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Performance analysis of coded cooperation diversity in wireless networks

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Summary

Diversity is an effective technique in enhancing the link quality and increasing network capacity. When multiple antennas cannot be used in mobile units, user cooperation can be employed to provide transmit diversity. In this paper, we analyze the error performance of coded cooperation diversity with multiple cooperating users. We derive the end-to-end bit error probability of coded cooperation (averaged over all cooperation scenarios). We consider different fading distributions for the interuser channels. Furthermore, we consider the case of two cooperating users with correlated uplink channels. Results show that more cooperating users should be allowed under good interuser channel conditions, while it suffices to have two cooperating users in adverse interuser conditions. Furthermore, under bad interuser conditions, more cooperating users can be accommodated as the fading distribution becomes more random. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: cooperative diversity; wireless; coding; union bound; error probability; Rayleigh; Rician; Nakagami; fading; convolutional codes

1. Introduction

Next generation wireless communication networks will be very different from second-generation cellular systems especially in the networking architecture. The mobile radio channel suffers from multipath fading, which causes random variations of the signal levels at the mobile units during a communication session. Diversity is considered as an effective tool for combating multipath fading [1]. Diversity is achieved by effectively transmitting or processing independently faded copies of the signal. Among diversity techniques, transmit diversity relies on the principle that signals transmitted from geographically separated transmitters experience independent fading, which results in a significantly improved performance compared to systems with no diversity [2,3]. Since most wireless networks operate in a multiuser mode, *user cooperation* [4,5] can be employed to provide diversity. In user cooperation, mobile units share their antennas to achieve uplink transmit diversity as illustrated in Figure 1. Since signals transmitted by different users undergo independent fading paths to the base station (BS), this approach achieves spatial

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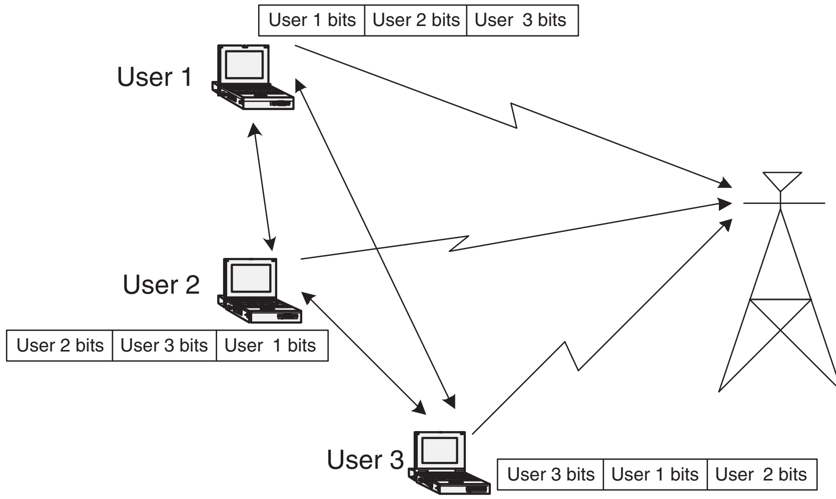

Fig. 1. Schematic diagram of a 3-user cluster employing coded cooperation.

diversity through the partner's antenna. The basic idea of user cooperation is based on the relay channel [6,7] and on the multiple access channel [8].

In conventional user cooperation, the partner repeats the received bits (via either forwarding or hard detection). Recently, a new framework for user cooperation was proposed [9–11] and is called *coded cooperation*. Unlike conventional user cooperation schemes, symbols in coded cooperation are not repeated by the partner. Instead, the codeword of each user is partitioned into two parts; one part is transmitted by the user, and the other part is sent by his partner. Coded cooperation provides significant performance gains for a variety of channel conditions. In addition, by allowing different code rates through rate-compatible coding [12], coded cooperation provides a great degree of flexibility to adapt to channel conditions.

In Reference [10], the performance of a two-user coded cooperation system was derived assuming that errors occurring in a codeword are equally distributed among the subframes sent by the cooperating users. This assumption is not necessarily true. Furthermore, the approach of Reference [10] becomes inaccurate and complicated when the number of cooperating users exceeds two. In this paper, we propose an analytical framework for deriving and evaluating the error performance of coded cooperation with multiple cooperating users. In this framework, the end-to-end probability of error averaged over different cooperation scenarios is derived. In addition, the bit error probability is derived for specific cooperation scenarios. Moreover, we consider the scenario of two cooperating users with correlated uplink channels.

