Rules of Inferences

Section 1.5

Definitions

- Argument: is a sequence of propositions (premises) that end with a proposition called conclusion.
- Valid Argument: The conclusion must follow from the truth of the previous premises, i.e.,

  all premises $\rightarrow$ conclusion
- Fallacy: is an invalid argument or incorrect reasoning.
- Rules of inference: rules we follow to construct valid arguments.

Valid Arguments in Propositional Logic

- If we rewrite all premises (propositions) in any argument using only variables and logical connectors then we get an argument form.
- Thus, an argument is valid when its form is valid.
- Valid argument doesn't mean the conclusion is true.

Example

- Argument:
  - If you have a password, then you can login to the network.
  - You have a password
  - Therefore you can login to the network.
- Argument Form:

$p \to q$

p

$\therefore q$

- So it is a valid argument with correct conclusion

Example

- Argument:
  - If $x \neq 0$, then $x^2 > 1$
  - But ½ $\neq 0$
  - Thus ¼ > 1
- Argument Form:

$$
\begin{array}{l} p \rightarrow q \\ p \\ \text{---------} \\ \therefore q \end{array}
$$

- So it is a valid argument with wrong conclusion

Rules of Inference:
correct argument forms

Rule	Name	Tautology
p $p \rightarrow q$ ----------- $\therefore q$	Modus Ponens	$p \wedge (p \rightarrow q) \rightarrow q$
$\neg q$ $p \rightarrow q$ ----------- $\therefore \neg p$	Modus Tollens	$\neg q \wedge (p \rightarrow q) \rightarrow \neg p$

Rules of Inference:
correct argument forms

Rule	Name	Tautology
$p \to q$ $q \to r$ ----------- $\therefore\ p \to r$	Hypothetical Syllogism	$(p \to q) \wedge (q \to r) \to (p \to r)$
$p \vee q$ $\neg p$ ----------- $\therefore\ q$	Disjunction Syllogism	$(p \vee q) \wedge \neg p \to q$

Rules of Inference:
correct argument forms

Rule	Name	Tautology
p ----------- $\therefore\ p \lor q$	Addition	$p \to p \lor q$
$p \land q$ ----------- $\therefore\ p$	Simplification	$p \land q \to p$

Rules of Inference:
correct argument forms

Rule	Name	Tautology
p q ----------- $\therefore\ p \land q$	Conjunction	$p \land q \rightarrow p \land q$
$p \lor q$ $\neg p \lor r$ ----------- $\therefore\ q \lor r$	Resolution	$(p \lor q) \land (\neg p \lor r) \rightarrow q \lor r$

Examples

- "It's below freezing and raining now. Therefore it's below freezing"
- Argument form:

  $p \wedge q$

  $\therefore\ p$
- Simplification Rule

Examples

- "If $x>1$, then $1/x \in (0,1)$. If $x \in (0,1)$, then $x^2 < x$. Therefore, if $x > 1$, then $1/x^2 < 1/x$."
- Argument Form:

  $p \rightarrow q$

  $q \rightarrow r$

  $\therefore\ p \rightarrow r$
- Rule: Hypothetical Syllogism

Using Rules of Inference to Build Arguments

Show that the hypotheses

- It's not sunny this afternoon and it's colder than yesterday.
- We will go swimming only if it's sunny
- If we don't go swimming, then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

lead to the conclusion "we will be home by sunset"

Using Rules of Inference to Build Arguments

the hypotheses

- It's not sunny this afternoon and it's colder than yesterday.
- We will go swimming only if it's sunny
- If we don't go swimming, then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

the conclusion

"we will be home by sunset"

Hypothesis:

$\neg s \wedge c$

$w \rightarrow s$

$\neg w \rightarrow t$

$t \rightarrow h$

Conclusion: h

Using Rules of Inference

$\neg s \wedge c$	hypo
$\neg s$	simplification
$w \rightarrow s$	hypo
$\neg W$	Modus Tollens
$\neg w \rightarrow t$	hypo
t	Modus Ponens
$t \rightarrow h$	hypo

$\therefore h$	Modus Ponens

Using Rules of Inference to Build Arguments

Show that the hypotheses

- If you send me an email message, then I'll finish writing the program.
- If you don't send me an email, then I'll go to sleep early.
- If I go to sleep early, then I'll wake up feeling refreshed.

lead to the conclusion "if I don't finish writing the program then I'll wake up feeling refreshed"

Using Rules of Inference to Build Arguments

the hypotheses

- If you send me an email message, then I'll finish writing the program.
- If you don't send me an email, then I'll go to sleep early.
- If I go to sleep early, then I'll wake up feeling refreshed.

the conclusion

"if I don't finish writing the program then I'll wake up feeling refreshed"

Hypothesis:

$s \rightarrow f$

$\neg s \rightarrow p$

$p \rightarrow w$

Conclusion:

$\neg f \rightarrow w$

$s \rightarrow f$	hypo
$\neg f \rightarrow \neg s$	Contrapositive
$\neg s \rightarrow p$	hypo
$p \rightarrow w$	hypo

$\therefore \neg f \rightarrow w$	Hypothetical Syllogism

Fallacies

- Incorrect reasoning based on contingencies and not tautologies.

1. Fallacy of affirming the conclusion:

$$(p \rightarrow q) \land q \rightarrow p$$

- Example: If you solve every problem in this book, then you'll pass the course. You did passed the course. Therefore, you did solved every problem in this book.

Fallacies

2. Fallacy of denying the hypothesis:

   $(p \rightarrow q) \wedge \neg p \rightarrow \neg q$

- Example:
- Since you didn't pass the course, then you didn't solve every problem. ✓
- Since you didn't solve every problem, then you didn't pass the course. ✗

Rules of Inference for Quantified Statements

- Universal Instantiation:

  $\forall x \ p(x)$

  $\therefore \ p(c)$

- Universal Generalization:

  p(c) for arbitrary c

  $\therefore \ \forall x \ p(x)$

Rules of Inference for Quantified Statements

- Existential Instantiation:

$$\exists x \; p(x)$$

$\therefore$ p(c) for some c

- Existential Generalization:

p(c) for some c

$$\therefore \exists x \; p(x)$$

Combining Rules of Inference

- Universal Modus Ponens:

Universal Instantiation + Modus Ponens

$$\forall x \ (P(x) \rightarrow Q(x))$$

$$P(a)$$

$$\therefore \ Q(a)$$

Combining Rules of Inference

- Universal Modus Tollens:

Universal Instantiation + Modus Tollens

$$\forall x \ (P(x) \rightarrow Q(x))$$

$$\neg Q(a)$$

$$\therefore \neg P(a)$$