this new approach provides substantial performance gains over limited feedback beamformers designed for uncorrelated channels.

It is of interest to find efficient ways to search over these subspace codebooks. Currently, a brute force search is used by computing the beamforming gain for each possible codebook vector. It might be possible to use other coding techniques to localize the beamforming vector required for feedback to a small search sphere in the Grassmann manifold.

References

[1] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, "Efficient use of side information in multiple-antenna data transmission over fading channels," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1423–1436, Oct. 1998.

[2] G. Jöngren and M. Skoglund, "Utilizing quantized feedback information in orthogonal space–time block coding," in *Proc. IEEE Global Telecommunications Conf.*, San Francisco, CA, Nov./Dec. 2000, vol. 2, pp. 995–999.

[3] E. G. Larsson, G. Ganesan, P. Stoica, and W.-H. Wong, "On the performance of orthogonal space–time block coding with quantized feedback," *IEEE Commun. Lett.*, vol. 6, no. 11, pp. 487–489, Nov. 2002.

[4] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.

[5] D. J. Love, R. W. Heath, Jr., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2735–2747, Oct. 2003.

[6] S. Zhou, Z. Wang, and G. B. Giannakis, "Quantifying the power loss when transmit beamforming relies on finite-rate feedback," *IEEE Trans. Wireless. Commun.*, vol. 4, no. 4, pp. 1948–1957, Jul. 2005.

[7] W. Santipach and M. L. Honig, "Asymptotic performance of MIMO wireless channels with limited feedback," in *Proc. IEEE Military Communications Conf.*, Boston, MA, Oct. 2003, vol. 1, pp. 141–146.

[8] G. Jöngren and M. Skoglund, "Quantized feedback information in orthogonal space–time block coding," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2473–2486, Oct. 2004.

[9] V. Lau, Y. Liu, and T.-A. Chen, "On the design of MIMO block-fading channels with feedback-link capacity constraint," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 62–70, Jan. 2004.

[10] R. S. Blum, "MIMO with limited feedback of channel state information," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, Hong Kong, Apr. 2003, vol. 4, pp. 89–92.

[11] P. Xia and G. B. Giannakis, "Design and analysis of transmit-beamforming based on limited-rate feedback," in *Proc. IEEE Vehicular Technology Conf.*, Los Angeles, CA, Sep. 2004, vol. 3, pp. 1653–1657.

[12] M. K. Simon and M. S. Alouini, *Digital Communications Over Fading Channels*. New York: Wiley, 2000.

[13] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, planes, etc.: Packings in Grassmannian spaces," *Exp. Math.*, vol. 5, no. 2, pp. 139–159, 1996.

[14] T. Ericson and V. Zinoviev, *Codes on Euclidean Spheres*. Amsterdam, The Netherlands: North-Holland, 2001.

[15] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Boston, MA: Kluwer, 1992.

[16] B. M. Hochwald, T. L. Marzetta, T. J. Richardson, W. Sweldens, and R. Urbanke, "Systematic design of unitary space–time constellations," *IEEE Trans. Inf. Theory*, vol. 46, no. 6, pp. 1962–1973, Sep. 2000.

[17] D. Agrawal, T. J. Richardson, and R. L. Urbanke, "Multiple-antenna signal constellations for fading channels," *IEEE Trans. Inf. Theory*, vol. 47, no. 6, pp. 2618–2626, Sep. 2001.

[18] T. Strohmer and R. W. Heath, Jr., "Grassmannian frames with applications to coding and communications," *Appl. Comput. Harmon. Anal.*, vol. 14, no. 3, pp. 257–275, May 2003.

[19] A. R. Calderbank, R. H. Hardin, E. M. Rains, P. W. Shor, and N. J. A. Sloane, "A group-theoretic framework for the construction of packings in Grassmannian spaces," *J. Algebr. Comb.*, vol. 9, no. 2, pp. 129–140, 1999.

[20] E. Visotsky and U. Madhow, "Space–time transmit precoding with imperfect feedback," *IEEE Trans. Inf. Theory*, vol. 47, no. 6, pp. 2632–2639, Sep. 2001.

[21] V. Erceg *et al.*, *Indoor MIMO WLAN Channel Models*, Sep. 2003. IEEE 802.11-03/161r2.

[22] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct. 1999.

[23] E. G. Larsson and P. Stoica, *Space–Time Block Coding for Wireless Communications*. New York: Cambridge Univ. Press, 2003.

[24] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 6, pp. 1211–1226, Aug. 2002.

