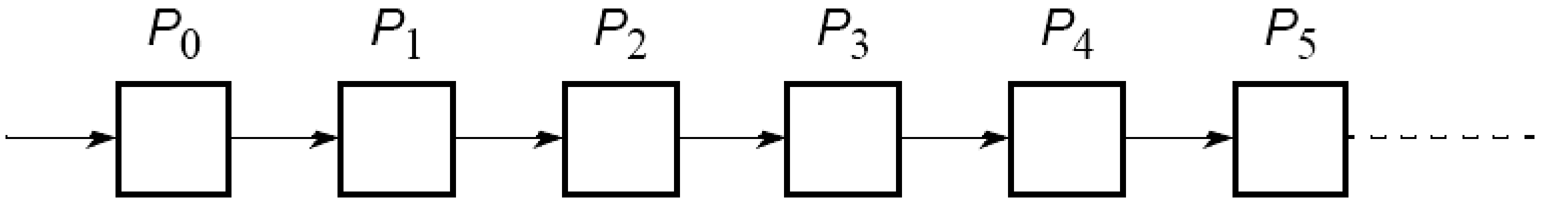
Chapter 5

Pipelined Computations

Pipelined Computations

Problem divided into a series of tasks that have to be completed one after the other (the basis of sequential programming). Each task executed by a separate process or processor.



Example

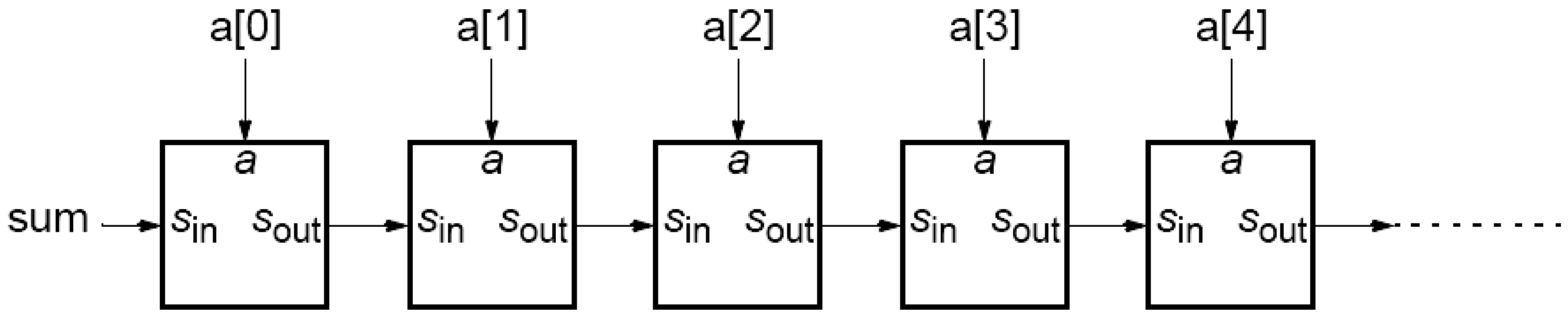
Add all the elements of array **a** to an accumulating sum:

```
for (i = 0; i < n; i++)
sum = sum + a[i];
```

The loop could be “unfolded” to yield

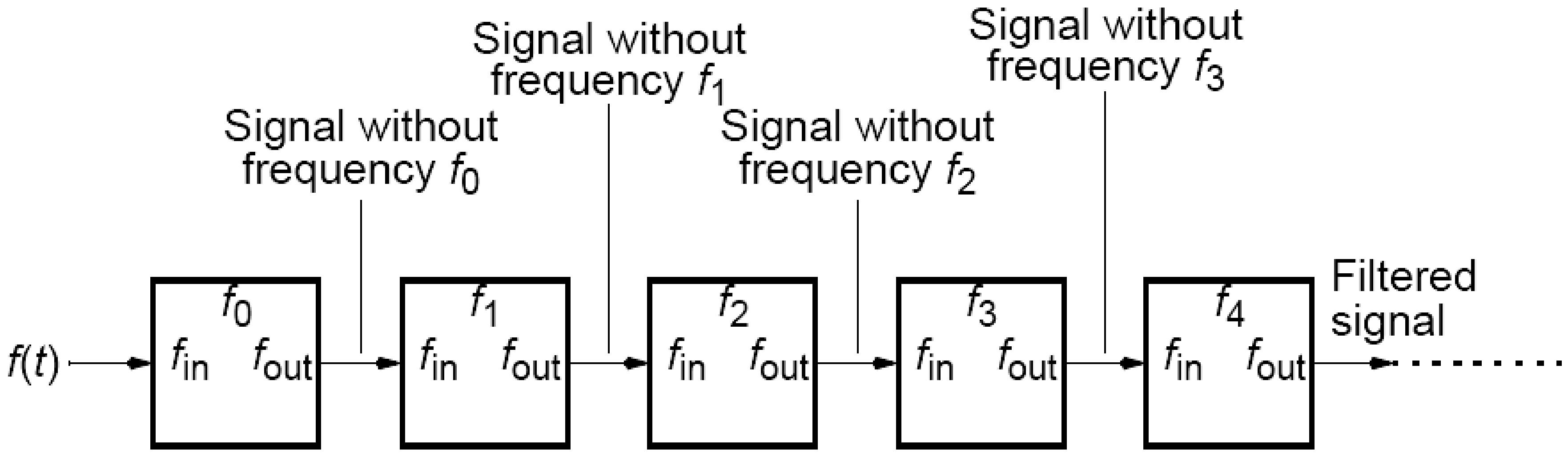
```
sum = sum + a[0];
sum = sum + a[1];
sum = sum + a[2];
sum = sum + a[3];
sum = sum + a[4];
          .
          .
          .
```

Pipeline for an unfolded loop



Another Example

Frequency filter - Objective to remove specific frequencies (f_0, f_1, f_2,f_3, etc.) from a digitized signal, $f(t)$.
Signal enters pipeline from left:



