

# Lexicalized and Probabilistic Parsing – Part 1

## ICS 482 Natural Language Processing

Lecture 14: Lexicalized and Probabilistic  
Parsing – Part 1

Husni Al-Muhtaseb

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# ICS 482 Natural Language Processing

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Parsing – Part 1

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# NLP Credits and

# Acknowledgment

These slides were adapted from presentations of the Authors of the book

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SPEECH and LANGUAGE PROCESSING:

An Introduction to Natural Language Processing,  
Computational Linguistics, and Speech Recognition

and some modifications from presentations found in the WEB by several scholars including the following

# NLP Credits and Acknowledgment

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# NLP Credits and Acknowledgment

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# Previous Lectures

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- Introduction and Phases of an NLP system
- NLP Applications - Chatting with Alice
- Finite State Automata, Regular Expressions & languages
- Morphology: Inflectional & Derivational
- Parsing and Finite State Transducers
- Stemming & Porter Stemmer
- Statistical NLP – Language Modeling
- N Grams, Smoothing : Add-one & Witten-Bell
- Parts of Speech - Arabic Parts of Speech
- Syntax: Context Free Grammar (CFG) & Parsing
- Parsing: Top-Down, Bottom-Up, Top-down parsing with bottom-up filtering
- Earley's Algorithm – Pop quiz on Earley's Algorithm

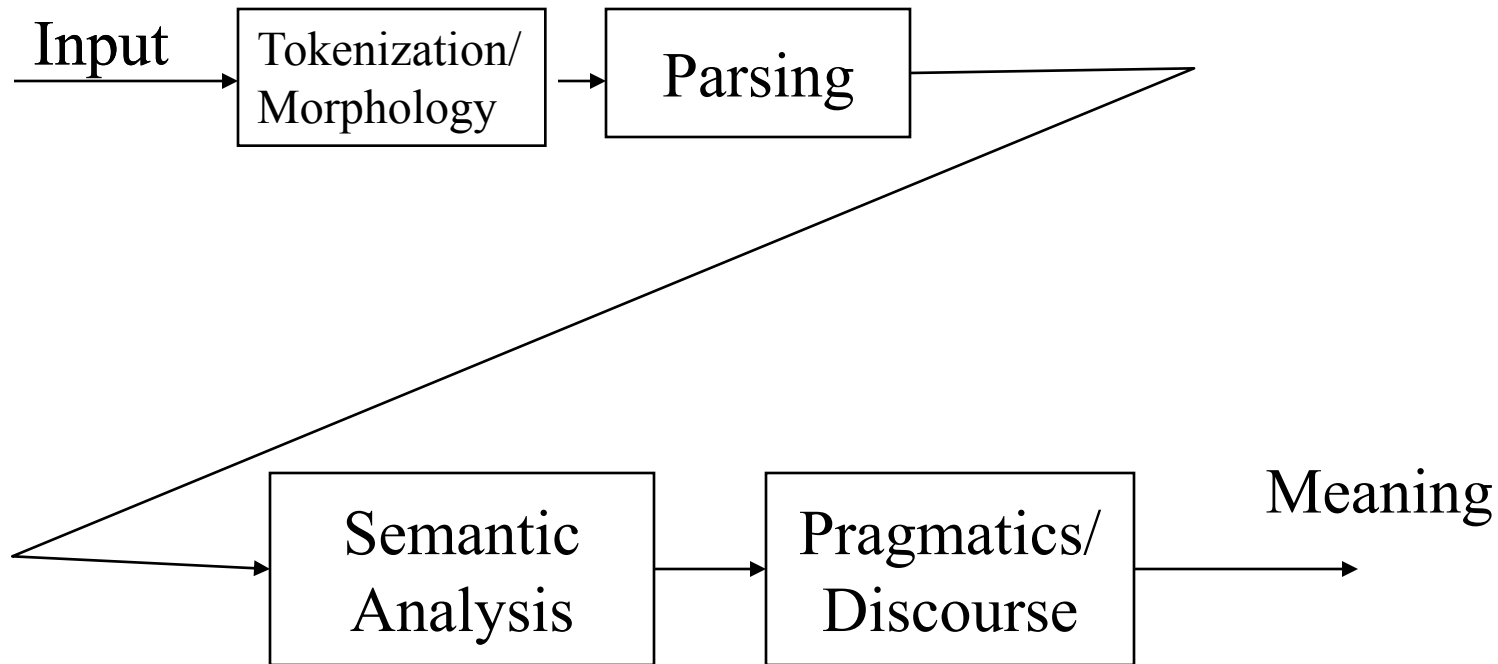
# Today's Lecture

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- Quiz 2 – 25 minutes
- Lexicalized and Probabilistic Parsing