Error Probability of Bit-Interleaved Coded Modulation in Wireless Environments

Ping-Cheng Yeh, Salam A. Zummo, and Wayne E. Stark

***Abstract*—The bit-interleaved coded modulation (BICM) method is efficient in mitigating multipath fading by providing time diversity. In this paper, union bounds on the bit and packet error probabilities of the BICM are derived. In the derivation, the authors assume the uniform interleaving of coded bits prior to mapping them onto the signal constellation. This results in a random distribution of the error bits in a codeword over the transmitted symbols. This distribution is evaluated, and the corresponding pairwise error probability is derived. Union bounds are functions of the distance spectrum of the channel code and the signal constellation used in the BICM system. The authors consider BICM systems operating over additive white Gaussian noise (AWGN), Rayleigh, Rician, and Nakagami fading channels. Results show that the new bounds are tight to simulation curves for different channel models. The proposed bounds are general for any coding scheme with a known distance spectrum.**

***Index Terms*—Additive white Gaussian noise (AWGN), bit interleaved, bit-interleaved coded modulation (BICM), coded modulation, convolutional codes, fading channels, generalized fading, Nakagami, Rayleigh, Rician, turbo codes, union bound.**

I. Introduction

The growing demand for data communications require bandwidth-efficient transmission techniques. A serious challenge to reliable communication in wireless systems is the time-varying multipath fading environment, which causes the received SNR to vary randomly. The fading distribution depends on the environment. For example, if a line of site (LOS) exists between the transmitter and the receiver in addition to the multipath reception, the fading process can be modeled by a Rician distribution [1]. Another popular fading model is the Nakagami distribution [2], which provides a family of distributions that match measurements in different propagation environments [3].

Coding and diversity techniques are methods used to mitigate the effects of multipath fading. Coded modulation [4] jointly considers error control coding and modulation to achieve high transmission rates

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with good quality. The basic idea is to partition the signal space into sets and use coding to maximize the distance measure between coded signals. For example, the Euclidean distance is maximized for additive white Gaussian noise (AWGN) channels, whereas the symbolwise Hamming distance is the appropriate distance measure for Rayleigh fading channels. The symbolwise Hamming distance is defined as the number of symbols in which two vectors of symbols are different. In fading environments, the symbolwise Hamming distance can be increased by interleaving coded bits prior to mapping them onto the signal constellation [5], [6]. This method is referred to as the bit-interleaved coded modulation (BICM).

Because of the bit interleaver used in the transmitter, each constellation symbol is composed of bits that are located far from each other in the codeword (from the decoder point of view). Thus, a symbol error does not cause consecutive error bits in the codeword, which improves performance significantly. However, the random nature of distributing the error bits over different symbols causes the performance analysis to be difficult. An expurgated (EX) bound of the bit error probability of the BICM was presented in [6] for Gray-labeled signal constellations and in [7] for square quadratic amplitude modulation (QAM) constellations. The EX bound is based on an equivalent channel model that converts the M-ary channel of the BICM into $m = \log_2 M$ parallel binary channels connected via a random switch. As a result of the model, the EX bound was derived assuming each error bit belongs to different symbols. However, in reality, it is possible to have more than one error bit residing in one symbol due to the bit-interleaving method used. Moreover, the EX bound was derived using the log–sum approximation $\log \sum_j \alpha_j \simeq \max_j \log \alpha_j$, which causes the EX bound to become loose as the size of the signal constellation increases.

In this paper, we derive union bounds on the bit and packet error probabilities of the BICM over the AWGN and fading channels. In the new bounds, we assume the uniform interleaving of coded bits prior to mapping them onto the signal constellation. The distribution of error bits in a codeword among the symbols is derived and the corresponding pairwise error probability is evaluated for the AWGN and fading channels with Rayleigh, Rician, and Nakagami distributions. The union bounds are general for any coding scheme with a known distance spectrum and any signal constellation with Gray labeling. Simulation results show that the proposed bound is tighter than the EX bound of [6], and it is tight for different coding schemes, signal constellations, and channel models. In addition, the weight-spectrum-estimation algorithm for turbo codes [8], is combined with our BICM bound, resulting in a very accurate approximation of the packet error probability of turbo coded BICM under various fading channel models.

The outline of this paper is as follows. The BICM system model is described in the next section. In Section III, the new union bounds are presented. Expressions for the pairwise error probability corresponding to different channel distributions are derived in Section IV. The analytical and simulation results are presented and discussed therein. Finally, conclusions are presented in Section V.