Where pipelining can be used to good effect

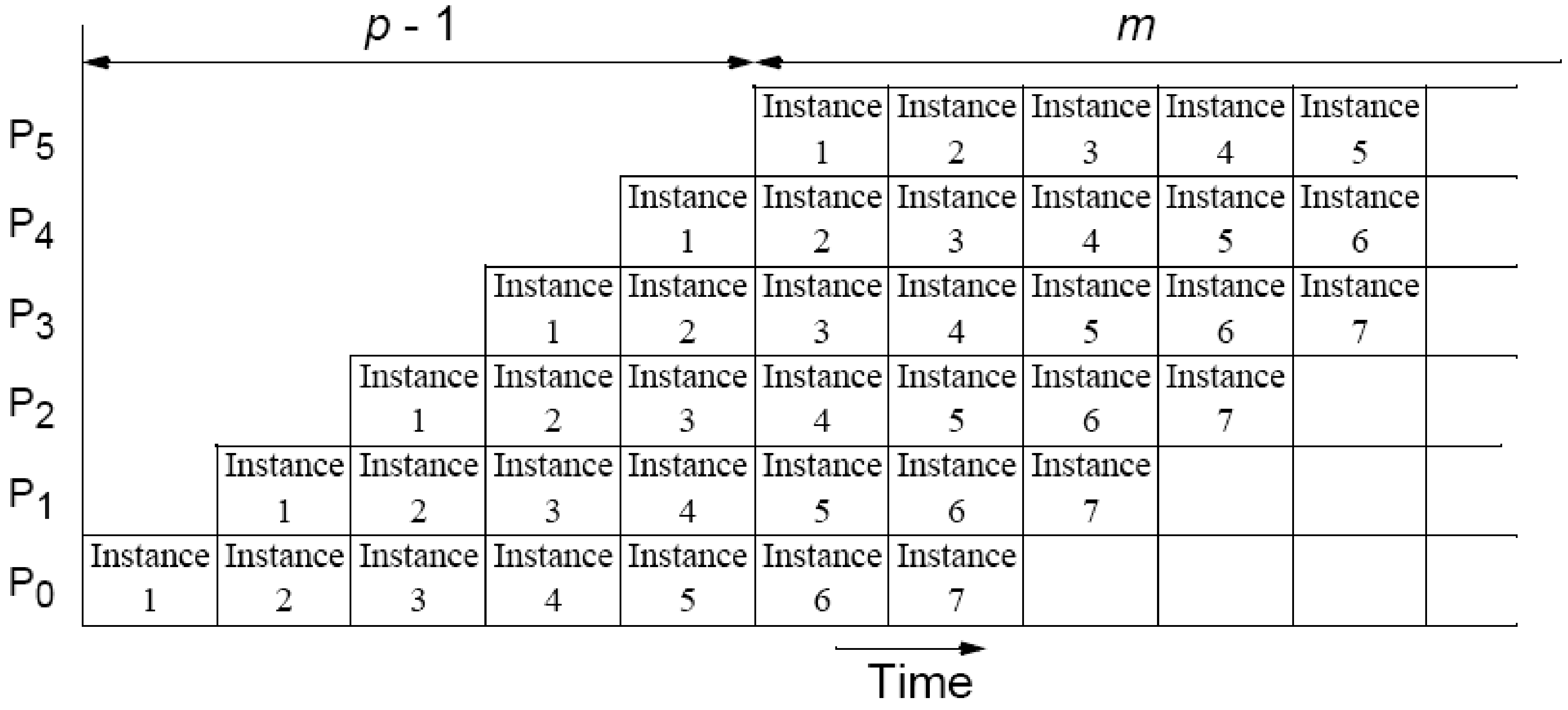
Assuming problem can be divided into a series of sequential tasks, pipelined approach can provide increased execution speed under the following three types of computations:

1. If more than one instance of the complete problem is to be Executed

2. If a series of data items must be processed, each requiring multiple operations

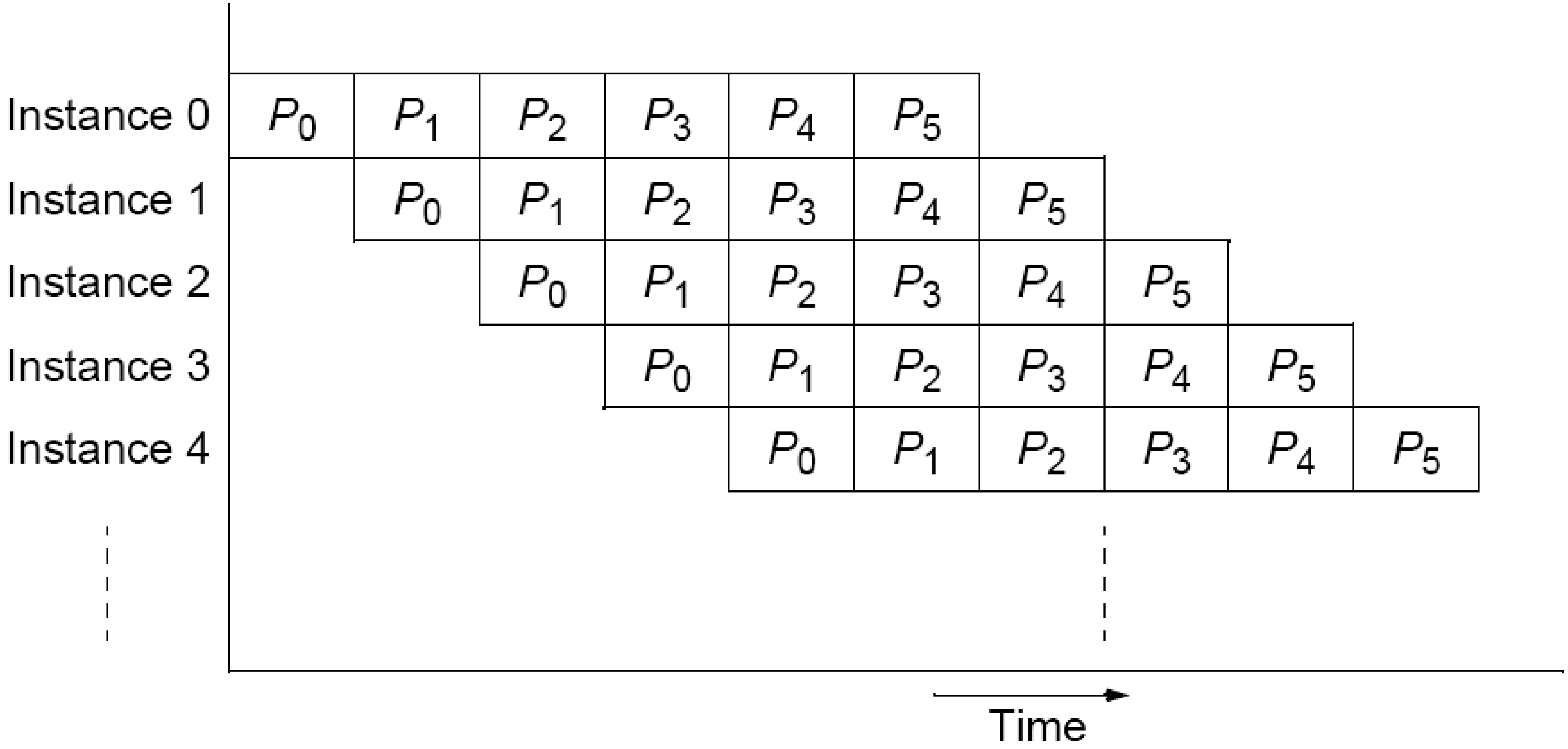
3. If information to start next process can be passed forward before process has completed all its internal operations

