

ICS 252–Discrete Structures

Major Exam 1

Date: Oct 12th, 2005

Duration: 90 minutes

Please write your name, ID and section numbers.

Name:

ID#:

Section #:

This is a closed-book and closed-notes exam. No calculators are allowed. You may use any of the following notations in your final answers without any farther simplifications:

$$\lfloor n/m \rfloor, \lceil n/m \rceil, \binom{n}{r}, C(n, r), P(n, r), n!.$$

Q.#	Marks	Scores	Remarks
1	30		
2	12		
3	10		
4	20		
5	16		
6	12		
Total	100		

Question 1: [3×10 marks]

- a) How many binary strings are there of length 12?
- b) How many binary strings are there of length at least 10 and at most 12?
- c) How many binary strings are there of length 12 that
- 1- start with 10 and end with 01?
 - 2- start with 10 or end with 01?
 - 3- begin and end with the same bit?
 - 4- are palindromes (i.e., they can be read from both sides)?
 - 5- contain exactly 3 ones?
 - 6- contain at most 3 ones?
 - 7- contain at least 1 zero?
 - 8- contain equal number of ones and zeros?

Question 2: [4×3 marks]

How many integers between 100 and 1000 inclusive that are divisible

a) by 3 and 7?

b) by 3 or 7?

c) by 3 but not by 7?

Question 3: [5×2 marks]

a) In a group of 200 students, 80 are taking a math class, 60 are taking a chemistry class, and 30 are taking both classes.

i. How many students are taking either a math class or a chemistry class?

ii. How many students are not taking either classes?

b) How many binary strings are there of length 10 that contain at least 7 consecutive ones?

Question 4: [5×4 marks]

A certain club consists of 9 men and 6 women.

- a) How many ways are there to form a committee of 3 people?

- b) How many ways are there to form a committee consisting of 3 men and 4 women?

- c) How many ways are there to form a committee of 6 people if a certain pair of women refuse to serve in the same committee?

- d) How many ways are there to arrange all of the men and women in a circular table such that any two women are separated by at least one man (i.e., no two women are allowed to set close to each other)?

Question 5: [4×4 marks]

- a) What is the smallest number of balls that we have to throw into n different boxes in order to be certain that there is a box with at least two balls?

- b) What is the smallest number of balls that we have to throw into n different boxes in order to be certain that there is a box with at least 4 balls?

c) Suppose we select books at random (without looking at them) from a collection of books that contains 5 chemistry, 18 math, 12 biology, and 15 physics books. How many books must we choose from this collection so it is guaranteed that there will be

i. at least 3 books of one type? Explain your answer.

ii. at least 10 books of one type? Explain your answer.

Question 6: [4 × 3 marks]

a) Find the coefficient of the term x^5y^7 in the expansion of $(x + y)^{12}$.

b) Find the coefficient of the term x^9y^{14} in the expansion of $(2x^3 - y^2)^{10}$.

c) By showing your steps for credit, evaluate the following sum.

$$\sum_{i=0}^n \binom{n}{i} 2^{-i} (-2)^i =$$