II. System Model

The block diagram of a BICM with iterative decoding is shown in Fig. 1. The information sequence $\mathbf{U} = \{u_l\}_{l=1}^{K}$ is encoded by a rate-R_c encoder to yield a codeword $\mathbf{C} = \{c_l\}_{l=1}^{N}$ with a length of N bits. The rate of the code is given by $R_c = K/N$. The codeword is interleaved using a random bit interleaver. Groups of m bits are mapped onto a signal point from a signal constellation. The mapping rule is a one-to-one mapping $f : \{0,1\}^m \rightarrow \mathcal{S}$, where $\mathcal{S}$ is a signal space of size $M = 2^m$ and dimension D. The signal points in $\mathcal{S}$ are scaled to have the average energy equal to 1. In general, the input to the mapper is a vector $\underline{c} = [c_1, \ldots, c_m]$, and the output is a signal point $s = f(\underline{c})$. The transmission frame consists of $J = \lceil N/m \rceil$ symbols and is denoted by $\mathbf{S} = \{s_l\}_{l=1}^{J}$. Note that the throughput of the system is mR_c/D bits/dimension.

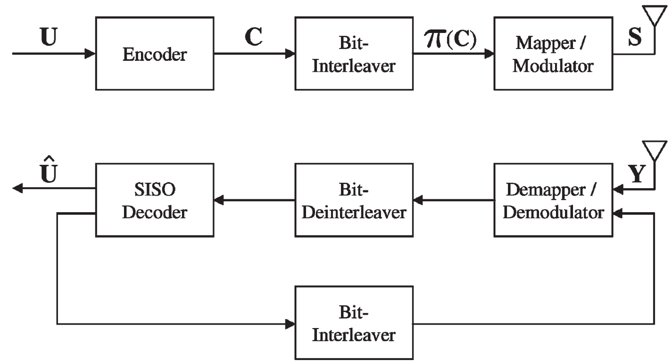

Fig. 1. Block diagram of a BICM system employing iterative detection and decoding.

At the receiver, the sampled matched-filter output corresponding to a transmitted symbol in the time interval l is given by

$$y_l = \sqrt{E_s} h_l s_l + z_l \tag{1}$$

where E_s is the average received signal energy, and z_l is a noise sample of a zero-mean complex Gaussian random variable with a variance N_0, i.e., $z_l \sim \mathcal{CN}(0, N_0)$. The coefficient h_l is the channel gain affecting the lth transmitted symbol and is written as $h_l = a_l \exp(j\theta_l)$, where θ_l is a uniformly distributed phase and a_l is the channel amplitude. In this paper, we assume that a_l is either a constant resulting in an AWGN channel or is distributed according to Rician or Nakagami distributions. Moreover, we assume an infinite channel interleaving, resulting in each symbol being affected by an independent fading realization from other symbols in the frame. Throughout this paper, the channel side information (amplitude and phase) is assumed to be known at the receiver.

The receiver consists of a demodulator/demapper, a deinterleaver, and a soft-input-soft-output (SISO) decoder. The demodulator/demapper computes the log-likelihood ratio (LLR) of the jth coded bit in the lth symbol of the codeword as

$$L(c_j) = \log \left(\frac{\sum_{s_l : c_j = 1} \exp\left(\lambda(y_l, s_l)\right)}{\sum_{s_l : c_j = 0} \exp\left(\lambda(y_l, s_l)\right)} \right) \tag{2}$$

where $\lambda(y_l, s_l)$ with perfect channel side information is given by

$$\lambda(y_l, s_l) = -a_l^2 \frac{E_s}{N_0} \|s_l\|^2 + \frac{2\sqrt{E_s}}{N_0} \mathrm{Re}\left\{h_l \; s_l \; y_l^*\right\} \tag{3}$$

and $(.)^*$ denotes the conjugate operator. The decoder accepts the LLRs of all coded bits and employs an MAP algorithm [9] to compute the LLRs of information bits, which are used for the decision. Note that iterative detection and decoding can be applied to improve the performance of the BICM [10]. The performance analysis of the BICM is presented in the following section.

III. Union Bounds

In this section, union bounds on the bit and packet error probabilities of the BICM are derived. Throughout this paper, the subscripts $\mathrm{c}, \mathrm{u}, \mathrm{b}$, and p are used to denote the conditional, unconditional, bit, and packet error probabilities, respectively. In the following, BICM systems employing convolutional and turbo codes are considered.