"Type 1" Pipeline Space-Time Diagram

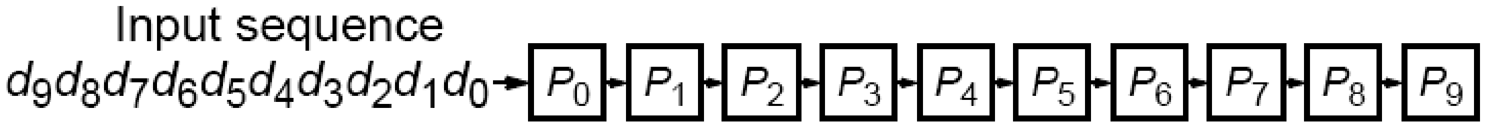


P_5						Instance 1	Instance 2	Instance 3	Instance 4	Instance 5
P_4					Instance 1	Instance 2	Instance 3	Instance 4	Instance 5	Instance 6
P_3				Instance 1	Instance 2	Instance 3	Instance 4	Instance 5	Instance 6	Instance 7
P_2			Instance 1	Instance 2	Instance 3	Instance 4	Instance 5	Instance 6	Instance 7	
P_1		Instance 1	Instance 2	Instance 3	Instance 4	Instance 5	Instance 6	Instance 7		
P_0	Instance 1	Instance 2	Instance 3	Instance 4	Instance 5	Instance 6	Instance 7			

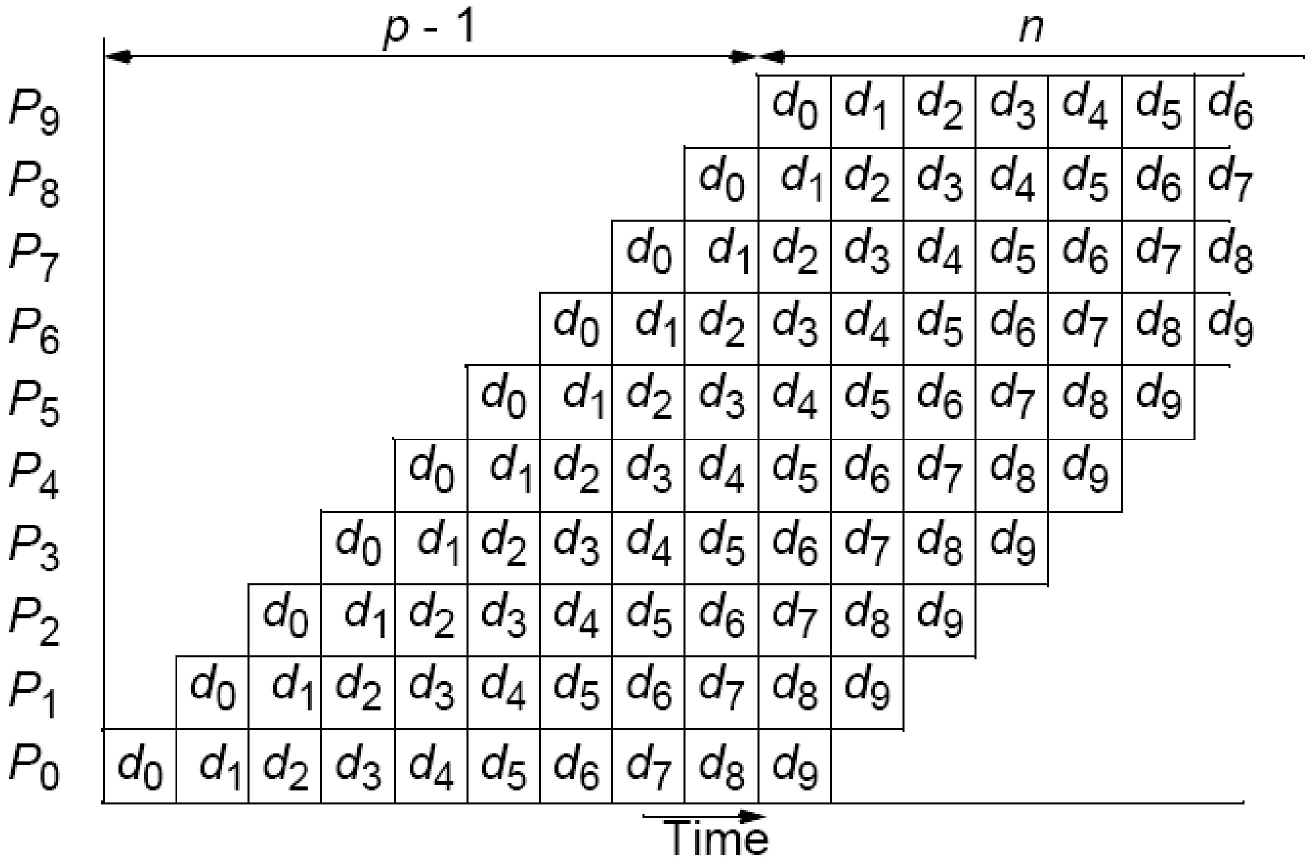
Alternative space-time diagram


Instance 0
P_0 P_1 P_2 P_3 P_4 P_5
Instance 1
P_0 P_1 P_2 P_3 P_4 P_5
Instance 2
P_0 P_1 P_2 P_3 P_4 P_5
Instance 3
P_0 P_1 P_2 P_3 P_4 P_5
Instance 4
P_0 P_1 P_2 P_3 P_4 P_5
Time

