\,(\sec x\tan x\,dx)$

$= \int (\sec x)^{3/2}(\sec^2 x - 1)(\sec x\tan x\,dx)$

Let $u = \sec x \Rightarrow du = \sec x\tan x\,dx \Rightarrow$

$I = \int u^{3/2}(u^2 - 1)\,du = \int (u^{7/2} - u^{3/2})\,du$

$= \frac{2}{9}u^{9/2} - \frac{2}{5}u^{5/2} + C$

$= \frac{2}{9}(\sec x)^{9/2} - \frac{2}{5}(\sec x)^{5/2} + C$.