The paper is organized as follows. In Section 2, the system model of coded cooperation with multiple cooperating users is described. The end-to-end error performance of coded cooperation is derived in Section 3. The bit error probability corresponding specific cooperation scenarios is derived in Section 4. Results are presented and discussed in Section 5. The main outcomes of the paper are summarized in Section 6.

2. System Model

2.1. Network Architecture

The coded cooperation scenario is illustrated in Figure 1. Coded cooperation starts by forming *clusters* of users, where users in a cluster cooperate to transmit their information to a common BS. The users within a cluster are called *partners*. The selection of users to join or leave a cluster can be based on the quality of the interuser channels or any other factor. In this paper, we limit our attention to the performance of a single cluster once it is formed, not concerned about the protocols used to set up a cluster. Users in a cluster are assumed to operate in a full-duplex mode, that is, they can transmit and receive simultaneously.

Let J be the number of cooperating users in a cluster. For each user in the cluster the transmission of each frame spans NT seconds, where N is the number of bits in the frame and T is the bit duration. A frame is formed by encoding K bits (information bits and cyclic redundancy check (CRC) bits) into $N = K/R$ bits, where R is the code rate of the error-correcting code. Partners cooperate by dividing their

N-bit frames into J *subframes* containing $N_1, N_2, \ldots, N_J$ bits, where $N = N_1 + N_2 + \ldots + N_J$. The distribution of coded bits over the subframes depends on the coding technique used. In the first $N_1 T$ seconds of each frame, each user transmits his first subframe composed of $N_1 = K/R_1$ coded bits, where R_1 is the code rate of the codeword in the first subframe, obtained by puncturing N-bit codeword. Clearly, $R_1 > R_J = R$. Upon the end of the first subframe, each user decodes the rate-R_1 codewords of his partners.

In the remaining $J-1$ subframes, each user in the cluster transmits one subframe for each of his $J-1$ partners. Each of these subframes contains parity bits of one of his partners which were not sent yet to the BS. Figure 1 shows the contents of the J subframes of each user in a 3-user cluster, that is, $J = 3$. If a user was not able to decode the first subframe of his partner, whom he should send his parity in a given subframe, then he sends his next parity subframe, that is, the parity subframe that was not yet sent by any of his partners. Thus each user transmits a total of N bits per source block over the J subframes. The *cooperation level* is defined as the percentage of the total bits per each source block that each user transmits for his partners, that is, $N - N_1/N$.

The partitioning of the coded bits in the J subframes may be achieved through puncturing a mother code as in Reference [9], where rate-compatible punctured convolutional (RCPC) codes [12] were used to implement coded cooperation. In this implementation, the rate-R code is selected from a given RCPC code family (e.g., the mother code). In this paper, we follow the same approach to obtain high-rate codewords from a rate-R mother code. The parity bits to be transmitted in each subframe are selected according to the puncturing matrix of the RCPC code, which is known and fixed to all partners in a cluster. The receiver combines all the received subframes for a user to produce a codeword of a more powerful code (a lower code rate) [12]. The code rates corresponding to different cooperation levels are $R_1 > R_2 > \ldots > R_J = R$.

2.2. Physical Link

After encoding the information block, the coded bits are modulated using BPSK. The matched filter output at user k due to user l in the time interval t in the first subframe is modeled by

$$y_{l,k}(t) = \sqrt{E_i} a_{l,k} s_l(t) + z_k(t) \tag{1}$$

where $s_l(t)$ is the signal transmitted from user l in time instance t in the first subframe and $z_k(t)$ is an AWGN sample at user k with a Normal distribution given by $N(0, \frac{N_0}{2})$. Here, E_i is the average received energy through the interuser channel and the average interuser signal-to-noise ratio (SNR) is $\gamma_i = E_i/N_0$. The coefficient $a_{l,k}$ is the gain of the interuser channel between user l and user k. The interuser channels are assumed to be independent and identically distributed (iid) with a Rician or a Nakagami distribution. Rician fading channels arise if a line-of-site (LOS) exists between the transmitter and the receiver [13]. In this model, the received signal is composed of two signal-dependent components; namely, the LOS and multipath components. In this case, the pdf of the interuser SNR [14] is given by

$$f_\gamma(\gamma) = \frac{(1+\kappa)}{\gamma_i} \exp\left[-\kappa - \frac{(1+\kappa)\gamma}{\gamma_i}\right] \times I_0\left(2\sqrt{\frac{\kappa(1+\kappa)\gamma}{\gamma_i}}\right), \quad \gamma \geq 0 \tag{2}$$