"Type 2" Pipeline Space-Time Diagram

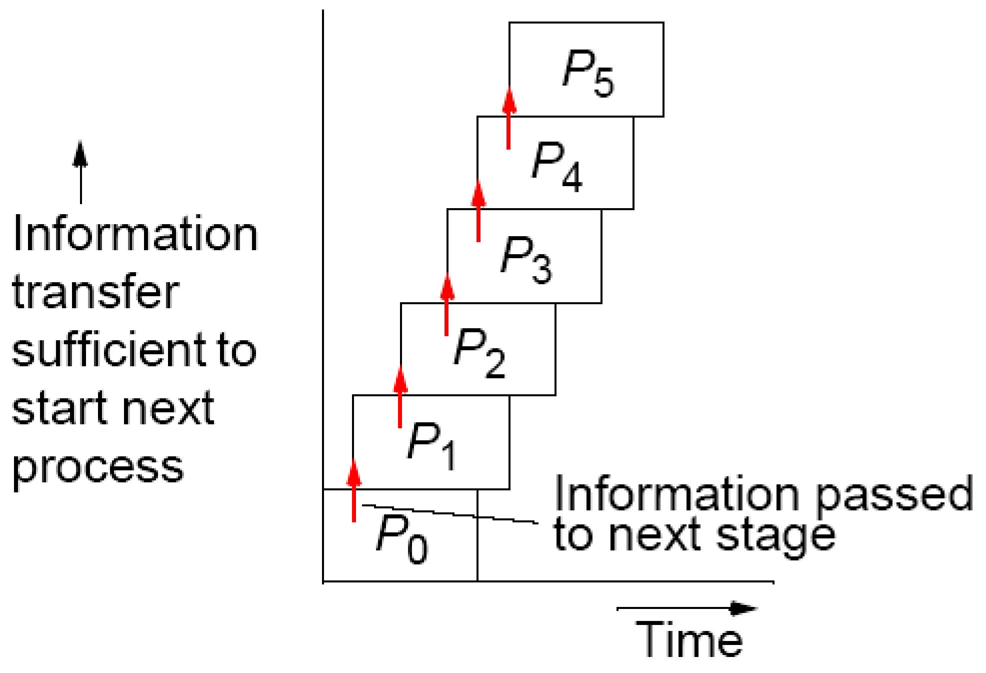


(a) Pipeline structure

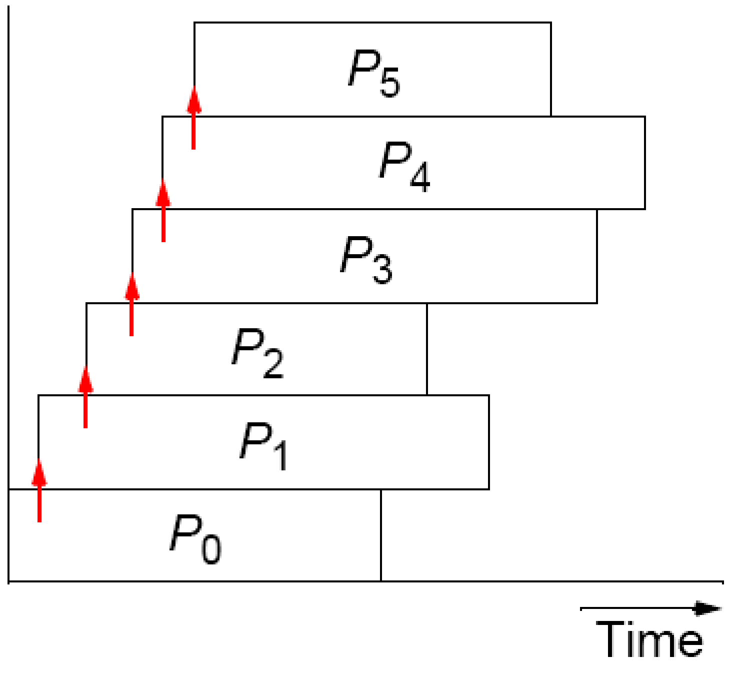


(b) Timing diagram

“Type 3” Pipeline Space-Time Diagram


Information transfer sufficient to start next process
P_5
P_4
P_3
P_2
P_1
P_0
Information passed to next stage
Time

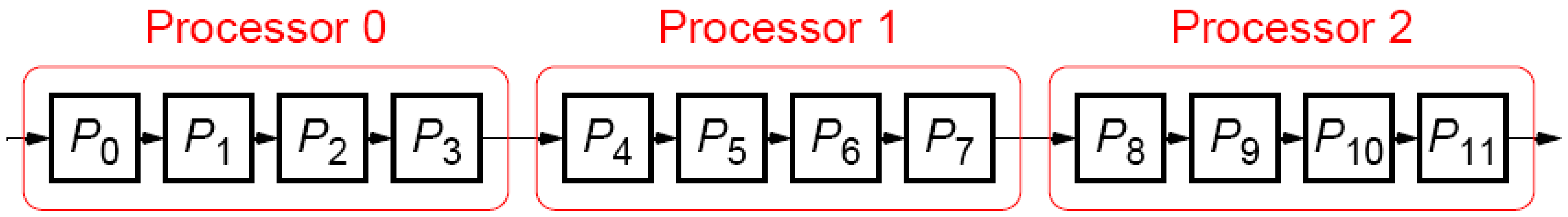
(a) Processes with the same execution time



(b) Processes not with the same execution time

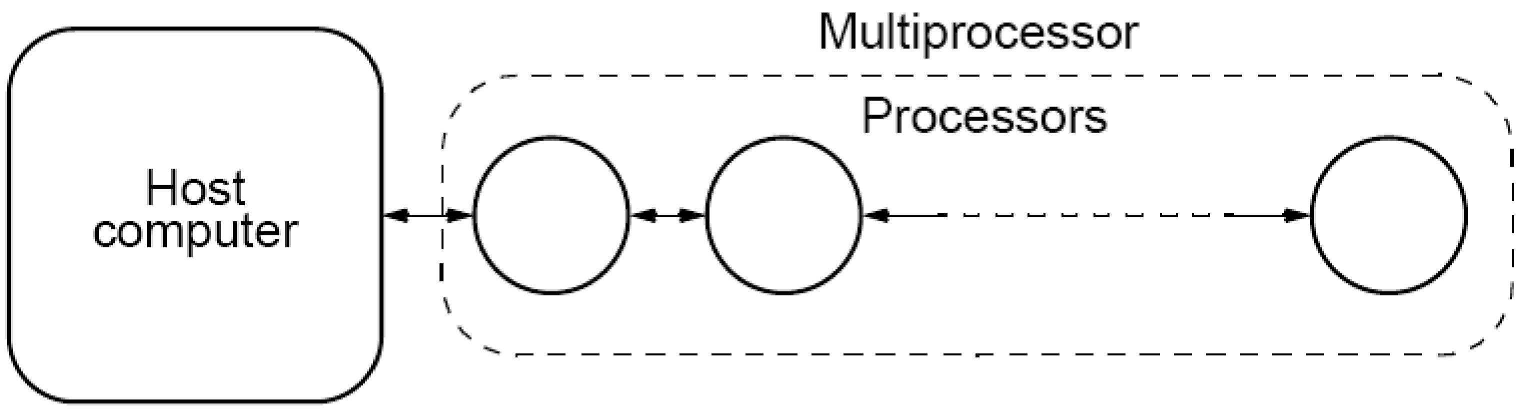
Pipeline processing where information passes to next stage before previous state completed.

