Discrete Math I – Practice Problems for Exam I

The upcoming exam on Thursday, January 12 will cover the material in Sections 1 through 6 of Chapter 1. There may also be one question from Section 7. If there is, it will not be ask you to prove any statement, but rather a short answer question about proofs.

You will be provided with a sheet containing the laws of logical equivalences and the rules of inference (and you can find it as page 3 of this practice exam). Do NOT print the one provided here. I will give each of you one during the exam.

Note that this practice exam is NOT "synchronized" with what you will see on exam day. I won't purposely present problems here and just give you the same problem with the numbers changed. That is, the following problems do not represent all of the possible types of problems that could appear on the exam. Problems chosen for the exam will be similar to homework problems, the quizzes, and examples done in class. Also note that the number of problems presented in this practice exam may not represent the actual length of the exam you see on the exam day. You should be prepared for a lengthy exam.

IMPORTANT! First try these problems as if it were the real exam; work by yourself without the text or your notes. This is supposed to be a gauge on what you need to work on to prepare for the exam. Answering these problems as you might handle homework problems won't necessarily give you much of a clue on what you need to work on.

Instructions: Provide all steps necessary to solve the problem. *Unless otherwise stated, your answer must be exact and reasonably simplified*. Additionally, clearly indicate the value or expression that is your final answer. Calculators are NOT allowed.

1. Find the truth table of the compound proposition $(p\vee q)\rightarrow(p\wedge\neg r)$.

2. Give the converse, the contrapositive, and the inverse of the statement "If it rains today, then I will drive to work."

3. Show that $\neg p\rightarrow(q\rightarrow r)$ and $q\rightarrow (p\vee r)$ are logically equivalent using the laws of logical equivalences. Be sure to cite each law whenever used.

4. Use the table of logical equivalences to simplify the compound proposition $[(p\vee q)\wedge\neg p]\rightarrow q$. Be sure to justify your answers.

5. Let $P(m,n)$ be the statement "$m \mid n$," where the domain for both variables consists of all positive integers. [By "$m \mid n$", which we say as "m divides n", we mean that $n = km$ for some integer k.] Determine the truth values of each of these statements.

(***a***) $P(4,5)$ (***b***) $P(2,4)$ (***c***) $\forall m\forall n\, P(m,n)$
(***d***) $\exists m\forall n\, P(m,n)$ (***e***) $\exists n\forall m\, P(m,n)$ (***f***) $\forall n\, P(1,n)$

6. Consider the compound proposition $(\forall m\exists n\, [P(m,n)])\rightarrow(\exists n\forall m[P(m,n)])$ where both m and n are integers. Determine the truth value of the proposition if
(***a***) $P(m,n)$ is the statement "$m < n$".
(***b***) $P(m,n)$ is the statement "$m \mid n$".

7. Suppose that the variable x represents students, $F(x)$ means "x is a freshman," and $M(x)$ means "x is a math major". For each of the three statements (***a***), (***b***), and (***c***), determine which of the symbolic statements are equivalent. (Note: Each statement may have multiple answers.)

I. $\forall x\, [M(x)\rightarrow\neg F(x)]$ **II.** $\neg\exists x\, [M(x)\wedge\neg F(x)]$ **III.** $\forall x\, [F(x)\rightarrow\neg M(x)]$ **IV.** $\forall x\, [M(x)\rightarrow F(x)]$
V. $\exists x\, [F(x)\wedge M(x)]$ **VI.** $\neg\forall x\, [\neg F(x)\vee\neg M(x)]$ **VII.** $\forall x\, [\neg(M(x)\wedge\neg F(x))]$ **VIII.** $\forall x\, [\neg M(x)\vee\neg F(x)]$
IX. $\neg\exists x\, [M(x)\wedge\neg F(x)]$ **X.** $\neg\exists x\, [M(x)\vee F(x)]$ **XI.** $\neg\forall x\, [F(x)\rightarrow\neg M(x)]$

(***a***) Some freshmen are math majors. **Answer: ________________________**

(***b***) Every math major is a freshman. **Answer: ________________________**

(***c***) No math major is a freshman. **Answer: ________________________**

8. Determine whether the following argument is valid.

$$\begin{array}{ll} & p \to r \\ & q \to r \\ & \neg(p \vee q) \\ & \text{--------} \\ \therefore & \neg r \end{array}$$

- If the argument if valid, provide a valid proof of the result (that is, use the laws of logical equivalences and the rules of inference to demonstrate that the conclusion is valid).
- If the argument is not valid, provide specific truth values of p, q, and r in which the premises are true, but the conclusion is false.