However, the results are general to any coding scheme with a known distance spectrum. In general, the bit error probability of binary convolutional and turbo codes is upper bounded [11] by

$$P_{\rm b} \leq \sum_{d=d_{\min}}^{N} \sum_{j=1}^{K} \frac{j}{K} w_{j,d}\, P_{\rm u}(d) \tag{4}$$

whereas the packet error probability can be upper bounded by

$$P_{\rm p} \leq \sum_{d=d_{\min}}^{N} \sum_{j=1}^{K} w_{j,d}\, P_{\rm u}(d) \tag{5}$$

where $d_{\min}$ is the minimum distance of the code, $P_{\rm u}(d)$ is the unconditional pairwise error probability defined as the probability of decoding a received sequence as a weight-d codeword, given that the all-zero codeword is transmitted. In (4) and (5), $w_{j,d}$ is the number of codewords with an input weight j and an output weight d, which is obtained from the weight enumerator of the code [11].

One thing to note here is that the channel is not symmetric. For instance, if 16 QAM is used for modulation, it is not necessarily true that any symbol pair with Hamming distance 2 will have the same Euclidean distance as the symbol pair 0011 and 0000. Therefore, strictly speaking, (4) and (5) may not be able to characterize the system performance when codewords other than the all-zero codeword are transmitted. This problem was solved in [6] using the random-modulation concept, in which the channel is modeled by parallel binary channels with random labeling maps. Furthermore, this equivalent model was used in [6] to derive an EX bound of the bit error probability. On the other hand, the channel-symmetry problem can be solved when computing the squared Euclidean-distance distribution between symbol pairs. Instead of computing the squared Euclidean-distance distribution between a random choice of symbols and the all-zero symbol, we compute it for a pair of random symbols. By applying the squared Euclidean-distance distribution to (4) and (5), we are able to characterize the system performance even when a nonzero codeword is transmitted.

In the BICM, the unconditional pairwise error probability $P_{\rm u}(d)$ is a function of the distribution of the d error bits over the J symbols in the frame. This distribution is quantified by assuming the uniform interleaving of coded bits over the symbols. We then denote the number of symbols with v error bits by j_v and define $w = \min(m, d)$. Then, the symbols are distributed according to the pattern $\mathbf{j} = \{j_v\}_{v=0}^{w}$, where

$$J = \sum_{v=0}^{w} j_v, \quad d = \sum_{v=1}^{w} v j_v. \tag{6}$$

$P_{\rm u}(d)$ is obtained by averaging over all possible symbol patterns

$$P_{\rm u}(d) = \sum_{j_w=0}^{L_w} \sum_{j_{w-1}=0}^{L_{w-1}} \cdots \sum_{j_1=0}^{L_1} P_{\rm u}(d|\mathbf{j}) p(\mathbf{j}) \tag{7}$$

where $L_w = \max\{0, \lfloor d/w \rfloor\}$, and

$$L_v = \max\left\{0, \left\lfloor \frac{d - \sum_{r=v+1}^{w} r j_r}{v} \right\rfloor\right\}, \qquad 1 \leq v < w \tag{8}$$

and $P_{\rm u}(d|\mathbf{j})$ is the pairwise error probability conditioned on the symbol distribution pattern $\mathbf{j}$. The probability of a symbol pattern $p(\mathbf{j})$ is computed using combinatorics as

$$p(\mathbf{j}) = \frac{\binom{m}{1}^{j_1}\binom{m}{2}^{j_2}\cdots\binom{m}{w}^{j_w}}{\binom{mJ}{d}} \cdot \frac{J!}{j_0! j_1! \cdots j_w!} \cdot \delta\left(J - \sum_{v=0}^{w} j_v\right). \tag{9}$$

The left term of $p(\mathbf{j})$ in (9) is the probability of distributing d error bits over J symbols with j_v symbols having v error bits for all possible values of v. The middle term of $p(\mathbf{j})$ is the number of combinations $\mathbf{j} = \{j_v\}_{v=0}^{w}$ among the J symbols. Using (7)–(9), the union bounds of the BICM can be obtained by substituting (7) in (4) and (5).