where κ denotes the ratio of the LOS energy to the multipath energy and $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. Nakagami distribution was shown to fit measurements in micro-cellular systems [15], where the received SNR has the pdf [16]

$$f_\gamma(\gamma) = \left(\frac{m}{\gamma_i}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-\frac{m\gamma}{\gamma_i}\right), \quad \gamma \geq 0, \quad m \geq 0.5 \tag{3}$$

where $\Gamma(.)$ is the Gamma function and $m = \gamma_i^2/\mathrm{Var}[\sqrt{\gamma}]$ is the Nakagami parameter that indicates the fading severity.

When $k = 0$, the signal model in Equation (1) represents the uplink channel from user l to the BS, where the received average energy is denoted by E_s and the average uplink SNR is $\gamma_s = E_s/N_0$. The uplink channels from different users are assumed to be iid with a Rayleigh distribution. Moreover, the interuser channels and the uplink channels are assumed to be mutually independent and slow enough such that the fading process stays fixed within a frame. This is a reasonable assumption for slowly moving mobile units that are separated enough in the space [17]. In addition, we assume that the interuser channels are reciprocal as in [4,5]. At the receivers of users and the BS, coherent detection is employed using perfect channel side information.

3. End-to-End Probability of Error

In this section, we derive the end-to-end bit error probability for users in a coded cooperation network. Throughout the paper, the subscripts c, u, and b are used to denote conditional, unconditional, and bit error probabilities, respectively. In a cluster, each user acts independently from his partners, not knowing whether his partners have decoded successfully his first subframe. Hence, there are different scenarios for the transmission in the subsequent $J-1$ subframes for each user in the cluster. This makes the error probability of a user depends on two factors; namely, the number of partners who were able to decode his first subframe successfully, and the number of partners whose first subframes were decoded successfully by this user. These numbers define the *cooperation level* between partners in a cluster. The end-to-end error probability is obtained by averaging the error probability (of a specific cooperation scenario) over the different cooperation scenarios, which was derived for the case of two cooperating users in Reference [10]. For two cooperating users, there are four cooperation scenarios [10]; namely, either both users cooperate or do not-cooperate or only one of them cooperates.

In a cluster of size J, there are J^2 possible cooperation scenarios. The end-to-end error probability of a user is obtained by averaging the probability of error over two random variables. The first random variable, U indicates the number of partners who were able to decode the first subframe of the user. The second variable, V indicates the number of partners whose first subframes were decoded successfully by the user. For example, if a user was able to decode the first subframes of v users, then he would use the remaining $J-1-v$ subframes to send his parity subframes that were not sent by his partners. This makes his code stronger since more parity bits are received at the BS. Furthermore, if u partners were able to decode the first subframe of a user, then the codeword of this user would consist of $(u+1)$ subframes, each suffering from an independent fading realization. In order to simplify analysis, we assume that the effect of duplicate reception of subframes (from the user and one of his partners) is negligible, that is, subframes are transmitted once through the cluster.

The end-to-end bit error probability averaged over all cooperation scenarios is given by

$$P_b = \sum_{v=0}^{J-1}\sum_{u=0}^{J-1}\binom{J-1}{v}\binom{J-1}{u}p_{v,u}P_b(v,u) \qquad (4)$$

where $P_b(v,u)$ is the conditional bit error probability of a user given that u partners decoded his first subframe successfully, and he decoded v of his partners, and $p_{v,u}$ is the probability of such event and given by

$$p_{v,u} = E_{h_i}\left\{[1-P_B(h_i)]^{v+u}P_B(h_i)^{2J-2-v-u}\right\} \qquad (5)$$

where h_i is the gain of the interuser channel and $P_B(h_i)$ is the packet error probability of the first subframe, which is upper bounded [18] as

$$P_B(h_i) \leq 1-[1-P_E(h_i)]^B \qquad (6)$$

where B is the number of trellis branches in the rate-R_1 codeword of the first subframe. In generaly, for a rate-$1/n$ convolutional code (or obtained by puncturing a rate-$1/n$ code), B is equal to the source block length K [19]. In Equation (6), $P_E(h_i)$ is the error event probability that is evaluated using the *limiting-before-averaging* approach [20] as

$$P_E(h_i) \leq \min\left\{1, \sum_{d=d_{\min}}^{N_1} a_d P_c(d|h_i)\right\} \qquad (7)$$

where a_d is the number of error events with a Hamming distance d from the all-zero codeword and $P_c(d\,|\,h_i) = Q\left(\sqrt{2d|h_i|^2}\right)$ is the conditional pairwise error probability of a weight-d codeword over the interuser channel with a channel gain of h_i. Note that $P_c(d\,|\,h_i)$ is the probability of decoding a received sequence as a weight-d codeword in a rate-R_1 code given that the all-zero codeword was transmitted.