# Natural Language Understanding

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# Lexicalized and Probabilistic Parsing

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- Resolving structural ambiguity:  
choose the most probable parse
- Use lexical dependency (relationship between words)

# Probability Model (1)

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- A derivation (tree) consists of the set of grammar rules that are in the tree
- The probability of a derivation (tree) is just the product of the probabilities of the rules in the derivation

# Probability Model (1.1)

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- The probability of a word sequence (sentence) is the probability of its tree in the unambiguous case
- It's the sum of the probabilities of the trees in the ambiguous case

# Formal

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$$P(T, S) = \prod_{n \in T} p(r(n))$$

$$P(T, S) = P(T)P(S | T)$$

Since  $P(S | T) = 1$ ,  $P(T, S) = P(T)$

$T$       Parse tree

$r$       rule

$n$       node in the pars tree

$p(r(n))$  probability of the rule expanded from node  $n$

# Probability Model

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- Attach probabilities to grammar rules
- The expansions for a given non-terminal sum to 1

VP  $\rightarrow$  Verb .55

VP  $\rightarrow$  Verb NP .40

VP  $\rightarrow$  Verb NP NP .05

# Probabilistic Context-Free Grammars

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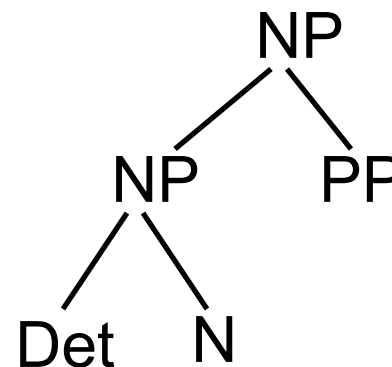
NP  $\rightarrow$  Det N : 0.4

NP  $\rightarrow$  NP<sub>poss</sub> N : 0.1

NP  $\rightarrow$  Pronoun : 0.2

NP  $\rightarrow$  NP PP : 0.1

NP  $\rightarrow$  N : 0.2



$P(\text{subtree above}) = 0.1 \times 0.4 = 0.04$

# Probabilistic Context-Free Grammars

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- PCFG
- Also called Stochastic CFG (SCFG)
- $G = (N, \Sigma, P, S, D)$ 
  - A set of non-terminal symbols (or variables)  $N$
  - A set of terminal symbols  $\Sigma$  ( $N \cap \Sigma = \emptyset$ )
  - A set of productions  $P$ , each of the form  $A \rightarrow \alpha$ , where  $A \in N$  and  $\alpha \in (\Sigma \cup N)^*$ 
    - \* denotes finite length of the infinite set of strings  $(\Sigma \cup N)$
  - A designated start symbol  $S \in N$
  - A function  $D$  that assigns a probability to each rule in  $P$
- $P(A \rightarrow \alpha)$  or  $P(A \rightarrow \alpha | A)$