Exercises from the text. I would STONGLY recommend that you try as many of these problems as you can. Any of these problems (or ones similar to them) could appear on the exam.

Chapter 1 Supplementary Exercises (pg 111-113): 3, 6, 20, 23, 25

EQUIVALENCES AND IMPLICATION EQUIVALENCES

Double negation law: $\neg(\neg p) \equiv p$

Identity laws: $p \lor \mathbf{F} \equiv p$, $p \land \mathbf{T} \equiv p$

Domination laws: $p \lor \mathbf{T} \equiv \mathbf{T}$, $p \land \mathbf{F} \equiv \mathbf{F}$

Negation laws: $p \lor \neg p \equiv \mathbf{T}$, $p \land \neg p \equiv \mathbf{F}$

Idempotent laws: $p \lor p \equiv p$, $p \land p \equiv p$

Commutative laws: $p \lor q \equiv q \lor p$, $p \land q \equiv q \land p$

Associative laws: $p \lor (q \lor r) \equiv (p \lor q) \lor r$,
$p \land (q \land r) \equiv (p \land q) \land r$

Distributive laws: $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$,
$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

Absorption laws: $p \lor (p \land q) \equiv p$, $p \land (p \lor q) \equiv p$

DeMorgan's laws: $\neg(p \lor q) \equiv \neg p \land \neg q$,
$\neg(p \land q) \equiv \neg p \lor \neg q$

1. $p \rightarrow q \equiv \neg p \lor q$
2. $p \rightarrow q \equiv \neg q \rightarrow \neg p$
3. $p \lor q \equiv \neg p \rightarrow q$
4. $p \land q \equiv \neg(p \rightarrow \neg q)$
5. $\neg(p \rightarrow q) \equiv p \land \neg q$
6. $(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$
7. $(p \rightarrow r) \land (q \rightarrow r) \equiv (p \lor q) \rightarrow r$
8. $(p \rightarrow q) \lor (p \rightarrow r) \equiv p \rightarrow (q \lor r)$
9. $(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$
10. $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$
11. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
12. $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
13. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

RULES OF INFERENCE

p

$\therefore\ p \lor q$ (*Addition*)

$p \land q$

$\therefore\ p$ (*Simplification*)

p
q

$\therefore\ p \land q$ (*Conjunction*)

p
$p \rightarrow q$

$\therefore\ q$ (*Modus ponens*)

$\neg q$
$p \rightarrow q$

$\therefore\ \neg p$ (*Modus tollens*)

$p \rightarrow q$
$q \rightarrow r$

$\therefore\ p \rightarrow r$ (*Hypothetical syllogism*)

$p \lor q$
$\neg p$

$\therefore\ q$ (*Disjunctive syllogism*)

$p \lor q$
$\neg p \lor r$

$\therefore\ q \lor r$ (*Resolution*)

Discrete Math I – Answers to Practice Problems for Exam I

1.

p	q	r	$(p\vee q)\to(p\wedge\neg r)$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

2. The converse is "If I drive to work today, then it will rain."
The contrapositive is "If I do not drive to work today, then it will not rain."
The inverse is "If it does not rain today, then I will not drive to work."

3.

$\neg p\to(q\to r)$	$\equiv \neg p \to (\neg q \vee r)$	[Law 1]
	$\equiv \neg(\neg p) \vee (\neg q \vee r)$	[Law 1]
	$\equiv p \vee (\neg q \vee r)$	[Double Negation Law]
	$\equiv (p \vee \neg q) \vee r$	[Associative Law]
	$\equiv (\neg q \vee p) \vee r$	[Commutative Law]
	$\equiv \neg q \vee (p \vee r)$	[Associative Law]
	$\equiv q \to (p \vee r)$	[Law 1]

4.