Among the different cooperation scenarios, it was found that the two extreme scenarios of *no cooperation* and *full cooperation* have the largest probabilities, denoted as $p_{0,0}$ and $p_{J-1,J-1}$, respectively. Thus the performance of coded cooperation is dominated by the performance of these two cooperation scenarios. The probabilities $p_{0,0}$ and $p_{J-1,J-1}$ are listed in Table I for different cluster sizes and interuser SNR values. We observe that for a fixed interuser channel quality, the probability of no cooperation increases as the cluster size increases, which causes the performance of large-size clusters to be worse than that of small-size clusters. As the uplink quality improves for a fixed interuser quality, small-size clusters are expected to outperform large-size clusters. This is because small-size clusters have a smaller probability of no cooperation which has a clear effect on the

Table I. The probabilities of *no cooperation* and *full cooperation* scenarios for a J-user cluster over Rayleigh interuser channels with an interuser SNR of γ_i.

γ_i (dB)	$p_{v,u}$	$J=2$	$J=3$	$J=4$
	$p_{0,0}$	0.5950	0.7249	0.8987
0	$p_{J-1,J-1}$	0.3491	0.1769	0.0481
	$p_{0,0}$	0.0869	0.1216	0.2053
10	$p_{J-1,J-1}$	0.8992	0.8389	0.7345
	$p_{0,0}$	0.0088	0.0124	0.0223
20	$p_{J-1,J-1}$	0.9897	0.9830	0.9698
	$p_{0,0}$	0	0	0
∞	$p_{J-1,J-1}$	1	1	1

performance especially at high uplink SNR as will be shown through the results in Section 5.

4. Bit Error Probability

In this section, we derive the bit error probability corresponding to a specific cooperation scenario. Given $U=u$ and $V=v$ for a user in a cluster, the bit error probability of the corresponding convolutional code is upper bounded [19] as

$$P_b(v,u) \leq \sum_{d=d_{\min}}^{N(v,u)} c_d P_u(v,u;d) \tag{8}$$

where $d_{\min}$ is the minimum distance of the code and c_d is the number of information bit errors corresponding to codewords with output weight d. In Equation (8), $P_u(v,u;d)$ is the unconditional pairwise error probability for a weight-d codeword given that u partners decoded correctly the first subframe of this user and he decoded the first subframe of v of his partners. Furthermore, $N(v,u)$ is the codeword length corresponding to $V=v$ and $U=u$.

Conditioning on $U=u$ and $V=v$ has two consequences on the error performance of a user. First, the received codeword at the BS has a rate R_ξ, where $\xi=\max(J-v,u+1)$. This is due to the negligible effect of duplicate transmission of subframes because of the dominant performance of the no and full cooperation scenarios as discussed above. In this case, $\{c_d\}$ used in Equation (8) are for the rate-R_ξ code. Second, given that $U=u$, each codeword is transmitted over $u+1$ subframes, whose lengths are $\{N_j\}_{j=1}^{u+1}$ bits. Recall that each subframe is transmitted over an independent fading channel via one of the partners in a cluster. Thus, the pairwise error probability $P_u(v,u;d)$ is a function of the distribution of the d error bits over the $u+1$ subframes transmitted by the $u+1$ partners. Since the coded bits of each subframes may not be consecutive bits due to the puncturing used, this distribution is quantified assuming uniform distribution of the coded bits over the subframes [21,22] and is derived as follows.