If the number of stages is larger than the number of processors in any pipeline, a group of stages can be assigned to each processor:



Computing Platform for Pipelined Applications

Multiprocessor system with a line configuration

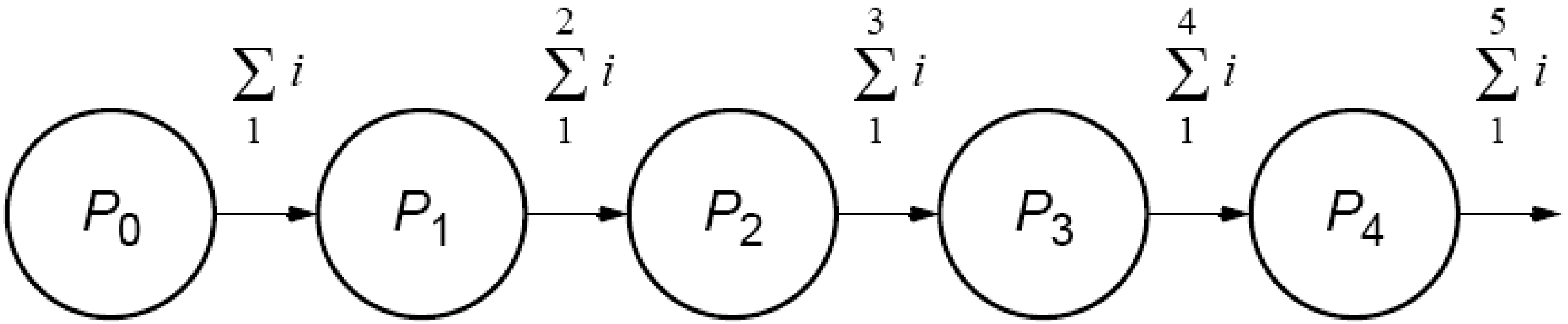


Strictly speaking pipeline may not be the best structure for a cluster - however a cluster with switched direct connections, as most have, can support simultaneous message passing.

Example Pipelined Solutions
(Examples of each type of computation)

Pipeline Program Examples

Adding Numbers



Type 1 pipeline computation

Basic code for process *Pi* :

```
recv(&accumulation, Pi-1);
accumulation = accumulation + number;
send(&accumulation, Pi+1);
```

except for the first process, *P*0, which is

```
send(&number, P1);
```

and the last process, *Pn*-1, which is

```
recv(&number, Pn-2);
accumulation = accumulation + number;
```

SPMD program

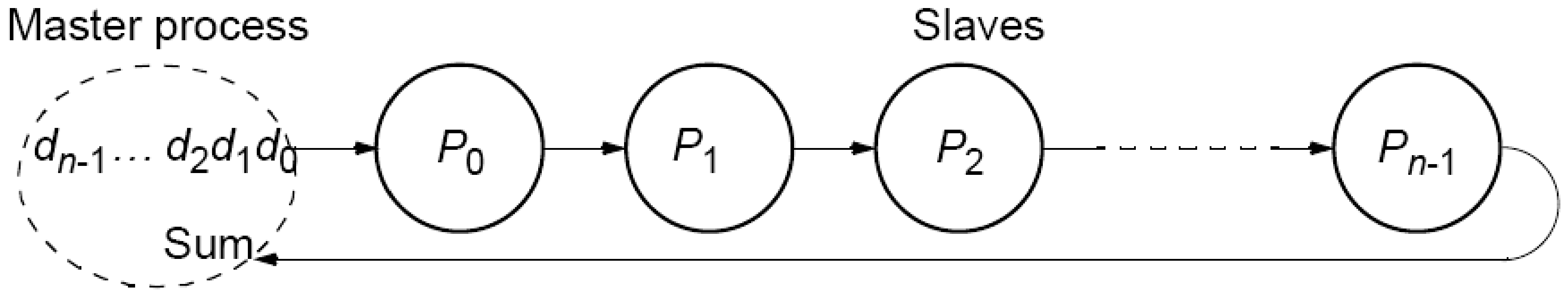
```
if (process > 0) {
        recv(&accumulation, Pi-1);
        accumulation = accumulation + number;
}
if (process < n-1)
        send(&accumulation, P i+1);
```

The final result is in the last process.

Instead of addition, other arithmetic operations could be done.

