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CHAPTER-7: Part 1

A. OBJECTIVE OF THIS CHAPTER

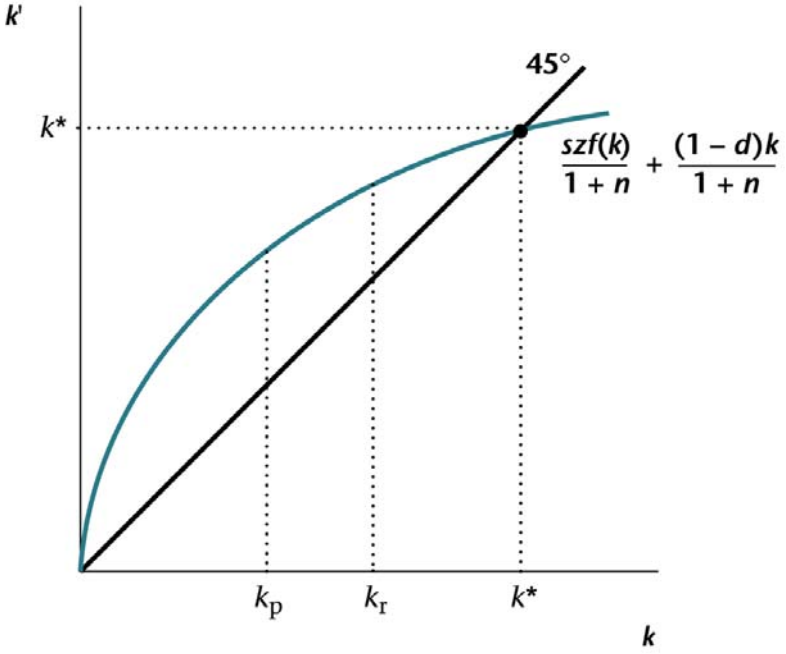
In this chapter, we will do the following:

- Analyze success and failure of Solow Growth model.
- Learn about endogenous growth model
- Do some policy analysis.
- Look at data

B. MAJOR IMPLICATION OF SOLOW GROWTH MODEL

The Solow model predicts that given the access to the same technology, all countries of the world (rich or poor) will converge to a similar standard of living.

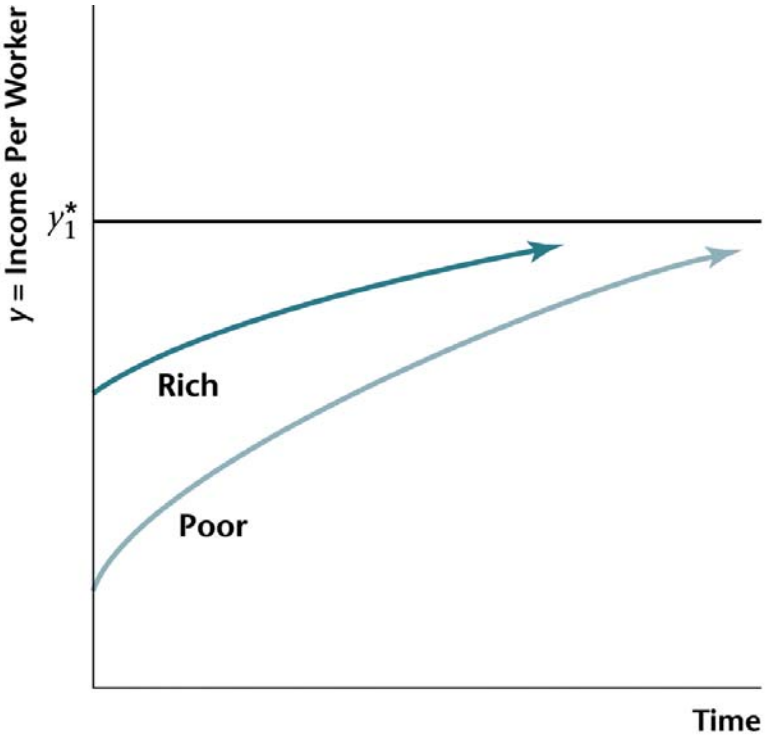
Figure 7.1 Rich and Poor Countries and the Steady State



- In the graph even if poor and rich countries start with different initial capital stock (k_p and k_r respectively), they would both converge to the same steady state capital stock k^* in the long run.