4.1. Uncorrelated Uplink Channels

Denote the weight of the jth subframe in the codeword by w_j such that $\sum_{j=1}^{u+1} w_j = d$, then the pairwise error probability averaged over the weight patterns $\mathbf{w}=\{w_j\}_{j=1}^{u+1}$ is given by

$$P_u(v,u;d) = \sum_{w_1,w_2,\ldots,w_{u+1}} \frac{\binom{N_1}{w_1}\binom{N_2}{w_2}\cdots\binom{N_{u+1}}{w_{u+1}}}{\binom{N}{d}} P_u(v,u;d\,|\,\mathbf{w}) \tag{9}$$

The pairwise error probability $P_u(v,u;d\,|\,\mathbf{w})$ is found by averaging $P_c(v,u;d\,|\,\mathbf{w})$ over the fading gains. The conditional pairwise error probability for BPSK with coherent detection is given by

$$P_c(v,u;d\mid\mathbf{w}) = \mathrm{Q}\left(\sqrt{2\gamma_s\sum_{j=1}^{u+1} w_j a_j^2}\right) \tag{10}$$

where $a_j = |h_j|$. An exact expression of the pairwise error probability can be found by using the integral expression of the Q-function, $Q(x)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{e}^{(-x^2/2\sin^2\theta)}d\theta$ [23] as

$$\begin{aligned} P_u(v,u;d\mid\mathbf{w}) &= \frac{1}{\pi}\mathrm{E}_a\left[\int_0^{\frac{\pi}{2}} \exp\left(-\beta_\theta\sum_{j=1}^{u+1} w_j a_j^2\right)d\theta\right] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{j=1}^{u+1}\frac{1}{1+w_j\beta_\theta}d\theta \end{aligned} \tag{11}$$

where $\boldsymbol{a}=\{a_j\}_{j=1}^{u+1}$, $\beta_\theta=\gamma_s/\sin^2\theta$ and the product results from the independence of the fading processes affecting different subframes.

When the subframes have the same length, that is, $N_1=N_2=\ldots N_{u+1}=n$, and the number of subframes with weight q is j_q, the probability $P_u(v,u;d)$ is averaged over all possible subframe patterns $\boldsymbol{j}=\{j_q\}_{q=0}^{w}$ [22] as

$$P_u(v,u;d) = \sum_{L=\lceil d/m\rceil}^{d}\sum_{j_1=0}^{L_1}\sum_{j_2=0}^{L_2}\cdots\sum_{j_w=0}^{L_w} P_u(v,u;d\,|\,\boldsymbol{j})p(\boldsymbol{j}\mid d) \tag{12}$$

where $w = \min(n, d), L = u + 1 - j_0$ is the number of subframes with nonzero weight and

$$L_q = \min\left\{L - \sum_{r=1}^{q-1} j_q, \frac{d - \sum_{r=1}^{q-1} r j_r}{q}\right\}, \quad 1 \leq q \leq w \quad (13)$$

The probability of a subframe pattern given d, that is, $p(\boldsymbol{j}\,|\,d)$ is computed using combinatorics as

$$p(\boldsymbol{j}\,|\,d) = \frac{\binom{n}{1}^{j_1}\binom{n}{2}^{j_2}\cdots\binom{n}{w}^{j_w}}{\binom{N}{d}} \cdot \frac{(u+1)!}{j_0! j_1! \ldots j_w!} \quad (14)$$

The left term of $p(\boldsymbol{j}\,|\,d)$ in Equation (14) is the probability of distributing d nonzero bits over $(u+1)$ subframes with j_q subframes having q bits for $1 \leq q \leq w$. The right term of $p(\boldsymbol{j})$ is the number of combinations $\boldsymbol{j} = \{j_q\}_{q=0}^{w}$ among the $(u+1)$ subframes. The conditional pairwise error probability is given by

$$P_c(v, u; d\,|\,\boldsymbol{j}) = Q\left(\sqrt{2\gamma_s \sum_{q=1}^{w} q \sum_{l=1}^{j_q} a_l^2}\right) \quad (15)$$

Averaging over the fading coefficients yields the pairwise error probability as

$$P_u(v, u; d\,|\,\boldsymbol{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{q=1}^{w} \left(\frac{1}{1 + q\beta_\theta}\right)^{j_q} d\theta \quad (16)$$

This simplification is particularly useful when the number of cooperating users is large. Note that due to the summation in Equations (9) and (12), the union bound in Equation (8) becomes complicated when d is large. Thus an approximation to the bit error probability is obtained by truncating Equation (8) to a distance $d_{\max}$.