For fading channels with coherent detection, the conditional pairwise error probability conditioned on the symbol pattern $\mathbf{j}$ and the fading variables $\{a_j\}_{j=1}^{J}$ is given by

$$P_{\rm c}(d|\mathbf{j}, a_1, \ldots, a_J) = {\rm E}_{d_1^2,\ldots,d_J^2}\left[{\rm Q}\left(\sqrt{\frac{mR_{\rm c}\gamma_{\rm b}}{2} \cdot \sum_{j=1}^{J} a_i^2 d_j^2}\right)\right] \tag{10}$$

where $\gamma_{\rm b} = E_{\rm b}/N_0$ is the SNR per information bit, and d_j^2 is the squared Euclidean distance between the jth symbol of the error codeword and that of the desired codeword. Note that the d error bits are distributed over the J symbols according to the pattern $\mathbf{j}$. The expectation in (10) is taken with respect to the squared Euclidean distances $\{d_j^2\}_{j=1}^{J}$. The unconditional pairwise error probability is found by averaging (10) over the fading variables $\{a_j\}_{j=1}^{J}$. It is written using the integral expression of the Q function $Q(x) = 1/\pi \int_0^{\pi/2} \exp^{(-x^2/2\sin^2\theta)}\, d\theta$ [12] as

$$P_{\rm u}(d|\mathbf{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{j=1}^{J} {\rm E}_{a_j}\left[{\rm E}_{d_1^2,\ldots,d_J^2}\left[\exp\left(-\beta(\theta) a_j^2 d_j^2\right)\right]\right] d\theta \tag{11}$$

where $\beta(\theta) = mR_{\rm c}\gamma_{\rm b}/4\sin^2\theta$ and the product is due to the independence of the fading variables affecting different symbols. By grouping symbols with the same number of error bits and using the independence of $\{d_j^2\}_{j=1}^{J}$, the probability in (11) can be written as

$$P_{\rm u}(d|\mathbf{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w} \left\{{\rm E}_a\left[\Psi_{d_v^2}\left(j\beta(\theta)a^2\right)\right]\right\}^{j_v} d\theta \tag{12}$$

where a is a random variable identically distributed as a_j's and $\Psi_{d_v^2}(\zeta) = {\rm E}_{d_v^2}[{\rm e}^{j\zeta d_v^2}]$ is the characteristic function of the random variable d_v^2, which is the squared Euclidean distance between a pair of symbols with Hamming distance v. It is easy to get the distribution of d_v^2 for a given m-ary signal constellation. Consider all $\binom{M}{2}$ possible distinct symbol pairs, count the number of symbol pairs $\{q_{v,i}\}$ with Hamming distance v and squared Euclidean distance $\xi_{v,i}$, $i = 1, 2, \ldots, k_v$, assuming there exists k_v possible distinct squared Euclidean distances between symbol pairs of Hamming distance v. The probability density function (pdf) of d_v^2 is then given by

$$p_{d_v^2}(x) = \sum_{i=1}^{k_v} p_{v,i}\delta(x - \xi_{v,i}) \tag{13}$$

where $p_{v,\xi_{v,i}} = q_{v,i}/\sum_{i=1}^{k_v} q_{v,i}$. The corresponding characteristic function is given by

$$\Psi_{d_v^2}(\zeta) = \sum_{i=1}^{k_v} p_{v,i}{\rm e}^{j\zeta\xi_{v,i}}. \tag{14}$$

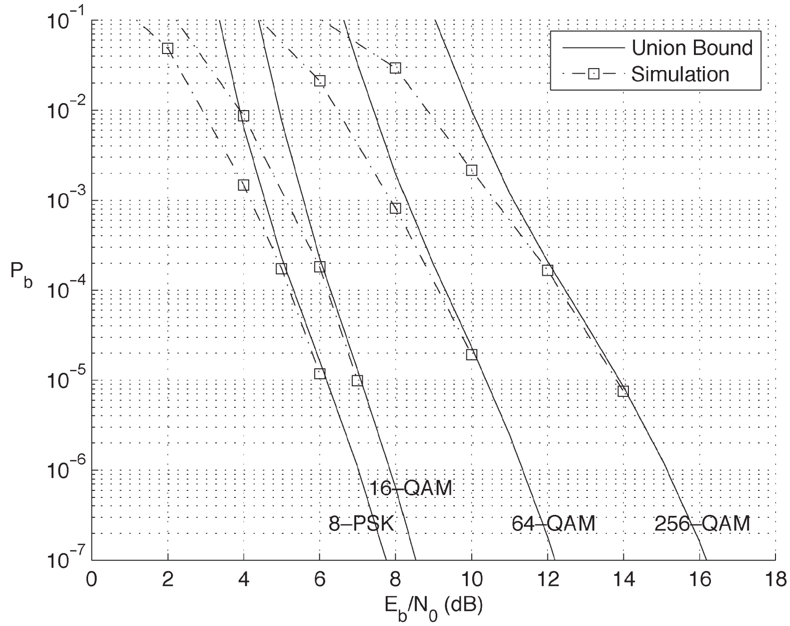

Fig. 2. Bit error probability of the BICM using a rate 1/2 (133, 171) convolutional code over AWGN channels with interleaver sizes of $N = 1024$ for 16 QAM, 256 QAM, and $N = 1026$ for 8 PSK and 64 QAM.

