KFUPM SEM II (Term 072) Name: Quiz #3 KEY

MATH 260-1-3 Quiz# 3 ID #: KEY Sec.#: ____ Serial #: ________

1. (8-points) Find a particular solution of $y'' - 4y' + 13y = 2\sin 3x$.

$\boxed{y_c}$ $r^2 - 4r + 13 = 0 \Rightarrow r = \dfrac{4 \pm \sqrt{16-52}}{2} \Rightarrow$

$r = 2 \pm 3i \Rightarrow$

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

$y_p = A\cos 3x + B\sin 3x$ is an appropriate trial

$\Rightarrow \quad y_p' = -3A\sin 3x + 3B\cos 3x$

& $\quad y_p'' = -9A\cos 3x - 9B\sin 3x$

Then substitute in the given DE $\Rightarrow$

$\underline{-9A\cos 3x} - 9B\sin 3x + 12A\sin 3x - \underline{12B\cos 3x}$

$+ \underline{13A\cos 3x} + 13B\sin 3x = 2\sin 3x \Rightarrow$

$(4A - 12B)\cos 3x + (12A + 4B)\sin 3x = 2\sin 3x$

$\Rightarrow \quad \begin{aligned} 4A - 12B &= 0 \\ 12A + 4B &= 2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} A - 3B &= 0 \\ 6A + 2B &= 1 \end{aligned} \quad \Rightarrow$

eliminate $A \Rightarrow 20B = 1 \Rightarrow \boxed{B = \frac{1}{20}} \Rightarrow$

$A = 3B \Rightarrow \boxed{A = \frac{3}{20}} \Rightarrow$

$y_p = \dfrac{3}{20}\cos 3x + \dfrac{1}{20}\sin 3x$

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2. (7-points) Use the meteod of variation of parameters to solve
$4y'' + 16y = \sec 2x \tan 2x$.

The DE in standard form is $y'' + 4y = \frac{1}{4} \sec 2x \tan 2x$

$\boxed{y_c} \Rightarrow r^2 + 4r = 0 \Rightarrow r = \pm 2i \Rightarrow$

$y_c = c_1 \cos 2x + c_2 \sin 2x$

Variation of Parameters Let

$$y_p = u_1 \cos 2x + u_2 \sin 2x$$

where $u_1 = \int \frac{-y_2 f}{W} dx$ & $u_2 = \int \frac{y_1 f}{W} dx$,

where $y_1 = \cos 2x$, $y_2 = \sin 2x$, $W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2$

& $f = \frac{1}{4} \sec 2x \tan 2x$.

$$\Rightarrow u_1 = - \int \frac{(\sin 2x)(\frac{1}{4} \sec 2x \tan 2x)}{2} dx = -\frac{1}{8} \int \tan^2 2x \, dx$$

$$\Rightarrow u_1 = -\frac{1}{8} \int (\sec^2 2x - 1) dx = -\frac{1}{16} \tan 2x + \frac{1}{8} x.$$

$$\& \quad u_2 = \frac{1}{8} \int (\cos 2x)(\sec 2x \tan 2x) dx = \frac{1}{8} \int \tan 2x \, dx$$

$$\Rightarrow u_2 = \frac{1}{16} \ln |\sec 2x|.$$

$\Rightarrow$ The G.S. is

$$y = c_1 \cos 2x + c_2 \sin 2x + \left(-\frac{1}{16} \tan 2x + \frac{1}{8} x\right) \cos 2x$$
$$+ \frac{1}{16} \ln |\sec 2x| \cdot \sin 2x$$

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1. (8-points) Find a particular solution of $y'' - 6y' + 13y = 3\cos 2x$.

$\boxed{y_c}$ $r^2 - 6r + 13 = 0 \Rightarrow r = \dfrac{6 \pm \sqrt{36-52}}{2}$

$\Rightarrow r = 3 \pm 2i \Rightarrow$

$y_c = e^{3x}(c_1 \cos 2x + c_2 \sin 2x)$

$y_p = A\cos 2x + B\sin 2x$ is an appropriate trial.

$\Rightarrow y_p' = -2A\sin 2x + 2B\cos 2x$

& $y_p'' = -4A\cos 2x - 4B\sin 2x$

Then substitute in the given DE $\Rightarrow$

$-4A\cos 2x - 4B\sin 2x + 12A\sin 2x - 12B\cos 2x$

$+ 13A\cos 2x + 13B\sin 2x = 3\cos 2x \Rightarrow$

$(9A - 12B)\cos 2x + (12A + 9B)\sin 2x = 3\cos 2x$

$\Rightarrow$ $9A - 12B = 3 \Rightarrow 3A - 4B = 1 \Rightarrow$

$12A + 9B = 0$ $\quad 4A + 3B = 0$

eliminate B we get: $25A = 3 \Rightarrow \boxed{A = \dfrac{3}{25}}$

$\Rightarrow 4B = 3A - 1 \Rightarrow 4B = \dfrac{-16}{25} \Rightarrow \boxed{B = \dfrac{-4}{25}}$

$\Rightarrow y_p = \dfrac{3}{25}\cos 2x - \dfrac{4}{25}\sin 2x$

Quiz#3 KEY

2. (7-points) Use the meteod of variation of parameters to solve $3y'' + 27y = \csc 3x \cot 3x$.

The DE in standard form is $y'' + 9y = \frac{1}{3}\csc 3x \cot 3x$

$\boxed{y_c}$ $r^2 + 9 = 0 \Rightarrow r = \pm 3i \Rightarrow$

$y_c = c_1 \cos 3x + c_2 \sin 3x$.

Variation of Parameters Let

$$y_p = u_1 \cos 3x + u_2 \sin 3x$$

where $u_1 = \int -\frac{y_2 f}{W} dx$ & $u_2 = \int \frac{y_1 f}{W} dx$

where $y_1 = \cos 3x$, $y_2 = \sin 3x$, $W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3$

and $f = \frac{1}{3}\csc 3x \cot 3x$

$$\Rightarrow u_1 = -\int \frac{(\sin 3x)(\frac{1}{3}\csc 3x \cot 3x)}{3} dx = -\frac{1}{9}\int \cot 3x \, dx$$

$$\Rightarrow u_1 = -\frac{1}{27}\ln|\sin 3x| = \frac{1}{27}\ln|\csc 3x|.$$

$$\& \; u_2 = \frac{1}{9}\int (\cos 3x) \csc 3x \cot 3x \, dx = \frac{1}{9}\int \cot^2 3x \, dx$$

$$= \frac{1}{9}\int (\csc^2 3x - 1) \, dx = -\frac{1}{27}\cot 3x - \frac{1}{9}x$$

$\Rightarrow$ The G.S. is

$$y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{27}\left(\ln|\csc 3x|\right)\cos 3x$$

$$- \left(\frac{1}{27}\cot 3x - \frac{1}{9}x\right)\sin 3x.$$