ICS 556–Parallel Algorithms, Winter 2007

Major Exam 1

Date: Apr. 9th, 2007 Duration: 2 hours

Name: ID#:

Please write clearly any assumption you make throughout your answers.

Question 1: **[25 marks]**

a) Explain the differences between the following measures of the performance of any parallel algorithm. Support your answer by giving examples.

   (a) The running time:

   (b) The cost:

   (c) The work:

   (d) The speedup:

   (e) The efficiency:

b) State and prove the speedup Folklore theorem.

Question 2: **[25 marks]**

a) Design a combinational circuit to solve the problem of computing the prefix sums of n numbers $x_1, x_2, \ldots, x_n$. State clearly the depth and the width of your circuit. Is it cost-optimal?

b) Explain how one can simulate the above combinational circuit on a PRAM model.

c) Design a cost-optimal PRAM algorithm that solves the prefix sums problem.

Question 3: [20 marks]

Consider an EREW PRAM computer with n processors. Explain how one can do the following efficiently.

a) Broadcast a datum that is stored in a memory location called x to all processors.

b) Check whether x belongs to an array A of size n stored in the memory or not. If A contains x we should output 1, otherwise we should output 0.

Question 4: [20 marks]

a) The inner product $\|x \cdot y\|$, for any n-dimensional vectors $x, y \in \mathbb{R}^n$, is defined by $\|x \cdot y\| = \sum_{i=1}^{n} x_i y_i$. Design a *cost-optimal* PRAM algorithm to compute the inner product of two given vectors $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ stored in the shared memory. Find the running time, the cost of your algorithm and prove that it is optimal.

b) Solve the same problem as in part a) but on a complete binary tree interconnected Network. Your algorithm again should have an optimal cost.

Question 5: [10 marks]

Given a binary string of length n where the 0s occurs before the 1s. (It is possible that the string consists of just 0s or just 1s.) Design a PRAM algorithm with N processors that counts the number of 1s in the binary string.