4.2. Correlated Uplink Channels

The mobile units might be located closely in the space which causes the uplink channels to be correlated. The effect of correlation in the uplink channels is investigated below. The conditional pairwise error probability in Equation (10) can be rewritten as

$$P_c(v, u; d\,|\,\boldsymbol{w}) = Q\left(\sqrt{2\gamma_s \tilde{\mathbf{h}} \cdot \tilde{\mathbf{h}}}\right) \quad (17)$$

where $\tilde{\boldsymbol{h}} = \left[\sqrt{w_1}h_1, \sqrt{w_2}h_2, \ldots, \sqrt{w_{u+1}}h_{u+1}\right]^T$. If the fading gains of the uplink channels are complex Gaussian (i.e., fading magnitude is Rician distributed), the vector $\boldsymbol{h}$ is a correlated complex Gaussian random vector with a covariance matrix $K_{\tilde{\mathbf{h}}}$ whose (i, j)th element is given by

$$K_{\tilde{\mathbf{h}}}(i,j) = E\left[\tilde{h}_{i,j}\tilde{h}_{i,j}^*\right] = \rho_{ij}\sqrt{w_i w_j} \quad (18)$$

where ρ_{ij} is the correlation coefficient between the uplink channels of the ith and the jth cooperating users. Clearly this probability is a function of the inner product $\sum_{j=1}^{u+1} |\tilde{h}_j|^2 = \tilde{\boldsymbol{h}} \cdot \tilde{\boldsymbol{h}}$. The unconditional error probability is found by averaging Equation (17) over the joint pdf of $\tilde{\boldsymbol{h}}$, which is difficult due to the complicated form of this pdf [24]. This problem is similar to the case of maximum-ratio combining (MRC) diversity with unequal-SNR branches [25]. By applying an appropriate linear transformation [25], an uncorrelated random vector $\boldsymbol{g}$ with a covariance matrix $K_{\boldsymbol{g}} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_{u+1}\}$ can generated, where $\{\lambda_i\}_{i=1}^{u+1}$ are the eigenvalues of $K_{\tilde{\mathrm{h}}}$. Thus Equation (10) becomes

$$P_c(v, u; d\,|\,\boldsymbol{w}) = \mathrm{Q}\left(\sqrt{2\gamma_s \sum_{j=1}^{u+1} \lambda_j\,|\,g_j\,|^2}\right) \quad (19)$$

By averaging Equation (19) over the distribution of $\boldsymbol{g}$, the unconditional pairwise error probability becomes

$$P_u(v, u; d\,|\,\boldsymbol{w}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{j=1}^{u+1} \frac{1}{1 + \lambda_j\beta_\theta} d\theta \quad (20)$$

When the SNR becomes high, the pairwise error probability approaches

$$P_u(v, u; d\,|\,\mathbf{w}) \sim \prod_{j=1}^{u+1} \frac{1}{\beta_\theta \lambda_j} \quad (21)$$

For the case of two cooperating users, the eigenvalues of the covariance matrix $K_{\tilde{\boldsymbol{h}}}$ are given by

$$\lambda_{1,2} = \frac{1}{2}(w_1 + w_2) \pm \frac{1}{2}\sqrt{w_1^2 + w_2^2 - 2w_1w_2 + 4\rho^2 w_1 w_2} \quad (22)$$

From Equation (21) it is clear that the diversity order of the coded cooperation with correlated uplink channels is maintained with a reduction in the SNR.

5. Numerical Results

In this section, we present numerical results based on the analysis derived above. We consider coded co-operation with cluster sizes $J = 1, 2, 3, 4$. Each user in the cluster employs a RCPC code from Reference [12] with four memory elements, a puncturing period $P = 8$ and a mother code of rate of $R = R_J = 1/4$. In all cases, the source block is $K = 128$ information bits. All the analytical results are obtained by truncating the union bound by including terms in Equation (8) up to $d_{\max} = 20$.

In Figure 2, the bit error probability is shown versus the uplink SNR assuming perfect Rayleigh interuser channels, that is, infinite interuser SNR. We observe that increasing the cluster size by one user results in significant performance gains, where the achieved performance gains decrease as the cluster size increases. Note that the performance gains of coded cooperation appears in the slope of the error probability curve versus the SNR. This is because more cooperating users increases the diversity order of the coded system.

Figure 3 shows the bit error probability for Rayleigh interuser channels with an SNR of 10 dB, where the approximation is shown for $\gamma_s > 10$ to reduce confusion resulting from the overlapping curves in the low-SNR region. We observe that the performance of clusters with four users is the best for low-to-medium SNR values, where the situation gets reversed as the as the uplink SNR increases. This is because at high SNR the performance becomes limited by the performance of the no cooperation scenario, whose probability increases with the cluster size as shown in Table I. For example, when the interuser SNR is 10 dB, four users provide the best performance for an uplink SNR lower than 7 dB.