- The convergence of the LR capital stock would also cause a convergence of the standard of living across all the countries of the world, poor and rich. This is seen in the graph below:

Figure 7.2 Convergences in Income per Worker across Countries in the Solow Growth Model



C. SUCCESS OF SOLOW GROWTH MODEL

We will analyze the success of the Solow growth models by asking some specific questions.

1. How can there be barriers to access to the technology?

There can be both domestic and international barriers to adopting new technology. Some of the restrictions are induced by the government and some are induced by the private sector. We analyze them as follows:

- Government laws to form labor unions may directly prevent adoption of new technology.
- Government might be motivated by the "**infant industry argument**" and protect domestic producers from international competition by providing them subsidy. This will take the **Incentive** away from the domestic firms to adopt new technology

2. Can Solow Model explain the differences in the standard of living between USA, Europe, New Zealand and the rest of the world (Stylized fact 02 and 05)?

It seems like Solow model can explain this puzzle. We see:

- The richest countries had access to the same technology.
- The poor countries had some kind barriers to adapting to new technology.

As a result, rich countries experienced similar growth and their growth experience differed from that of the rest of the world.

3. Are there real barriers to Technology in real world?

Of course there is!! Parante and Prescott (2000) showed that the income inequality across poor and rich countires is largely due to the barriers to the adoption of new technology

4. How can poor Countries catch up with the rich countries?

There are various ways the poor countries can catch up with the rich countries. They are as follows:

- If government promote domestic competition.
- If government promote free trade.
- If government natural monopoly in case where there is not economies of scale.

Critical Issue

There is one other alternative way the poor countries can catch up with the rich countries. If the government or the private sectors adopt strategy like provide public education or provide training for the employees, this will make the existing capital more productive and allow the poor countries to experience faster growth. Thus they will be able to catch up with the richer countries. This motivates the analysis of **Endogenous Growth model**

D. THE ENDOGENOUS GROWTH MODEL

1. Motivation

The motivation for the endogenous growth model originates from the failure of exogenous growth model. These failures are as follows:

- Solow model cannot explain growth properly. Growth in Solow model depends on Total Factor Productivity (z or Solow residual). But it is well known fact that TFP depends on R&D by the firms. Therefore:
    1. TFP gets effected by R&D
    2. R&D requires training, education of the employees and other economic environments.

But Solow growth model fails to provide any explanation as to **how do the education and training of the employees convert into improvement which can promote economic growth.**

- Solow model cannot explain how TFP reponds do to public spending on public education.
- Solow model cannot explain how government's policies such as tax credit directly effects economic growth.
- Solow model cannot explain whether government should intervene to promote economic growth.

Therefore, we need a better model that can explain the importance of all the above issues.

The key issue

- The conventional exogenous model cannot explain two important facts:

1. How learning effects economic growth
2. Whether growth can be achieved with out major technological improvement. If we can, we will be able to explain rapid growth of countries like Korea and India.

- The key issue that we will develop to explain endogenous growth is **Human Capital**

2. Basic Outline of the model

- The model that we would like to learn was developed by Robert Lucas. Some important contribution to this model was made by Paul Romer.
- We will still consider a representative agent model.
    a. The representative consumer allocates his time between supplying labor to the market and accumulated human capital (studying, going to college). **There will be no leisure decision.**
    b. The higher human capital the worker has, the more productive he is (skilled vs unskilled worker).
    c. The higher human capital the worker has, the more human capital he can produce (if you have a MA degree, you can get a PhD degree).
    d. The representative consumer works for two things, production of goods and production of future human capital.
    e. The representative consumer has three choices to make today:
    How much consume today?
    How much to save for tomorrow? (We will drop this)
    How much to accumulate human capital for tomorrow?