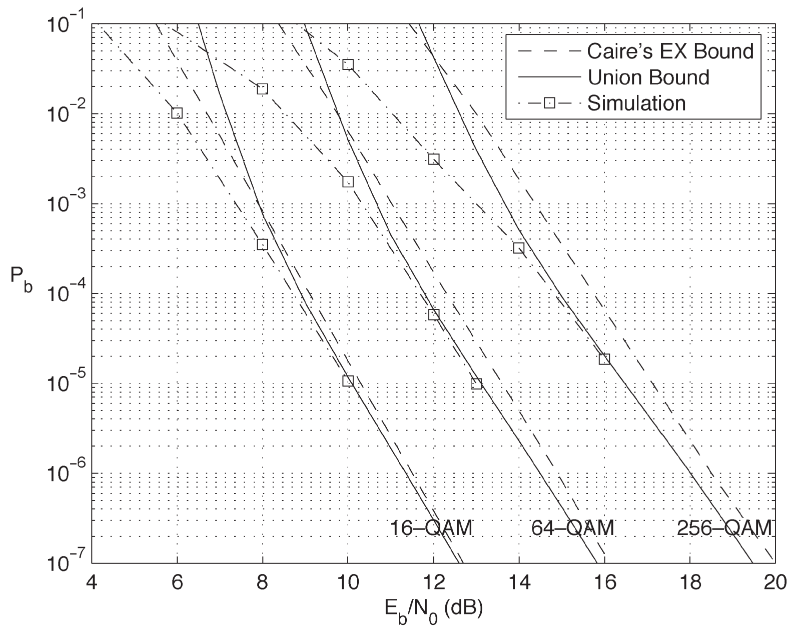

Fig. 3. Bit error probability of the BICM using a rate 1/2 (133, 171) convolutional code over Rayleigh fading channels with interleaver sizes of $N = 1024$ for 16 QAM, 256 QAM, and $N = 1026$ for 8 PSK and 64 QAM. The EX bound of [6] is also shown.

Expressions for the pairwise error probability of the BICM over the AWGN and various fading channels are derived in the following section.

IV. Pairwise Error Probability

In this section, we derive the pairwise error probability corresponding to a specific distribution pattern $\mathbf{j}$ of error bits over the J symbols in a packet. Furthermore, we demonstrate the numerical results of the BICM using convolutional and turbo codes and various modulation schemes over the AWGN channel and fading channels with different fading distributions.

A. AWGN

Combining (12) and (14), the pairwise error probability for the AWGN channels is given by

$$P_{\mathrm{u}}(d|\mathbf{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w} \left(\sum_{i=1}^{k_v} p_{v,i} \mathrm{e}^{-\xi_{v,i}\beta(\theta)} \right)^{j_v} d\theta. \qquad (15)$$

Substituting (15) in (7) and then in (4) and (5) afterward results in the union bounds of the BICM over the AWGN channel. For instance, BICM systems employing the standard rate 1/2 (131, 171)

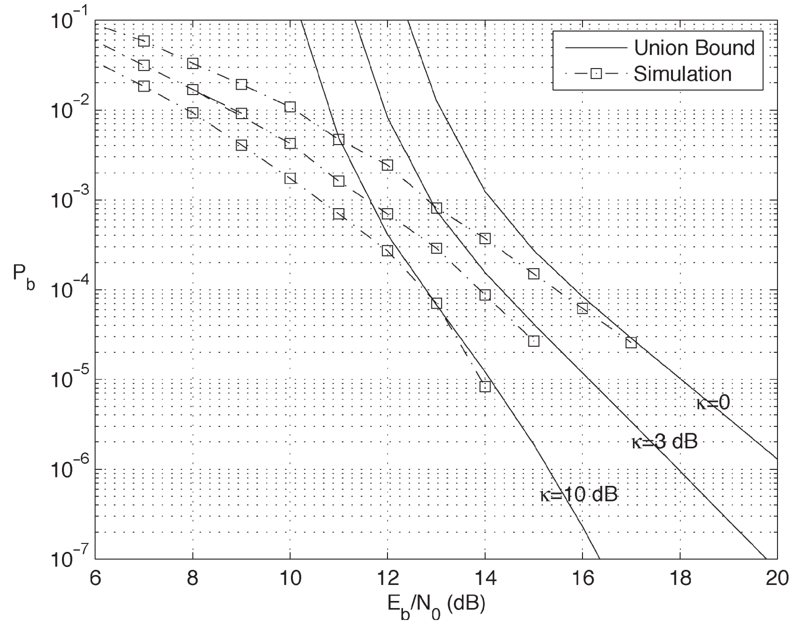

Fig. 4. Bit error probability of the BICM using a rate 1/2 (1, 5/7) convolutional code and 64-QAM signaling over Rician fading channels with different κ values and an interleaver size of $N = 2052$.