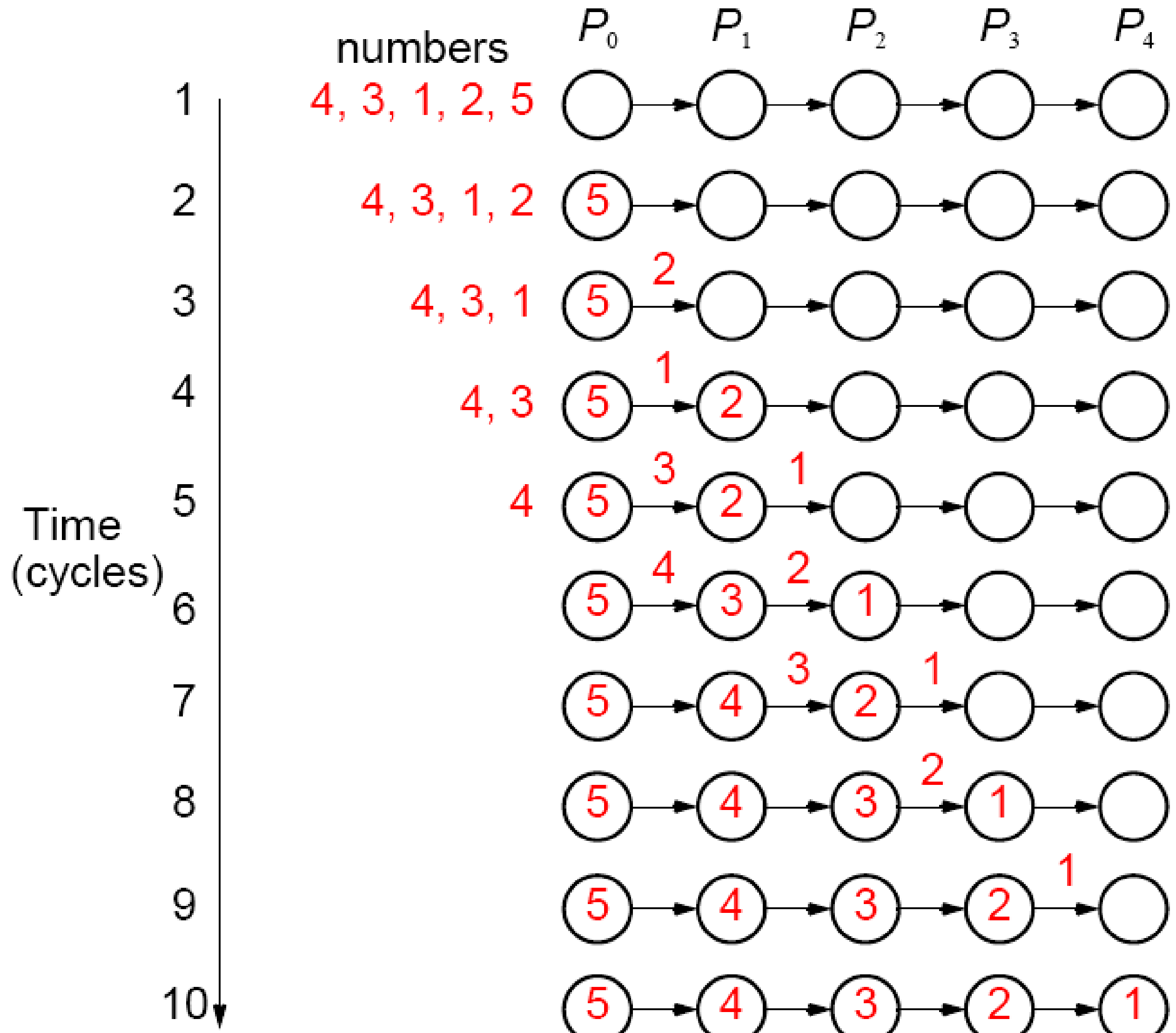
Pipelined addition numbers

Master process and ring configuration

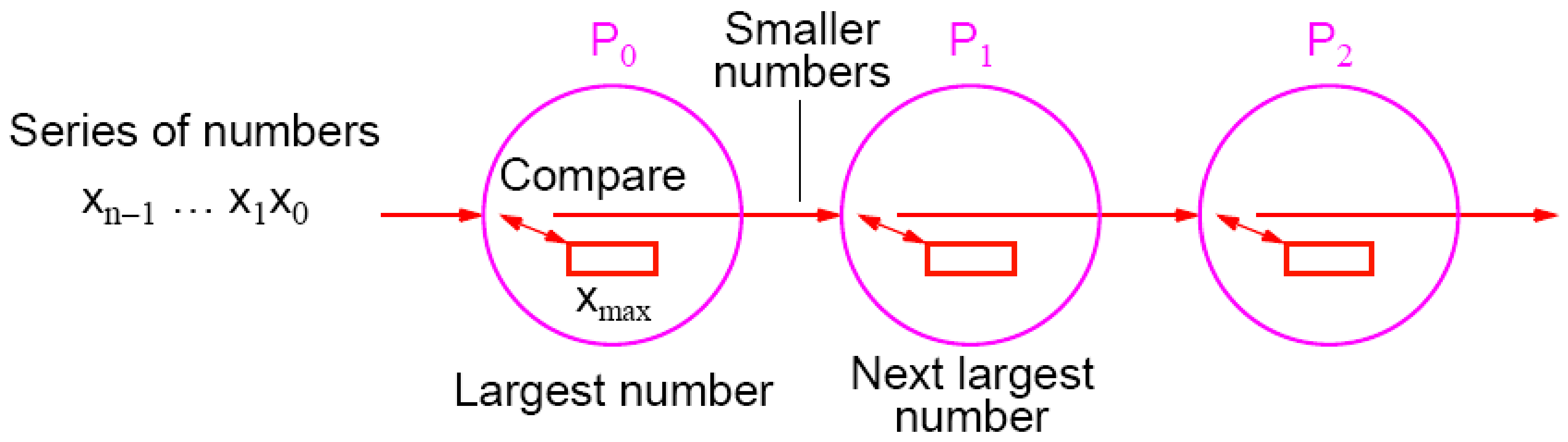


Sorting Numbers

A parallel version of *insertion sort*.