The key issue

- This model is very realistic because it captures both kinds of future decisions by the consumer. Acquiring human capital should be thought as investment to the production process. But whereas investment in physical is embodied in buying new machinery, investment in human capital is embodied in people. **Investment in human capital will always be thought as acquiring more education.**
- Unlike Physical capital, human capital is **not** subject to **Diminishing returns to scale** because there is apparently there is no limit to knowledge. This creates the prospect of unbounded growth in our economy.
- Romer assumes that human capital has **one** feature that physical capital does not have. It has **nonrivalry**. That means if knowledge is created, people cannot be prevented from acquiring that knowledge.

3. Representative Consumer

- We assume that the consumer starts with H^s units of human capital in his career.
- We assume that the consumer has 1 unit of time each period. He spends a fraction μ of that time working. So, the number of **efficiency units of labor** spent in working is μH^s (This is the labor that actually is productive. This is because the consumer also has to study). Thus the **Effective wage** that the consumer gets is $w\mu H^s$
- We assume for simplicity that the consumer cannot save (in terms of physical capital). Thus the budget constraint for the consumer is as follows:
  $C = w\mu H^s$ ------- (1)

- Although the consumer cannot save, he can save something for future consumption by accumulating human capital today which will enable him to earn more tomorrow. We assume the consumer allocates the remaining $1-\mu$ fraction of his time in accumulating human capital (studying). The technology for human capital is defined as follows:

  $H^{s'} = b(1-\mu)H^{s}$ ------ (2)

  Here, $H^{s'}$ is next or future period's capital and H^{s} is just this period's human capital. The term b captures the efficiency of the human capital accumulation and is assumed to be $b \succ 0$.

4. Representative Firm

- The representative has only one input, it uses efficiency unit of labor to produce output. This output is not storable. The production function is given by:

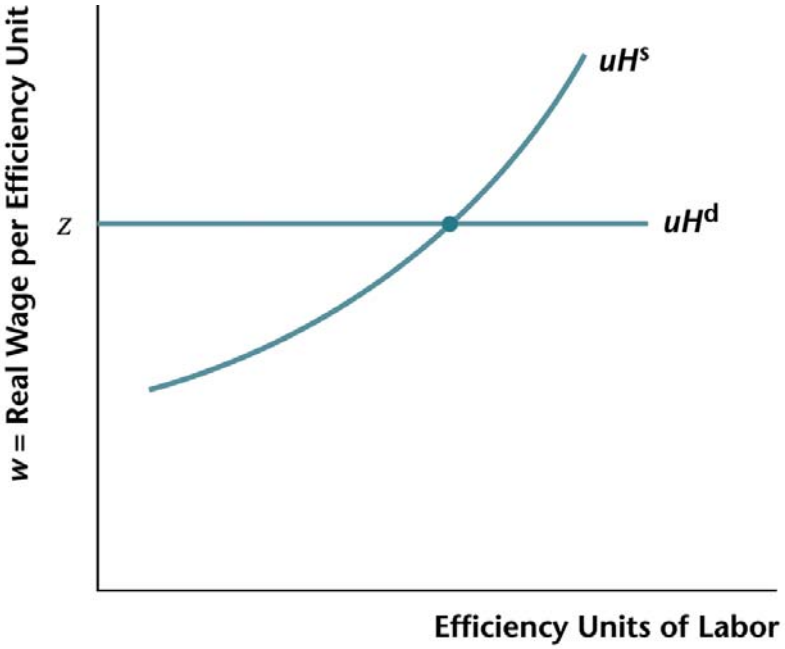
  $Y = z\mu H^{d}$ ------ (3)

  Here Y is the current output and z is the marginal product of efficiency unit of labor. μH^{d} is the labor demand by the firm
- The Production function is subject to **CRS.**
- The profit function of the firm is defined as follows:

  $\pi = Y - w\mu H^{d} = z\mu H^{d} - w\mu H^{d} = (z-w)\mu H^{d}$ ----- (4)

  a. If $(z-w) \prec 0$ - negative profits, so none are hired
  b. If $(z-w) \succ 0$ - positive profits, hire infinite amount of workers. Because no DRS
  c. If $(z-w) = 0$ - firm is indifferent in hiring
- When $(z-w) = 0 \Rightarrow w = z$, firm's demand for labor is indefinite. Hence at $(z-w) = 0 \Rightarrow w = z$, labor demand is perfectly elastic