# Probabilistic Context-Free Grammars

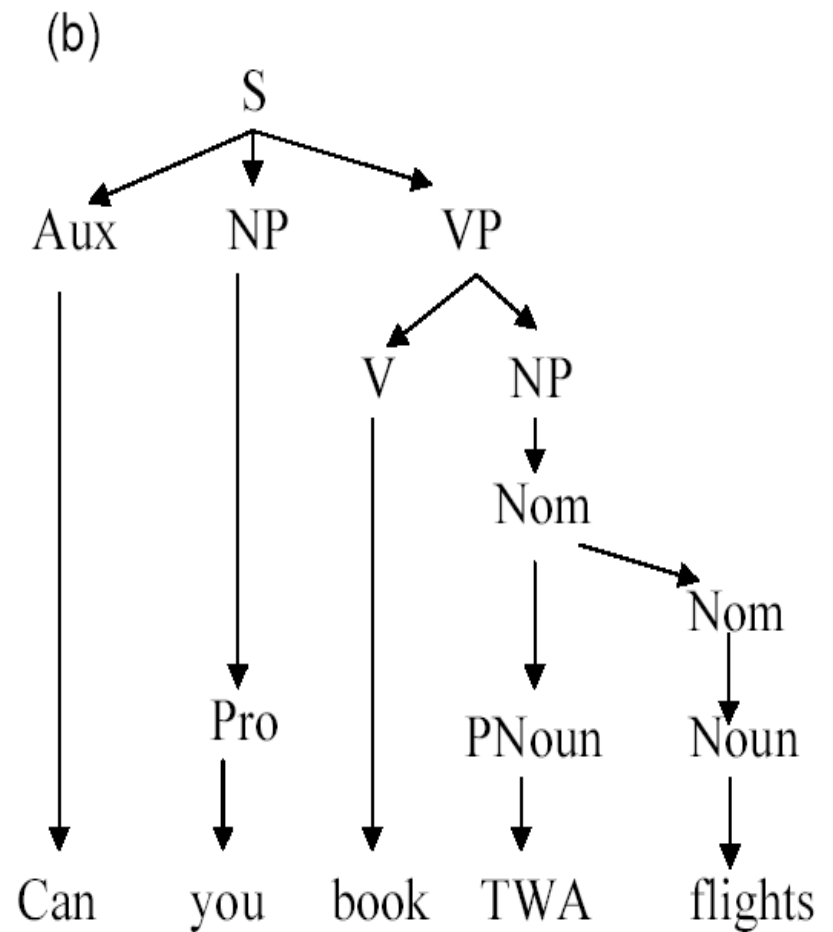
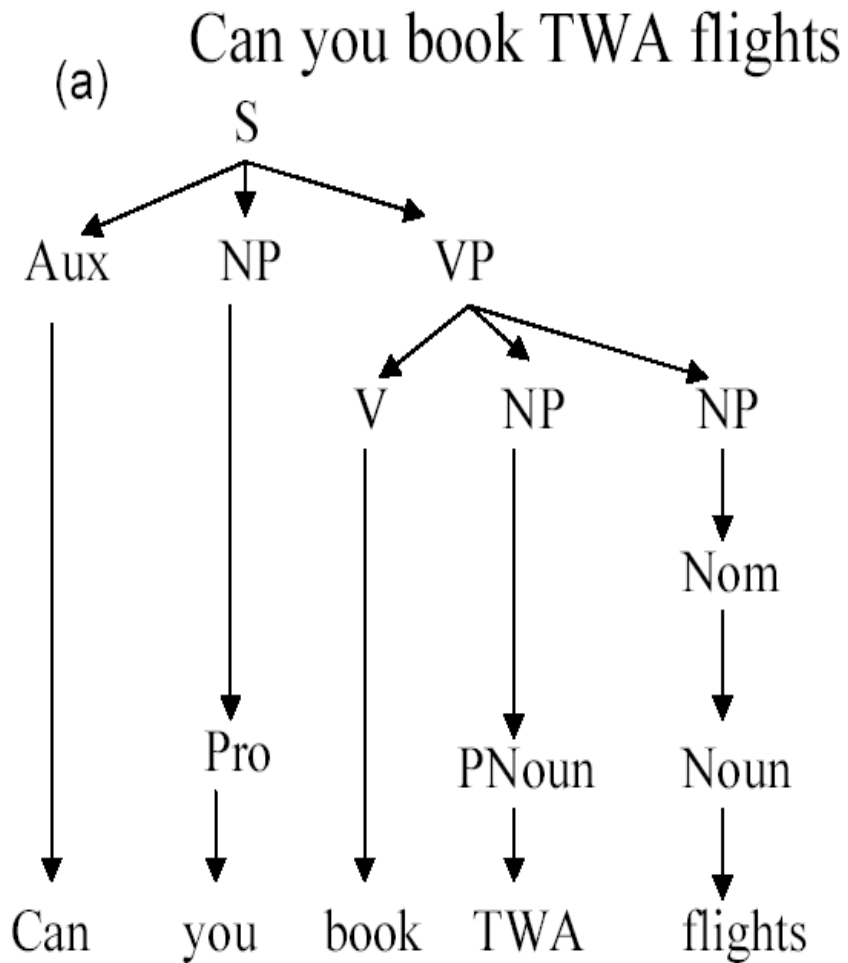
$S \rightarrow NP VP$	[.80]	$Det \rightarrow that$	[.05]		$the$	[.80]		$a$	[.15]
$S \rightarrow Aux NP VP$	[.15]	$Noun \rightarrow book$	[.10]						
$S \rightarrow VP$	[.05]	$Noun \rightarrow flights$	[.50]						
$NP \rightarrow Det Nom$	[.20]	$Noun \rightarrow meal$	[.40]						
$NP \rightarrow Proper-Noun$	[.35]	$Verb \rightarrow book$	[.30]						
$NP \rightarrow Nom$	[.05]	$Verb \rightarrow include$	[.30]						
$NP \rightarrow Pronoun$	[.40]	$Verb \rightarrow want$	[.40]						
$Nom \rightarrow Noun$	[.75]	$Aux \rightarrow can$	[.40]						
$Nom \rightarrow Noun Nom$	[.20]	$Aux \rightarrow does$	[.30]						
$Nom \rightarrow Proper-Noun Nom$	[.05]	$Aux \rightarrow do$	[.30]						
$VP \rightarrow Verb$	[.55]	$Proper-Noun \rightarrow TWA$	[.40]						
$VP \rightarrow Verb NP$	[.40]	$Proper-Noun \rightarrow Denver$	[.40]						
$VP \rightarrow Verb NP NP$	[.05]	$Pronoun \rightarrow you$	[.40]		$I$	[.60]			