$[(p\vee q) \wedge \neg p] \to q$	$\equiv \neg[(p\vee q) \wedge \neg p] \vee q$	[Law 1]
	$\equiv [\neg(p\vee q) \vee \neg\neg p] \vee q$	[DeMorgan's Law]
	$\equiv [\neg(p\vee q) \vee p] \vee q$	[Double Negation Law]
	$\equiv [(\neg p\wedge\neg q) \vee p] \vee q$	[DeMorgan's Law]
	$\equiv [p \vee (\neg p\wedge\neg q)] \vee q$	[Commutative Law]
	$\equiv [(p\wedge\neg p) \wedge (p\wedge\neg q)] \vee q$	[Distributive Law]
	$\equiv [\mathbf{F} \wedge (p\wedge\neg q)] \vee q$	[Negation Law]
	$\equiv [(p\wedge\neg q) \wedge \mathbf{F}] \vee q$	[Commutative Law]
	$\equiv \mathbf{F} \vee q$	[Domination Law]
	$\equiv q \vee \mathbf{F}$	[Commutative Law]
	$\equiv q$	[Identity Law]

5. ***(a)*** F; ***(b)*** T; ***(c)*** F; ***(d)*** T; ***(e)*** F; ***(f)*** T

6. ***(a)*** F; ***(b)*** T

7.

(a) V, VI, XI

Expression	Reason	Number
"Some freshmen are math majors."		
$\equiv \exists x\ [F(x)\wedge M(x)]$		V
$\equiv \exists x\ \neg\neg[F(x)\wedge M(x)]$	Double Negation Law	
$\equiv \neg\forall x\ \neg[F(x)\wedge M(x)]$	Negation of Quantifiers	
$\equiv \neg\forall x\ [\neg F(x)\vee\neg M(x)]$	DeMorgan's Law	VI

$\equiv \neg\forall x\ [F(x)\rightarrow\neg M(x)]$	Law 1	XI

(b) II, IV, VII, IX

Expression	Reason	Number
"Every math major is a freshman."		
$\equiv \forall x\ [M(x)\rightarrow F(x)]$		IV
$\equiv \forall x\ \neg[\neg(M(x)\rightarrow F(x))]$	Double Negation Law	
$\equiv \forall x\ \neg[M(x)\wedge\neg F(x)]$	Law 5	VII
$\equiv \neg\exists x\ [M(x)\wedge\neg F(x)]$	Negation of Quantifiers	IX and II

(c) I, III, VIII

Expression	Reason	Number
"No math major is a freshman."		
$\equiv \forall x\ [F(x)\rightarrow\neg M(x)]$		I
$\equiv \forall x\ [\neg\neg F(x)\rightarrow\neg M(x)]$	Double Negation Law	
$\equiv \forall x\ [M(x)\rightarrow\neg F(x)]$	Law 2	III
$\equiv \forall x\ [\neg M(x)\vee\neg F(x)]$	Law 1	VIII

The expression (X) $\neg\exists x\ [M(x)\vee F(x)]$ wasn't equivalent to any of the others.

Expression	Reason	Number
$\neg\exists x\ [M(x)\vee F(x)]$		X
$\equiv \forall x\ \neg[M(x)\vee F(x)]$	Negation of Quantifiers	
$\equiv \forall x\ [\neg M(x)\wedge\neg F(x)]$	DeMorgan's Law	*
$\equiv \forall x\ [M(x)\rightarrow\neg F(x)]$	Law 1	**

In the above table, the expression

- in the line marked with * is not equivalent to V because the quantifiers are different.
- in the line marked with * is not equivalent to VIII because the only difference is a disjunction operation replacing a conjunction operation.
- in the line marked with ** is not equivalent to IV because of an additional negation.

8. It is not valid when p is false, q is false, and r is true.