Figure 7.5 Determination of the Equilibrium Real Wage in the Endogenous Growth Model



- In the graph, the labor demand curve is perfectly elastic at $w = z$. But the labor supply curve is upward sloping. Hence we will have labor market equilibrium at a point where $w = z$. This means in equilibrium:
  a. $w = z$
  b. $H^d = H^s$
  c. $wH^d = zH^d$

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Critical Issue

- The Production function of the representative firm is as follows:

$$Y = z\mu H^d$$

Hence the **Marginal Product** of the efficiency unit of labor (μH^d) in the production process is just z. Thus the marginal product of efficiency unit of labor is constant.

- The firm makes decision about how much efficiency unit of labor (μH^d) to hire to maximize profit. The profit function is defined as:

$$\pi = Y - w\mu H^d = z\mu H^d - w\mu H^d = (z-w)\mu H^d$$

The first order condition of profit maximization thus reveals that, in equilibrium, $(z-w) = 0 \Rightarrow z = w$

- Looking at the profit function, we see:
  - If $(z-w) \prec 0$ - negative profits, so none are hired
  - If $(z-w) \succ 0$ - positive profits, hire infinite amount of workers.
  - If $(z-w) = 0$ - firm is indifferent in hiring. they can either hire zero or infinite number of efficiency unit of labor
- In figure 7.5, we the following:
  - At any real wage above z, the firm earns negative profit. Hence amount of labor hired is zero.
  - At $z = w$, the firm can either hire zero or infinite number of efficiency unit of labor
  - At any wage below z, the firm hires infinite number of labor units.

Thus graphically, the labor demand curve for the firm representative firm is given by a horizontal line.

- Since marginal product of the efficiency unit of worker is constant, it will also be represented by a horizontal line. Thus the labor demand curve in this case is identical to the marginal product of efficiency unit of worker.
- The supply curve of the efficiency unit of labor is an upward sloping graph I the (w, μH^d) space. Hence we have an intersection between the labor demand and labor supply curve at a point where the equilibrium wage is just equal to z, the marginal product of efficiency unit of worker.

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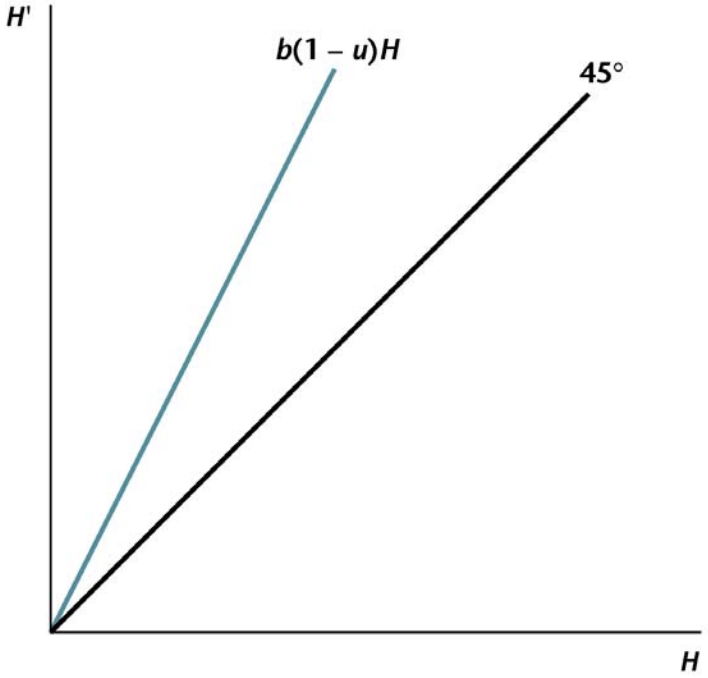
5. Definition of Competitive Equilibrium