Pipeline for sorting using insertion sort



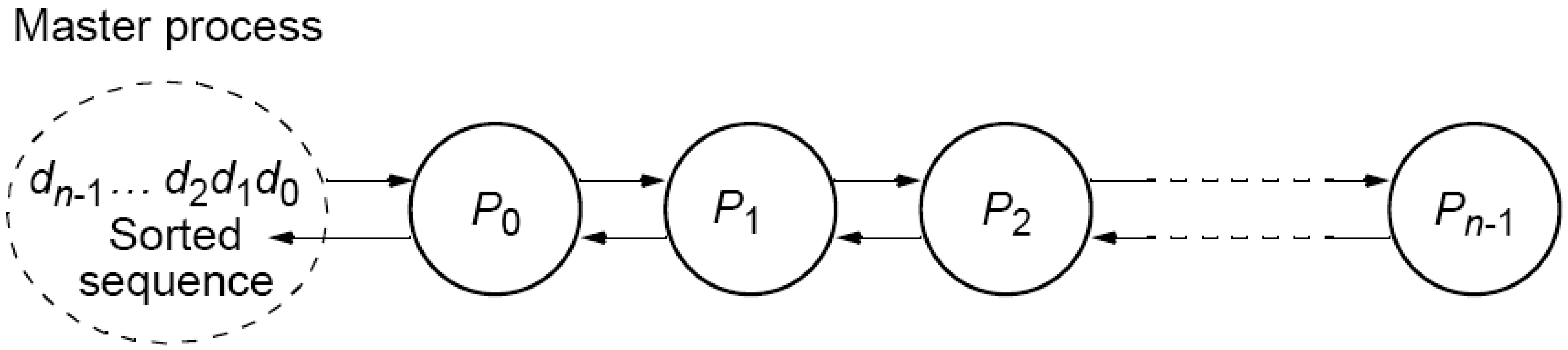
Type 2 pipeline computation

The basic algorithm for process *Pi* is

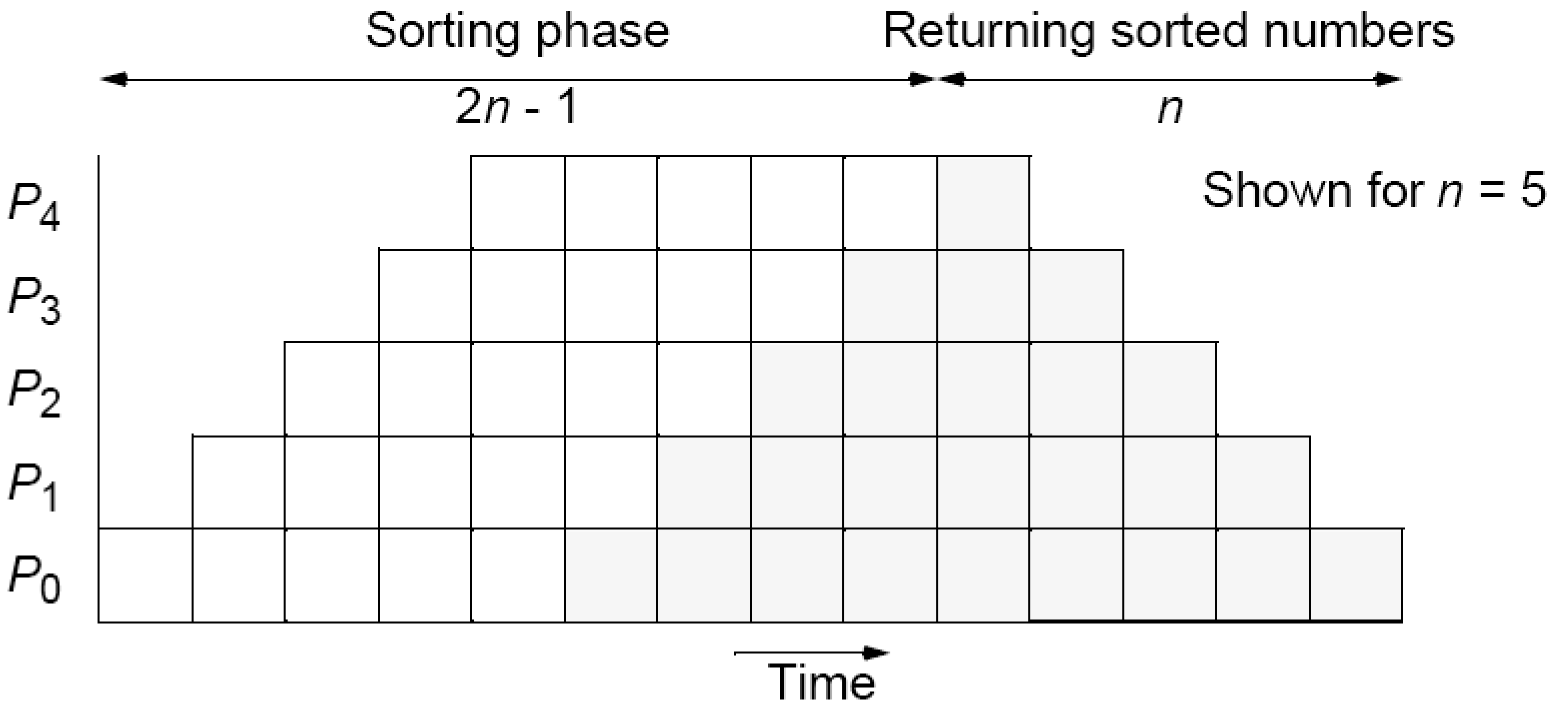
```
recv(&number, Pi-1);
if (number > x) {
        send(&x, Pi+1);
        x = number;
} else send(&number, Pi+1);
```

With *n* numbers, number *i*th process is to accept = *n* - *i*.
Number of passes onward = *n* - *i* - 1
Hence, a simple loop could be used.

Insertion sort with results returned to master process using bidirectional line configuration



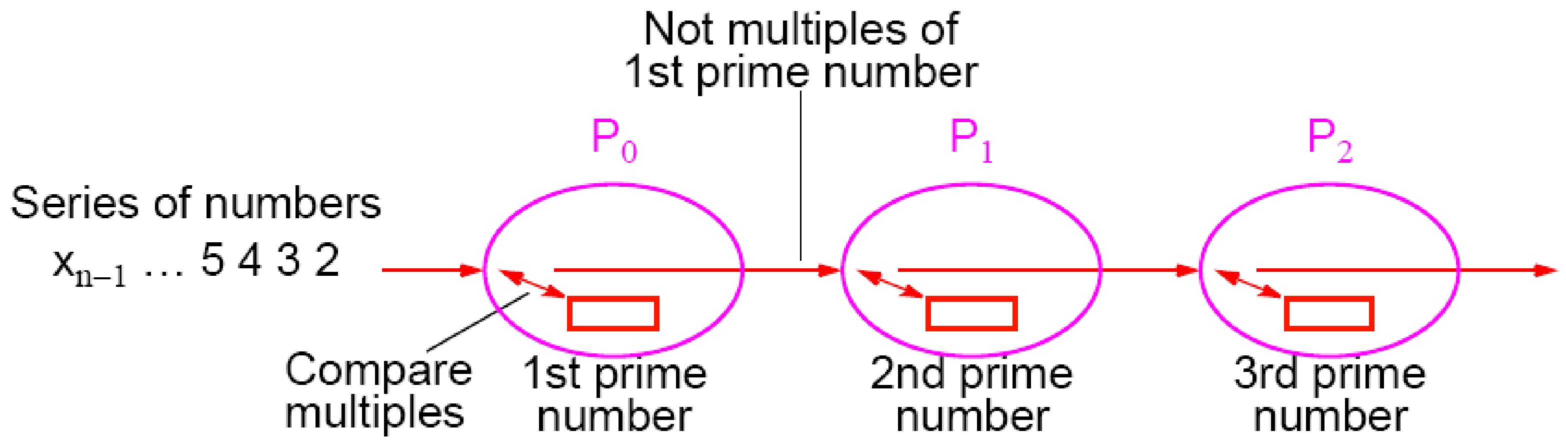
Insertion sort with results returned



Prime Number Generation

Sieve of Eratosthenes

- Series of all integers generated from 2.
- First number, 2, is prime and kept.
- All multiples of this number deleted as they cannot be prime.
- Process repeated with each remaining number.
- The algorithm removes non-primes, leaving only primes.



Type 2 pipeline computation

The code for a process, *Pi*, could be based upon

```
recv(&x, Pi-1);
/* repeat following for each number */
recv(&number, Pi-1);
if ((number % x) != 0) send(&number, P i+1);
```

Each process will not receive the same number of numbers and is not known beforehand. Use a “terminator” message, which is sent at the end of the sequence:

```
recv(&x, Pi-1);
for (i = 0; i < n; i++) {
      recv(&number, Pi-1);
      If (number == terminator) break;
       (number % x) != 0) send(&number, P i+1);
}
```

Solving a System of Linear Equations

Upper-triangular form

$$
\begin{array}{lr}
a_{n-1,0}x_0 + a_{n-1,1}x_1 + a_{n-1,2}x_2 \quad \ldots \quad + a_{n-1,n-1}x_{n-1} & = b_{n-1} \\
. & \\
. & \\
a_{2,0}x_0 + a_{2,1}x_1 + a_{2,2}x_2 & = b_2 \\
a_{1,0}x_0 + a_{1,1}x_1 & = b_1 \\
a_{0,0}x_0 & = b_0
\end{array}
$$

where *a’s* and *b’s* are constants and *x*’s are unknowns to be found.