For an uplink SNR between 7 and 14 dB, three users perform the best, where two users become the best for an uplink SNR greater than 14 dB. Figure 4 shows the uplink SNR, required to achieve $P_b = 10^{-4}$ over Rayleigh interuser channels versus the interuser SNR. We observe that two users perform the best for low interuser channel SNR, and the situation gets

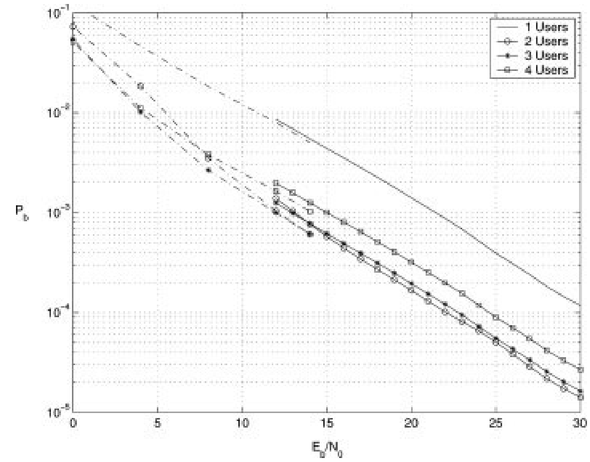

Fig. 3. Bit error probability of coded cooperation with Rayleigh uplink channels and 10-dB Rayleigh interuser channels. solid: approximation, dashed: simulation.

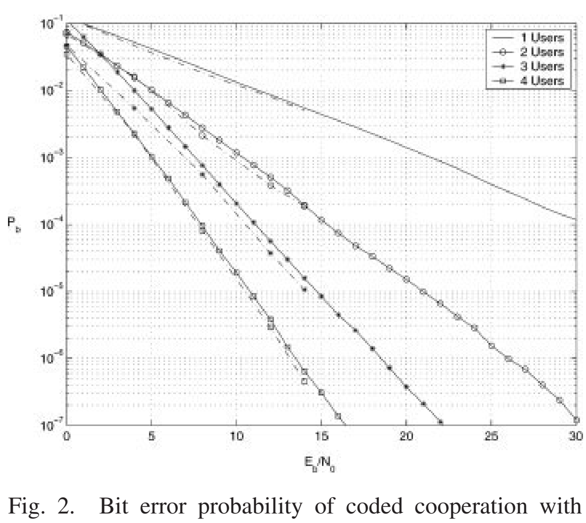

Fig. 2. Bit error probability of coded cooperation with Rayleigh uplink channels and perfect Rayleigh interuser channels, solid: approximation, dashed: simulation.

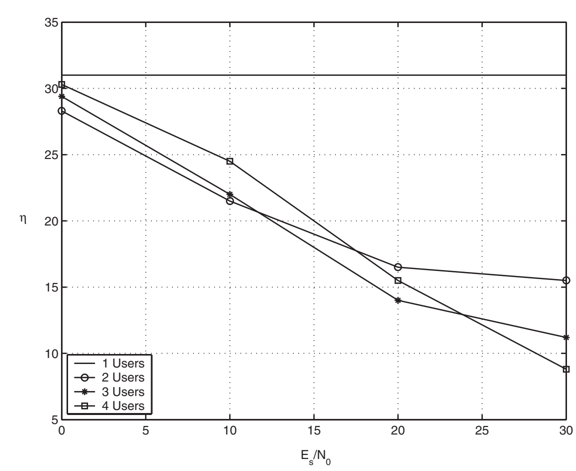

Fig. 4. Uplink SNR required to achieve $P_{\mathrm{b}} = 10^{-4}$ versus the interuser SNR for Rayleigh interuser channels.

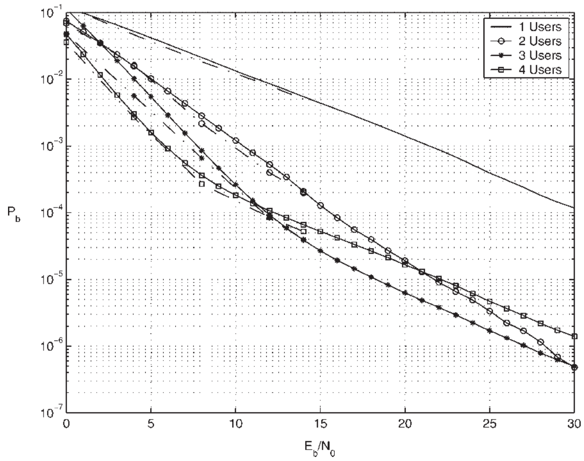

Fig. 5. Bit error probability of coded cooperation with Rayleigh uplink channels and 10-dB Rician interuser channels with $\kappa = 10\,\text{dB}$. solid: approximation, dashed: simulation.