- Competitive equilibrium is very easy to derive because:
    a. There is only one market where the consumption goods are traded for efficiency units of labor wage.
    b. The labor always clears at the equilibrium real wage: $w = z$
- Labor market clearing condition: $\mu H^s = \mu H^d$
  This means that: $H^s = H^d = H$ ---- (5)
- Subbing (5) into (1): $C = z\mu H$ ---- (6)
- Subbing (5) into (2): $H' = b(1-\mu)H$ ----(7)
  Hence in equilibrium, the growth rate of human capital is denoted as: $\frac{H'}{H} - 1 = b(1-\mu) - 1$---- (8)

Figure 7.6 Human Capital Accumulations in the Endogenous Growth Model



- The graph plots equation (7). We see the following:
  - If $b(1-\mu) \succ 1$, then $H' \succ H$.
  - If the above condition is satisfied, then there is no convergence of human capital in the long run. Hence growth can be unbounded. This is reflected in the graph
- Unbounded Growth can in this model can be achieved in two ways
- If b increases, then $b(1-\mu) \succ 1$. This means that **an increase in the efficiency of the human capital technology can lead to unbounded growth.**
- If μ goes down, then also $b(1-\mu) \succ 1$ and we have unbounded growth. Thus **an increase in time allocated to accumulating human capital ($(1-\mu)$) may also lead to unbounded growth.**
- If equation (6) holds in equilibrium for C, it also holds for C'. So, we can have: $C' = z\mu H'$ ----(9)

Using (6) and (9) together and make use of (8), we get:

$$\frac{C'}{C} = \frac{H'}{H} = \frac{C'}{C} - 1 = \frac{H'}{H} - 1 = b(1-\mu) - 1 \text{-------} \quad (10)$$

So, **the growth rate of consumption is equal to the growth rate of human capital accumulation.**

- Since in equilibrium, the income expenditure identity holds, $C = Y$, we see:

$$\frac{Y'}{Y} = \frac{H'}{H} = \frac{Y'}{Y} - 1 = \frac{H'}{H} - 1 = b(1-\mu) - 1$$

Thus, **the growth rate of output is equal to the growth rate of human capital accumulation.**

Very Very Very Important Critical Issue

- This model predicts that countries with more efficient education system can experience higher human capital accumulation and therefore, higher economic growth. This is consistent with the growth experience from India and Korea.
- In this model, growth is completely **endogenous.** There is no population growth, no Technological innovation (change in z). So technology remains unchanged over time and yet, we have economic growth.
- The growth process is controlled by **endogenous factors** such as b, μ.
- The main reason behind unbounded growth in Lucas Model is because Human capital does not exhibit **DRS**. So, although production technology has **CRS**, output increase in the same proportion with human capital and we have economic growth

C. GOVERNMENT POLICY AND ENDOGENOUS GROWTH MODEL: POLICY EXPERIEMNTS

1. Basic idea

- Government policy can directly effect economic growth. Examples are making public education more effective, introducing an efficient mixture of public and private education.
- Government policy can indirectly effect growth by effecting b, μ. We see:
  - Government could provide tax evasion or subsidy on education. Subsidy on education makes it more desirable to accumulate. This would increase $(1-\mu)$ and promote economic growth.
  - Government could create more jobs with different skill requirements or education requirements. This also makes education more attractive to acquire. But this will also make human capital accumulation more productive. Thus b would go up and promote economic growth.

2. Policy Experiment: Validity of Government intervention

- We will consider an experiment where government changes one of the efficiency parameter. Then we will analyze the consequences and make comments about the validity of government intervention.
- Suppose government can directly or indirectly increase $(1-\mu)$. This will have two effects:
  - As a **direct** effect, growth rate of consumption will go up according to equation (10):
    $$\frac{C'}{C} = \frac{H'}{H} = \frac{C'}{C} - 1 = \frac{H'}{H} - 1 = b(1-\mu) - 1$$
  - The aggregate level of **initial** consumption will go down according to equation (6):
    $$C = z\mu H$$
  - But although initial consumption goes down, consumption experiences a larger growth rate in future consumption:
    $$\left(\frac{C'}{C\downarrow}\right)\uparrow\uparrow = \frac{H'}{H} = \frac{C'}{C} - 1 = \frac{H'}{H} - 1 = b(1-\mu)\uparrow - 1$$

    Therefore, when μ goes up, there is a tradeoff between lower level of initial consumption vs higher level of consumption in future.