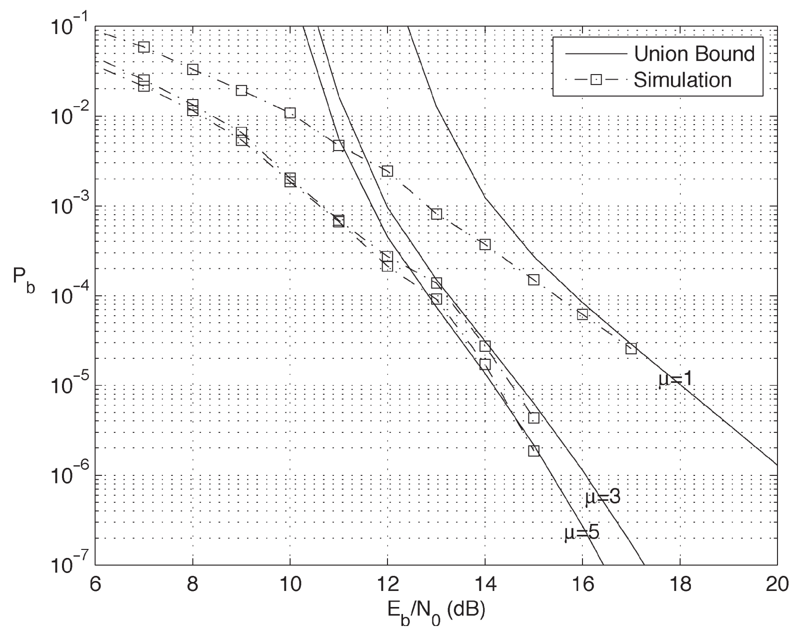

Fig. 5. Bit error probability of the BICM using a rate 1/2 (1, 5/7) convolutional code and 64-QAM signaling over Nakagami fading channels with different fading parameters μ and an interleaver size of $N = 2052$.

convolutional code with Gray-labeled signal constellations are considered. The interleavers used are of size $N = 1024$ bits for 16 QAM, 256 QAM, and $N = 1026$ for 8 phase-shift keying (PSK) and 64 QAM. The interleavers used are the S random interleavers presented in [13]. In order to simulate the effect of uniform bit interleaving, the bit interleaver is replaced for every 100 packets. Moreover, the iterative detection and decoding with three iterations is applied to improve the performance of the BICM. Fig. 2 shows the results for AWGN channels. We observe that the bound curves are tight to the simulation curves at medium-to-high SNR values.

In the experiments of the BICM with non-Gray labeling, it was found out that the proposed union bound does not work well for the non-Gray-labeled cases, which is the same restriction observed in the study of Caire *et al.* [6]. One reason behind this is the use of (7), in which symbol error patterns with more error bits per symbol are weighted less according to (9). Hence, for non-Gray labeling in which each symbol has a higher chance of having more error bits, the bound does not work well since it underestimates the probability of symbol errors with high bit error weights.

B. Rician Fading

If an LOS exists between the transmitter and the receiver, the amplitude of the channel gain can be modeled as a Rician random variable