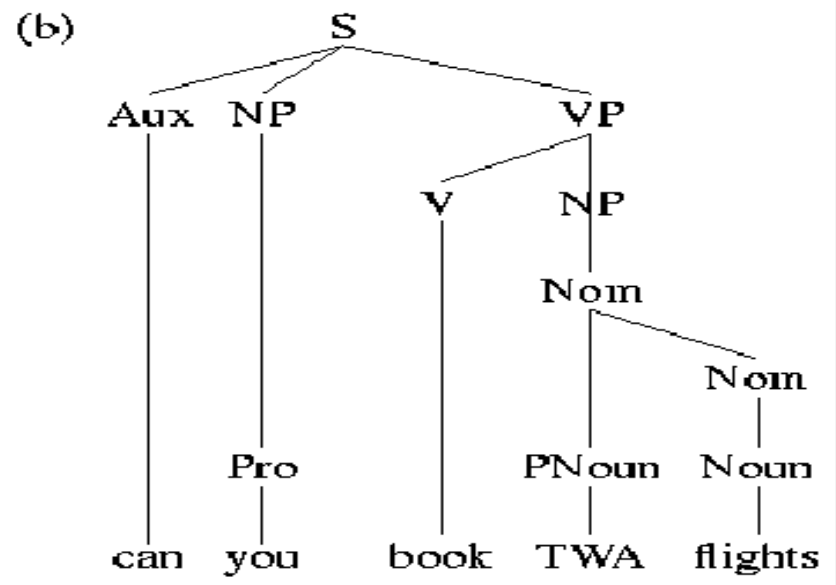
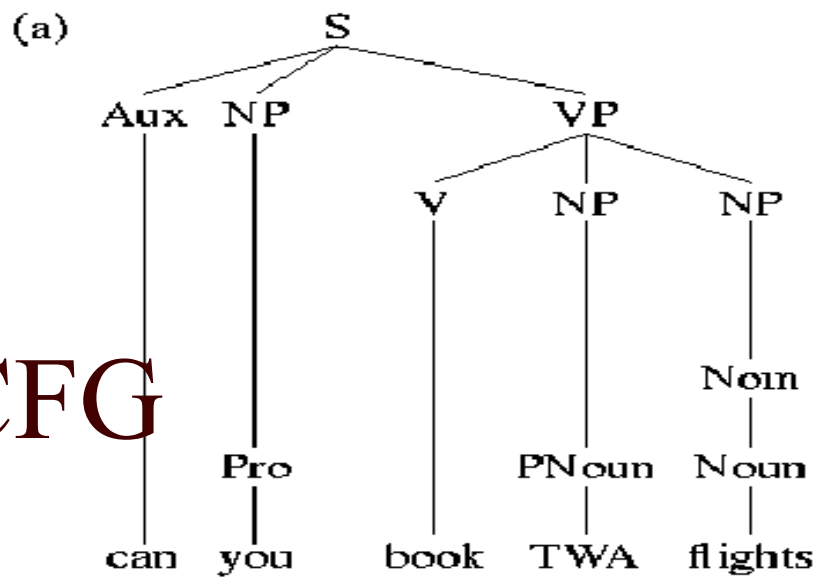
# English practice

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- What do you understand from the sentence:  
“Can you book TWA flights?”
  - Can you book flights on behalf of TWA?
    - → [TWA] [flights]
  - Can you book flights run by TWA?
    - → [TWA flights]