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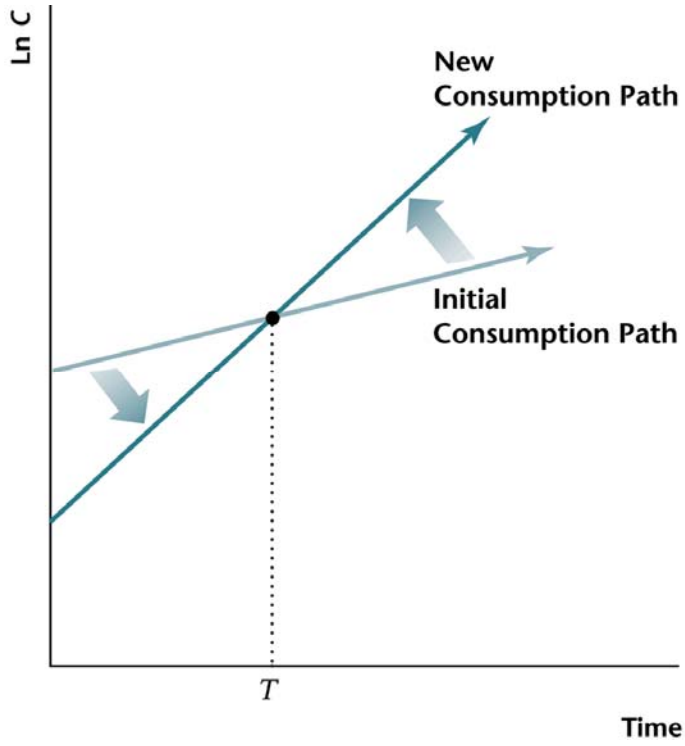
Figure 7.7 Effect of a Decrease in u on the Consumption Path in the Endogenous Growth Model



- In the graph above, because of an increase in μ, consumption path takes a downward shift. But as growth rate of consumption goes up, the new consumption path has a steeper slope indicating higher future consumption level.
- Whether the consumer will like the new or the old consumption path will depend on consumer preference. If he is a patient consumer, he would probably like the new path. Thus **justification of government intervention is questionable.**

Critical Question

Q: What happens when the government increases b?

Answer: Do it at home

D. IMPLICATIONS OF ENDOGENOUS GROWTH MODEL

- The endogenous growth model can explain the disparity among poor countries. (Why do some poor countries grow faster than other poor countries?)
- If two poor countries have different level of initial human capital stock but same of everything (even access to the same technology), their economic growth would still be different because they will experience different level of human capita accumulation.
- If a poor and a rich country have different level of initial human capital they would experience different level of economic growth. Thus **there would not be any convergence of economic growth across the world in the long run. This contradicts with Solow prediction and is more intuitive. This also explains data**(stylized facts of economic growth)
- But the model predicts that even two rich countries with different initial human capital might also not converge in terms of growth. This prediction also **contradicts** Solow Growth model but also is **inconsistent** with data.

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Critical Issue

- One possible explanation of how growth rate in rich countries can converge is by introducing **Human Capital Externality**. According to Lucas, Human capital externality refers to a situation where the presence of higher level of human capital in one country can also make the human capital in other country more productive. Examples of Human Capital Externality would the increase in Human capital in the presence of **cities.**
- But sometimes Human Capital Externality may **backfire**. If poor developing countries come in touch with richer countries, there is "**brain drain**" in the sense that human capital flights from poorer to richer countries. Thus the difference in the growth rate between rich and poor countries may persist even in the presence of Human Capital Externality

Growth and Education: Glomm & Ravikumar(1992)

- Education plays a pivotal role in promoting economic growth. Jones (1998) finds a positive correlation between educational attainment and growth rate of GDP.
- It is therefore absolutely crucial to have an efficient education system. There are debates to how can one make education more productive.
- Glomm and Ravikumar(1992) develops an endogenous model of economic growth where they have both public and private education as a propeller for human capital accumulation. They show that with an efficient public education system and with an efficient mechanism to finance public education, public education reduces inequality in terms of growth. But with private education system, long run growth is higher