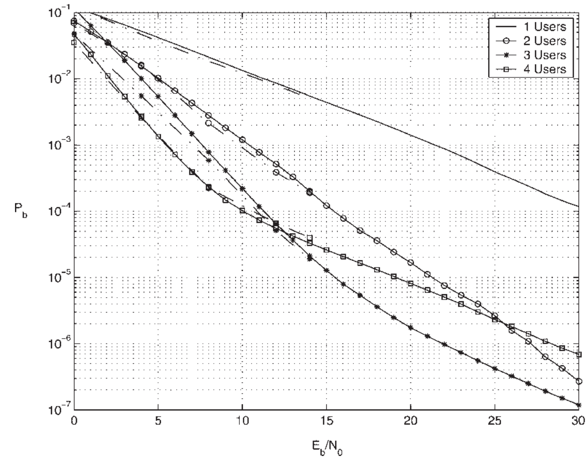

Fig. 6. Bit error probability of coded cooperation with Rayleigh uplink channels and 10-dB Nakagami interuser channels with $m = 3$. solid: approximation, dashed: simulation.

reversed as the interuser channel SNR increases. Note that the quality of the interuser channel is usually better than that of the uplink channels because the BS is usually located far away relative to the users within a cluster.

Figure 5 shows the performance of coded cooperation over Rician interuser channels with $\kappa = 10\,\text{dB}$ and an interuser SNR of 10 dB. Comparing with Figure 3, we observe that as the LOS power of the interuser channel increases, the performance of large clusters improves. This is because the probability of no cooperation decreases with increasing the LOS power of the interuser channel, which improves the performance of large clusters. The same observation is valid for the case of Nakagami interuser channels shown in Figure 6, where increasing the fading parameter m of the interuser channel improves the performance of large clusters more than it does for small clusters. This causes the large clusters to outperform small clusters for a wide range of uplink SNR.

In Figure 7, we show the effect of correlated uplink channels for a two-user cluster in a Rayleigh environment. We observe that the diversity order is maintained even in highly correlated uplink channels ($\rho = 0.9$). For example, coded cooperation with a correlation coefficient $\rho = 0.7$ provides an SNR gain of 9 dB over the single-user case at $P_b = 10^{-3}$, where it encounters an SNR loss of 2 dB compared to the uncorrelated case. This shows that coded cooperation is a powerful technique even when the mobile units are closely located.

6. Conclusions

In this paper, we analyzed the performance of coded cooperation diversity with multiple cooperating users. We derived a union bound on the end-to-end bit error probability averaged over different cooperation scenarios. We considered Rayleigh uplink channels with Rician and Nakagami interuser channels. Furthermore, the case of two cooperating users with correlated uplink channels was analyzed. The effect of the interuser channel quality as well as its distribution was investigated analytically. Results show that as the interuser channel quality improves, large clusters

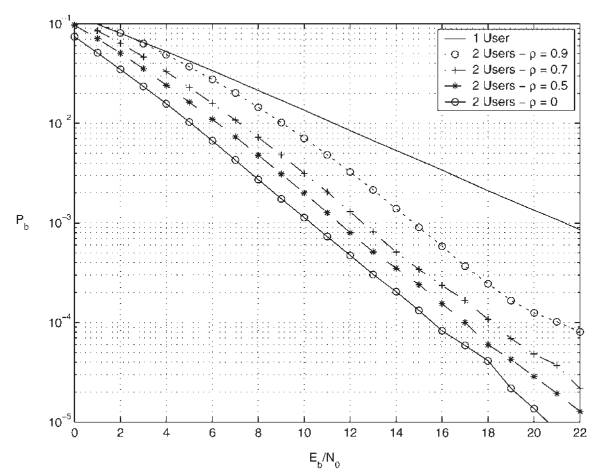

Fig. 7. Analytical bit error probability of a two-user coded cooperation with correlated Rayleigh uplink channels and perfect Rayleigh interuser channels.

outperform small clusters. The same observation applies as the Rician and Nakagami parameters of the interuser channels increase. The main conclusion is that two cooperating users provide the best performance when the interuser channel is bad. Furthermore, the number of cooperating users is optimized as a function of the quality of the interuser and uplink channels and the distribution of the multipath fading in the network.

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