# PCFG



	Rules	P
S	→ Aux NP VP	.15
NP	→ Pro	.40
VP	→ V NP NP	.05
NP	→ Nom	.05
NP	→ PNoun	.35
Nom	→ Noun	.75
Aux	→ Can	.40
<del>NP</del>	<del>→ Pro</del>	<del>.40</del>
Pro	→ you	.40
Verb	→ book	.30
PNoun	→ TWA	.40
Noun	→ flights	.50

	Rules	P
S	→ Aux NP VP	.15
NP	→ Pro	.40
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NP	→ Nom	.05
Nom	→ PNoun Nom	.05
Nom	→ Noun	.75
Aux	→ Can	.40
<del>NP</del>	<del>→ Pro</del>	<del>.40</del>
Pro	→ you	.40
Verb	→ book	.30
Pnoun	→ TWA	.40
Noun	→ flights	.50

# PCFG

---

$$P(T, S) = \prod_{n \in T} p(r(n))$$

$$P(T, S) = P(T)P(S | T)$$

Since  $P(S | T) = 1$ ,  $P(T, S) = P(T)$

$T$  Parse tree

$r$  rule

$n$  node in the pars tree

$p(r(n))$  propability of the role expanded from node  $n$

$$P(T_l) = .15 \times .40 \times .05 \times .05 \times .35 \times .75 \times .40 \times .40 \times .30 \times .40 \times .50 = 3.78 \times 10^{-7}$$

$$P(T_r) = .15 \times .40 \times .40 \times .05 \times .05 \times .75 \times .40 \times .40 \times .30 \times .40 \times .50 = 4.32 \times 10^{-7}$$

$$\hat{T}(S) = \arg \max_T P(T | S) = \arg \max_T \frac{P(T, S)}{P(S)} = \arg \max_T P(T, S) = \arg \max_T P(T)$$

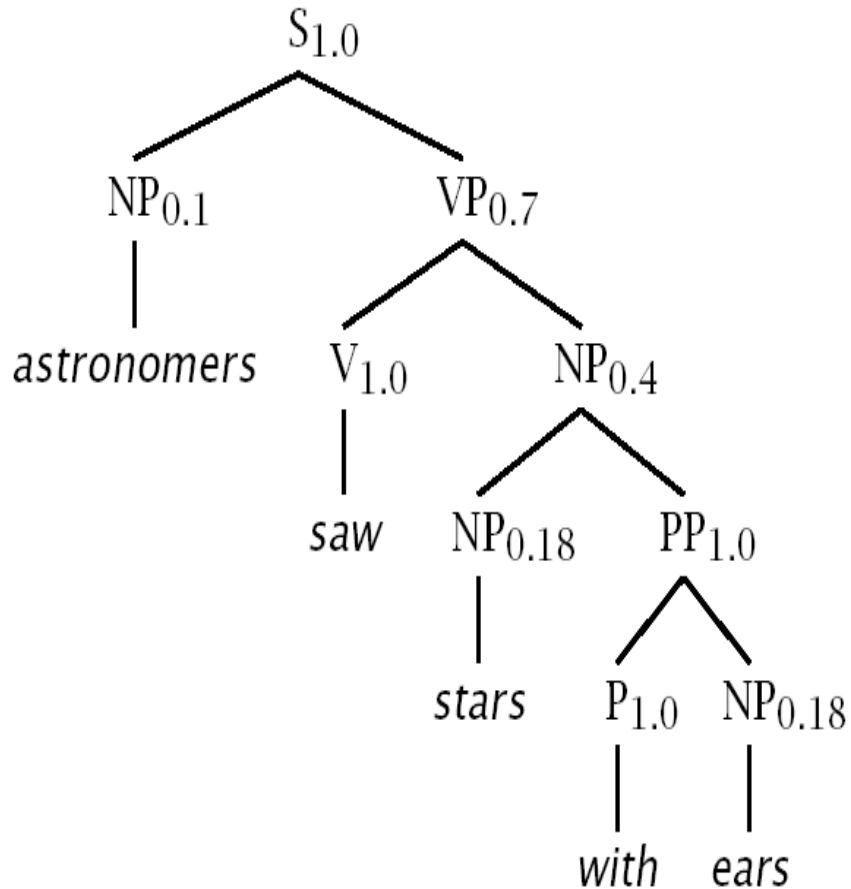
# A simple PCFG (in CNF)

