King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 –Calculus I
Exam I
Semester 071

November 6, 2007 Time: 7:00 - 8:45 PM

Section Number:_____

ID Number: _____.

Name: _____

Instructions :

- (1) Write neatly and eligibly. You may lose points for messy work.
  (2) Show all your work. No credits for answers without justification.
  (3) All types of calculators and mobiles are not allowed.
  (4) Make sure that you have 7 different Problem. (7 pages +cover)

Problem No	Student Grade	Maximum Points
1		15
2		15
3		14
4		15
5		15
6		12
7		14
Total		100

1. Evaluate the limit if it exists:

(a) $\lim_{x\to 2} \dfrac{x^2-4}{x^3-8}$. (5 points)

(b) $\lim_{x\to 1^-} \dfrac{|x^2-3x+2|}{x^2-1}$. (5 points)

(c) $\lim_{x\to\infty} (\sqrt{x^2+x}-x)$. (5 points)

2. (a) Let $f(x) = 5x + 2$. Find the largest value of δ such that $|f(x) - 12| < 0.01$ whenever $-\delta < x - 2 < \delta$. (5 points)

(b) Show that there is a root of the equation $\sqrt[3]{x} = 1 - x$, between 0 and 1. (5 points)

(c) Find the limit if it exists:

$$\lim_{x \to 0} x^2 \cos\left(\frac{\pi}{x}\right).$$

(5 points)

3. (a) Find all values of A and B which will make the following function continuous:

$$f(x) = \begin{cases} x^2 - A & \text{if } x < 1, \\ A + Bx & \text{if } 1 \leq x \leq 2, \\ B - x^3 & \text{if } 2 < x. \end{cases}$$

(8 points)

(b) Find all horizontal and vertical asymptotes of $f(x) = \dfrac{\sqrt{4x^2+1}}{x+1}$, if any exists.

(6 points)

4. (a) A stone is thrown upward from ground level. Its height (in meters) after t seconds is given by $s = 20t - 5t^2$. Find the following:

   (i) The average velocity of the stone during the first 3 seconds after being thrown. (2 points)

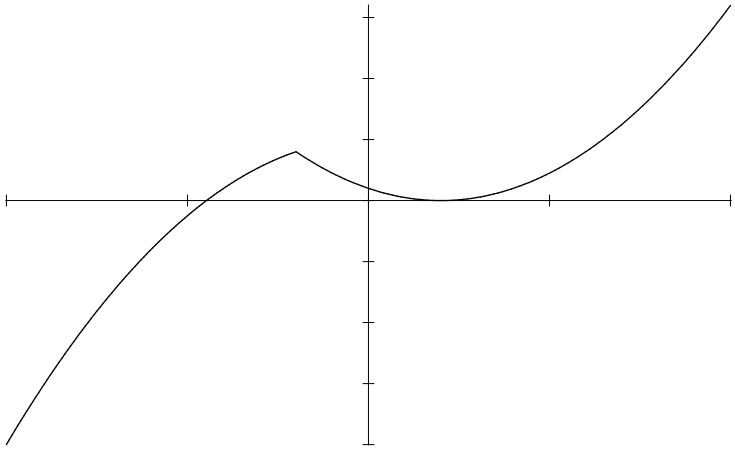
   (ii) The velocity of the stone after exactly 3 seconds of being thrown. (3 points)

   (iii) Find the maximum height reached by the stone. (3 points)

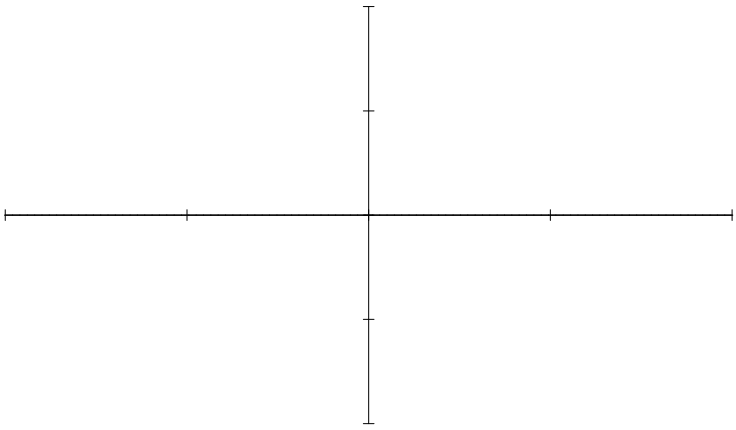
   (b) **Using the definition of the derivative**, find the slope of the tangent line of the curve $f(x) = \sqrt{2x+1}$ at $x = 4$. (7 points)

5. (a) Use the graph of $f(x)$, given below to sketch the graph of $f'(x)$. (7 points)

$f(x)$



$f'(x)$



(b) True or False? (8 points)

(i) Every differentiable function is continuous. **T** **F**

(ii) Every continuous function is differentiable. **T** **F**

(iii) The function $f(x) = |x^2 + 2|$ is continuous and differentiable. **T** **F**

(iv) The function $f(x) = \ln(4 - x^2)$ has vertical asymptotes. **T** **F**

6. (a) Let $f(x) = \dfrac{a+x}{b+x}$.

(i) Find $f'(x)$. (4 points)

(ii) Find a when $f'(0) = 4$ and $b = 2$. (2 points)

(b) Determine the point $P(a, b)$ on the graph of $f(x) = e^x + x$, which has the property that the tangent line at P is parallel to the line $y = 2x - 1$. (6 points)

7. (a) Find the derivative of $f(x) = \dfrac{\sqrt{x}\, e^x}{x+1}$ at $x = 1$. (6 points)

(b) Given $f(x) = \begin{cases} mx - 4, & \text{if } x < 2 \\ x^2 + m, & \text{if } x \geq 2 \end{cases}$,
find all values of m that make f differentiable at 2, if any exists. (8 points)