Back Substitution

First, unknown x_0 is found from last equation; i.e.,

$$x_0 = \frac{b_0}{a_{0,0}}$$

Value obtained for x_0 substituted into next equation to obtain x_1; i.e.,

$$x_1 = \frac{b_1 - a_{1,0}x_0}{a_{1,1}}$$

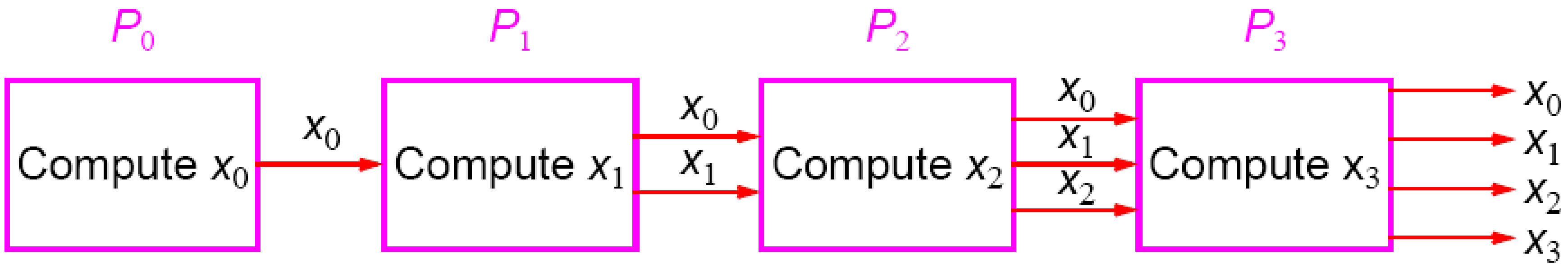
Values obtained for x_1 and x_0 substituted into next equation to obtain x_2:

$$x_2 = \frac{b_2 - a_{2,0}x_0 - a_{2,1}x_1}{a_{2,2}}$$

and so on until all the unknowns are found.

Pipeline Solution

First pipeline stage computes x_0 and passes x_0 onto the second stage, which computes x_1 from x_0 and passes both x_0 and x_1 onto the next stage, which computes x_2 from x_0 and x_1, and so on.



Type 3 pipeline computation

The *i*th process ($0 < i < n$) receives the values x_0, x_1, x_2, …, x_{i-1} and computes x_i from the equation:

$$x_i = \frac{b_i - \sum_{j=0}^{i-1} a_{i,j}x_j}{a_{i,i}}$$

Sequential Code

Given constants $a_{i,j}$ and b_k stored in arrays **a[ ][ ]** and **b[ ]**, respectively, and values for unknowns to be stored in array, **x[ ]**, sequential code could be

```
x[0] = b[0]/a[0][0];              /* computed separately */
for (i = 1; i < n; i++) {         /*for remaining unknowns*/
        sum = 0;
        For (j = 0; j < i; j++
                sum = sum + a[i][j]*x[j];
        x[i] = (b[i] - sum)/a[i][i];
}
```

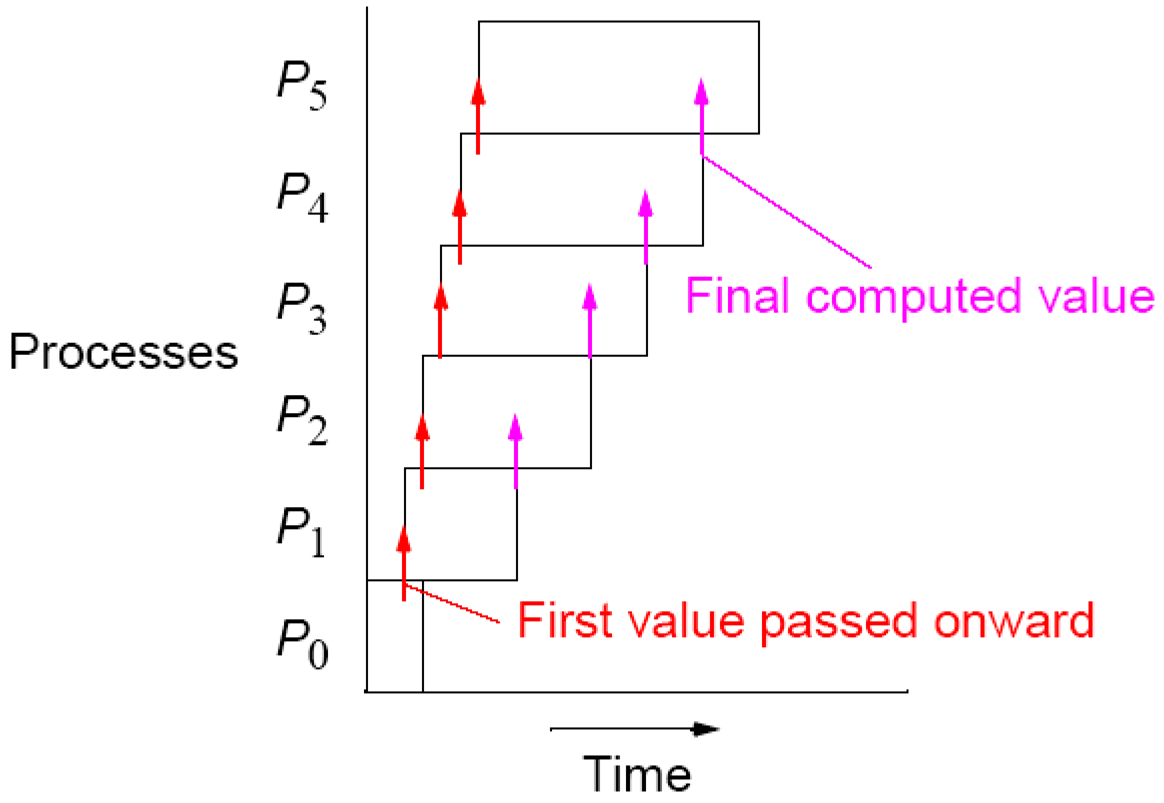
Parallel Code

Pseudocode of process P_i ($1 < i < n$) of could be

```
for (j = 0; j < i; j++) {
        recv(&x[j], Pi-1);
        send(&x[j], Pi+1);
}
sum = 0;
for (j = 0; j < i; j++)
        sum = sum + a[i][j]*x[j];
x[i] = (b[i] - sum)/a[i][i];
send(&x[i], Pi+1);
```

Now have additional computations to do after receiving and resending values.

Pipeline processing using back substitution



P_5
P_4
P_3
P_2
P_1
P_0
Processes
Final computed value
First value passed onward
Time