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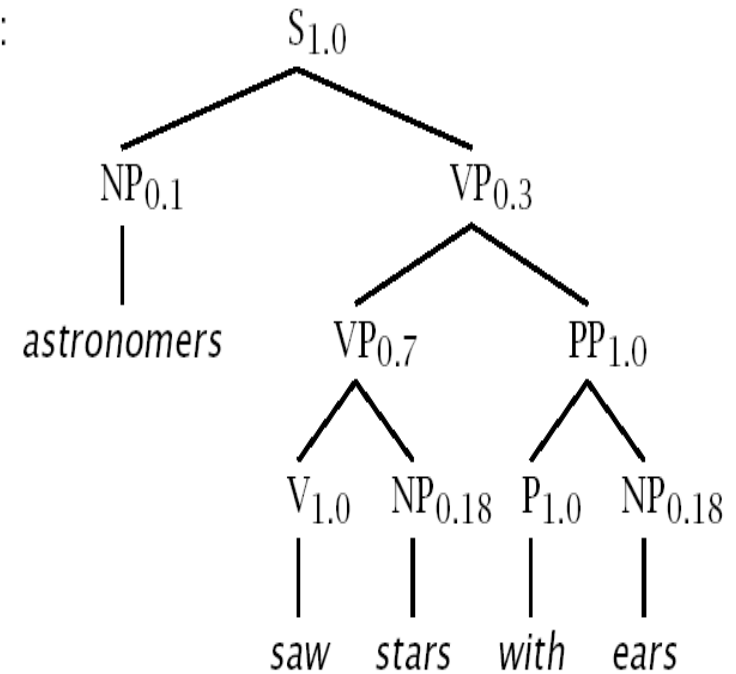
- $S \rightarrow NP VP$  1.0
- $NP \rightarrow NP PP$  0.4
- $PP \rightarrow P NP$  1.0
- $NP \rightarrow \text{astronomers}$  0.1
- $VP \rightarrow V NP$  0.7
- $NP \rightarrow \text{ears}$  0.18
- $VP \rightarrow VP PP$  0.3
- $NP \rightarrow \text{saw}$  0.04
- $P \rightarrow \text{with}$  1.0
- $NP \rightarrow \text{stars}$  0.18
- $V \rightarrow \text{saw}$  1.0
- $NP \rightarrow \text{telescopes}$  0.1

# Ex: Astronomers saw stars with ears

$t_1$ :



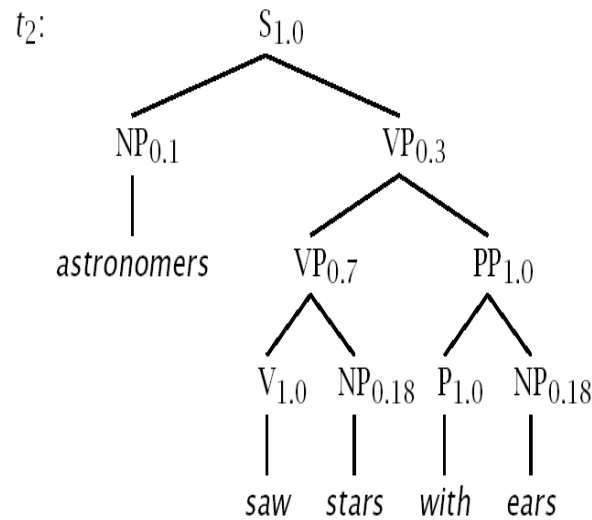
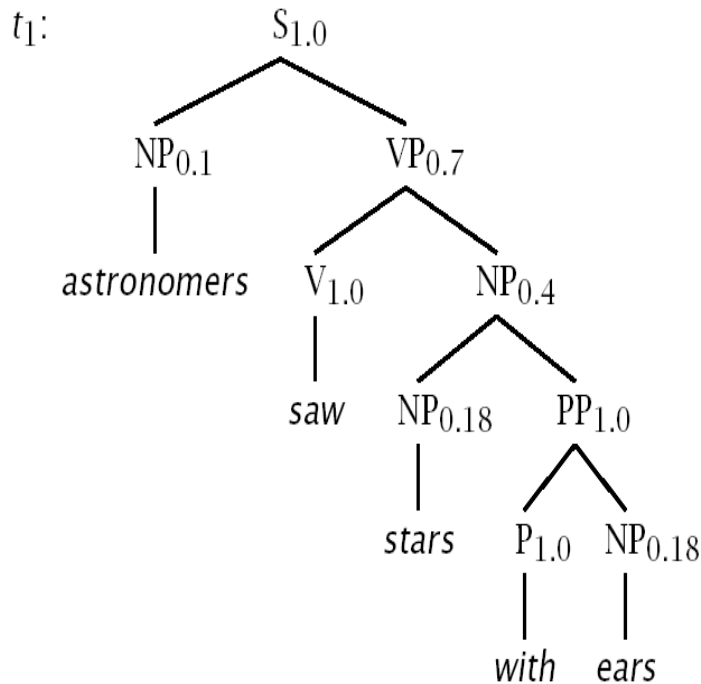
$t_2$ :



