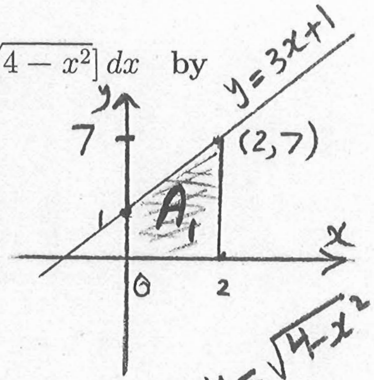
KFUPM	SEM II (Term 062)	Name: KEY	Serial #:
MATH 102	Quiz # 1	ID: #:	Section #:

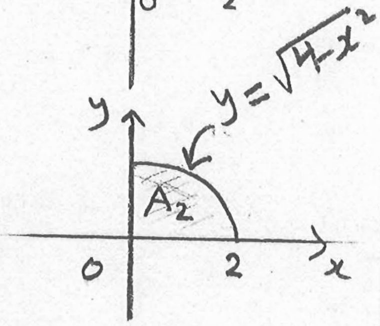
1. (4-points) Find the exact value of the integral $\int_0^2 [(3x+1) + 2\sqrt{4-x^2}]\,dx$ by **interpolating it in terms of area.**

$$I = \int_0^2 (3x+1)\,dx + 2\int_0^2 \sqrt{4-x^2}\,dx = A_1 + 2A_2$$



$$A_1 = \frac{1}{2}(2)(1+7) = 8$$

$$A_2 = \frac{1}{4}(\pi 4) = \pi$$

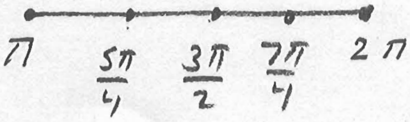


$$\Rightarrow \quad I = 8 + 2\pi.$$

2. (3-points) Evaluate the Riemann sum for $f(x) = \sin x$, $\pi \le x \le 2\pi$ with four subintervals, taking the sample points to be the right endpoints.

The required Riemann sum =

$$\frac{\pi}{4}\left[\sin\frac{5\pi}{4} + \sin\frac{3\pi}{2} + \sin\frac{7\pi}{4} + \sin 2\pi\right]$$



$$= \frac{\pi}{4}\left[-\frac{\sqrt{2}}{2} - 1 - \frac{\sqrt{2}}{2} + 0\right] = -\frac{\pi}{4}(\sqrt{2}+1).$$

3. (2-points) Express $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^{3/2} + \left(\frac{2}{n}\right)^{3/n} + \cdots + \left(\frac{n}{n}\right)^{3/2}\right]$ as an integral over the interval $[0,1]$, then find its value.

We can take $f(x) = x^{3/2}$ $\Longrightarrow$

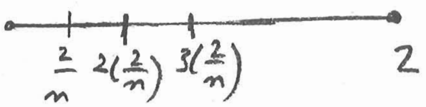
$$\text{The given Limit} = \int_0^1 x^{3/2}\,dx = \frac{2}{5}\left[x^{5/2}\right]_0^1$$

$$= \frac{2}{5}[1-0] = \frac{2}{5}.$$

4. (4-points) Express the integral $\int_0^2 (3x^2+4)\,dx$ as a limit of a sum over the interval $[0,2]$, then find the exact value of the limit.

$$I = \int_0^2 (3x^2+4)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(3x_i^{*2}+4\right)\Delta x$$

$$\Delta x = \frac{2}{n}, \quad x_i^* = i\left(\frac{2}{n}\right) \text{ (right end point)}$$



$$I = \lim_{n\to\infty} \sum_{i=1}^{n} \left[3\left(\frac{4i^2}{n^2}\right)+4\right]\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{24}{n^3}\sum_{i=1}^{n} i^2\right) + \lim_{n\to\infty}\left(\frac{8}{n}\sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \frac{24}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \lim_{n\to\infty}\frac{8}{n}(n)$$

$$= \lim_{n\to\infty} 4\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 8 = 8+8 = 16.$$

$$\frac{6}{2}x^2\Big]_0^{\frac{\pi}{2}}$$

5. (2-points) Show that $\int_0^{\pi/2} \frac{8}{3} x \sin x\,dx \le \frac{\pi^2}{3}$.

$\frac{8}{3}x\sin x \le \frac{8}{3}x$ because $\sin x \le 1$

$$\Longrightarrow \int_0^{\pi/2} \frac{8}{3}x\sin x\,dx \le \int_0^{\pi/2}\frac{8}{3}x\,dx = \frac{8}{3}\left[\frac{x^2}{2}\right]_0^{2}$$

$$= \frac{4}{3}\left[\frac{\pi^2}{4}\right] = \frac{\pi^2}{3}.$$