# The two parse trees' probabilities & the sentence probability

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- $P(t_1) = 1.0 \times 0.1 \times 0.7 \times 1.0 \times 0.4 \times 0.18 \times 1.0 \times 1.0 \times 0.18 = 0.0009072$
- $P(t_2) = 1.0 \times 0.1 \times 0.3 \times 0.7 \times 1.0 \times 0.18 \times 1.0 \times 1.0 \times 0.18 = 0.0006804$
- $P(w_{15}) = P(t_1) + P(t_2) = 0.0015876$



- S → NP VP 1.0
- PP → P NP 1.0
- VP → V NP 0.7
- VP → VP PP 0.3
- P → with 1.0
- V → saw 1.0
- NP → NP PP 0.4
- NP → astronomers 0.1
- NP → ears 0.18
- NP → saw 0.04
- NP → stars 0.18
- NP → telescopes 0.1

- $P(t_1) = 1.0 \times 0.1 \times 0.7 \times 1.0 \times 0.4 \times 0.18 \times 1.0 \times 1.0 \times 0.18 = 0.0009072$
- $P(t_2) = 1.0 \times 0.1 \times 0.3 \times 0.7 \times 1.0 \times 0.18 \times 1.0 \times 1.0 \times 0.18 = 0.0006804$
- $P(w_{15}) = P(t_1) + P(t_2) = 0.0015876$



# Probabilistic CFGs

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- The probabilistic model
  - Assigning probabilities to parse trees
- Getting the probabilities for the model
- Parsing with probabilities
  - Slight modification to dynamic programming approach
  - Task is to find the max probability tree for an input

# Getting the Probabilities

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- From an annotated database (a treebank)
- Learned from a corpus

# Treebank

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- Get a large collection of parsed sentences
- Collect counts for each non-terminal rule expansion in the collection
- Normalize
- Done

# Learning

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- What if you don't have a treebank (and can't get one)
- Take a large collection of text and parse it.
- In the case of syntactically ambiguous sentences collect all the possible parses
- Prorate the rule statistics gathered for rules in the ambiguous case by their probability
- Proceed as you did with a treebank.
- **Inside-Outside** algorithm

# Assumptions

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- We're assuming that there is a **grammar** to be used to parse with.
- We're assuming the existence of a large robust **dictionary** with parts of speech
- We're assuming the ability to parse (i.e. **a parser**)
- Given all that... we can parse probabilistically

# Typical Approach

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- Bottom-up dynamic programming approach
- Assign probabilities to constituents as they are completed and placed in the table
- Use the max probability for each constituent going up

# Max probability

---

- Say we're talking about a final part of a parse
  - $S_0 \rightarrow NP_i VP_j$

The probability of the S is...

$$P(S \rightarrow NP VP) * P(NP) * P(VP)$$

The green stuff is already known. We're doing  
bottom-up parsing

# Max

---

- The P(NP) is known.
- What if there are multiple NPs for the span of text in question ( $0$  to  $i$ )?
- Take the max (Why?)
- Does not mean that other kinds of constituents for the same span are ignored (i.e. they might be in the solution)



# Probabilistic Parsing

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- Probabilistic CYK (Cocke-Younger-Kasami) algorithm for parsing PCFG
- Bottom-up dynamic programming algorithm
- Assume PCFG is in Chomsky Normal Form (production is either  $A \rightarrow B C$  or  $A \rightarrow a$ )

# Chomsky Normal Form (CNF)

---

All rules have form:

$A \rightarrow BC$

and

$A \rightarrow a$

Non-Terminal    Non-Terminal<sub>1</sub>

terminal

# Examples:

---

$$S \rightarrow AS$$

$$S \rightarrow a$$

$$A \rightarrow SA$$

$$A \rightarrow b$$

Chomsky  
Normal Form

$$S \rightarrow AS$$

$$S \rightarrow AAS$$

$$A \rightarrow SA$$

$$A \rightarrow aa$$

Not Chomsky  
Normal Form

# Observations

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- Chomsky normal forms are good for parsing and proving theorems
- It is possible to find the Chomsky normal form of any context-free grammar

# Probabilistic CYK Parsing of PCFGs

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- CYK Algorithm: bottom-up parser
- Input:
  - A Chomsky normal form PCFG,  $G = (N, \Sigma, P, S, D)$   
Assume that the  $N$  non-terminals have indices  $1, 2, \dots, |N|$ , and the start symbol  $S$  has index 1
  - $n$  words  $w_1, \dots, w_n$
- Data Structure:
  - A dynamic programming array  $\pi[i, j, a]$  holds the maximum probability for a constituent with non-terminal index  $a$  spanning words  $i..j$ .
- Output:
  - The maximum probability parse  $\pi[1, n, 1]$

# Base Case

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- CYK fills out  $\pi[i,j,a]$  by induction
- Base case
  - Input strings with length = 1 (individual words  $w_i$ )
  - In CNF, the probability of a given non-terminal  $A$  expanding to a single word  $w_i$  must come only from the rule  $A \rightarrow w_i$  i.e.,  $P(A \rightarrow w_i)$

# Probabilistic CYK Algorithm [Corrected]

```
Function CYK(words, grammar)
    return the most probable parse and its probability
For  $i \leftarrow 1$  to num_words
    for  $a \leftarrow 1$  to num_nonterminals
        If ( $A \rightarrow w_i$ ) is in grammar then  $\pi[i, i, a] \leftarrow P(A \rightarrow w_i)$ 
For  $span \leftarrow 2$  to num_words
    For  $begin \leftarrow 1$  to  $num\_words - span + 1$ 
         $end \leftarrow begin + span - 1$ 
        For  $m \leftarrow begin$  to  $end - 1$ 
            For  $a \leftarrow 1$  to num_nonterminals
                For  $b \leftarrow 1$  to num_nonterminals
                    For  $c \leftarrow 1$  to num_nonterminals
                         $prob \leftarrow \pi[begin, m, b] \times \pi[m+1, end, c] \times P(A \rightarrow BC)$ 
                        If ( $prob > \pi[begin, end, a]$ ) then
                             $\pi[begin, end, a] = prob$ 
                             $back[begin, end, a] = \{m, b, c\}$ 
Return  $build\_tree(back[1, num\_words, 1], \pi[1, num\_words, 1])$ 
```

# The CYK Membership Algorithm

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## Input:

- Grammar  $G$  in Chomsky Normal Form
- String  $w$

## Output:

find if  $w \in L(G)$



# The Algorithm

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Input example:

- Grammar  $G$ :

$$S \rightarrow AB$$

$$A \rightarrow BB$$

$$A \rightarrow a$$

$$B \rightarrow AB$$

$$B \rightarrow b$$

- String :  $w$   $aabbb$

# *aabbb*

---

All substrings of length 1      a      a      b      b      b

All substrings of length 2      aa      ab      bb      bb

All substrings of length 3      aab      abb      bbb

All substrings of length 4      aabb      abbb

All substrings of length 5      aabbb

---

$S \rightarrow AB$

a

a

b

b

b

$A \rightarrow BB$

A

A

B

B

B

---

$A \rightarrow a$

aa

ab

bb

bb

$B \rightarrow AB$

aab

abb

bbb

$B \rightarrow b$

aabb

abbb

aabbb

$$S \rightarrow AB$$

$$A \rightarrow BB$$

$$A \rightarrow a$$

$$B \rightarrow AB$$

$$B \rightarrow b$$

---

a	a	b	b	b
A	A	B	B	B

---

aa	ab	bb	bb
----	----	----	----

	S,B	A	A
--	-----	---	---

---

aab	abb	bbb
-----	-----	-----

aabb	abbb
------	------

aabbb
-------

---

$S \rightarrow AB$

a	a	b	b	b
A	A	B	B	B

---

$A \rightarrow BB$

aa	ab	bb	bb
----	----	----	----

$A \rightarrow a$

S,B	A	A
-----	---	---

---

$B \rightarrow AB$

aab	abb	bbb
-----	-----	-----

S,B	A	S,B
-----	---	-----

---

$B \rightarrow b$

aabb	abbb
------	------

A	S,B
---	-----

---

aabbb

Ⓢ,B

Therefore:  $aabbb \in L(G)$

# CYK Algorithm for Deciding Context Free Languages

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IDEA: For each substring of a given input  $x$ , find all variables which can derive the substring. Once these have been found, telling which variables generate  $x$  becomes a simple matter of looking at the grammar, since it's in Chomsky normal form

# Thank you

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