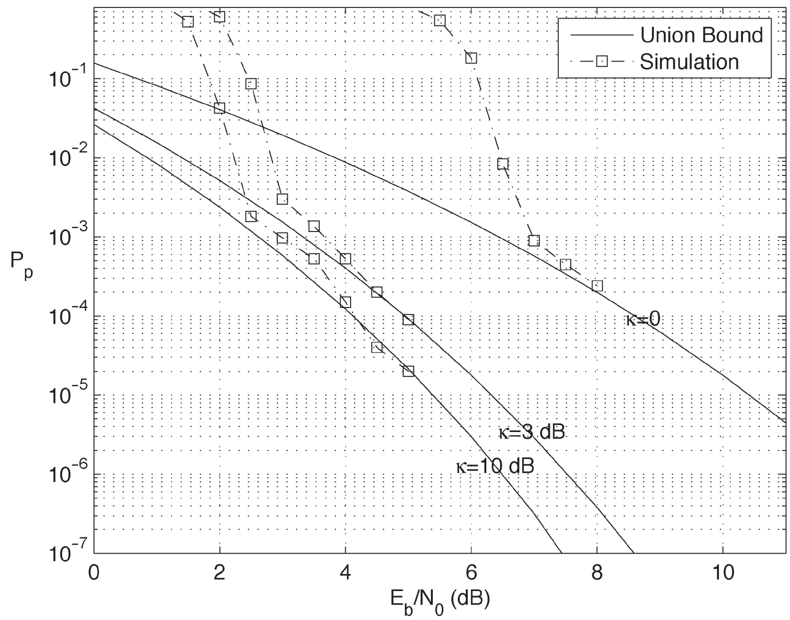

Fig. 6. Packet error probability of the BICM using a rate 1/3 (1, 33/37) turbo code and 8-PSK signaling over Rician fading channels with different κ values and an interleaver size of $N = 3000$.

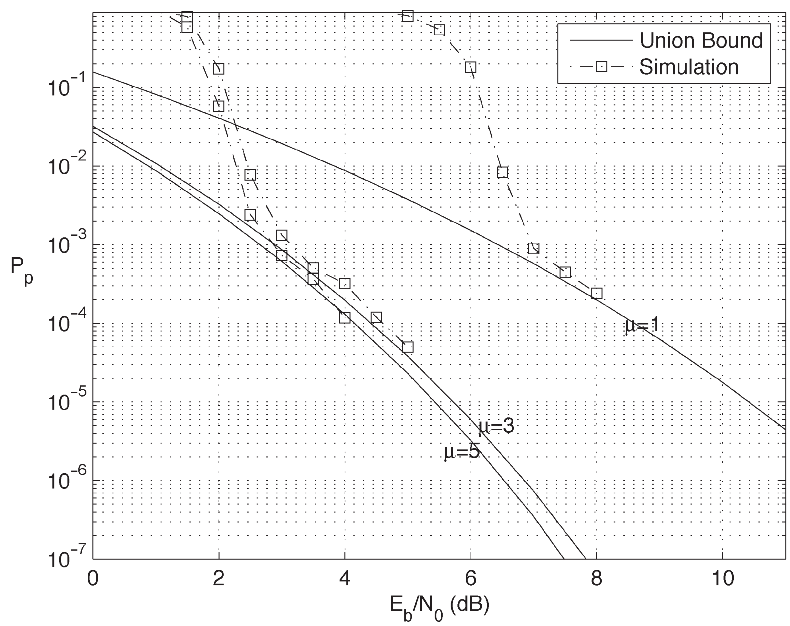

Fig. 7. Packet error probability of the BICM using a rate 1/3 (1, 33/37) turbo code and 8-PSK signaling over Nakagami fading channels with different fading parameters μ and an interleaver size of $N = 3000$.

[1]. In this model, the received signal is composed of two signal-dependent components; namely the specular and diffuse components. The specular component is due to the LOS reception and the diffuse component results from the multipath reception. Let κ denote the ratio of the specular component energy to the diffuse component energy of the channel. In Rician fading, the normalized channel gain h_l affecting each symbol is modeled by a complex Gaussian variable with a $\mathcal{CN}(\sqrt{\kappa/(1+\kappa)}, 1/(1+\kappa))$ distribution. The pdf of the normalized Rician random variable [14] is given by

$$f_a(a) = 2a(1+\kappa)\exp\left[-\kappa - a^2(1+\kappa)\right] \cdot I_0\left(2a\sqrt{\kappa(1+\kappa)}\right), \qquad a \geq 0 \quad (16)$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. While the channel approaches the AWGN case as κ increases toward infinity, setting $\kappa = 0$ results in the Rayleigh fading.

The pairwise error probability for the BICM over the Rician fading channel with perfect side information is obtained by averaging (12) over the channel statistics in (16) as

$$P_{\mathrm{u}}(d|\mathbf{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w} \left[\sum_{i=1}^{k_v} \frac{p_{v,i}\cdot(1+\kappa)}{1+\kappa+\xi_{v,i}\beta(\theta)} \cdot \exp\left(-\frac{\kappa\xi_{v,i}\beta(\theta)}{1+\kappa+\xi_{v,i}\beta(\theta)}\right)\right]^{j_v} d\theta. \quad (17)$$

For Rayleigh fading channels, the pairwise error probability can be obtained by setting $\kappa = 0$ in (17)

$$P_{\mathrm{u}}(d|\mathbf{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w}\left[\sum_{i=1}^{k_v}\frac{p_{v,i}}{1+\xi_{v,i}\beta(\theta)}\right]^{j_v} d\theta. \tag{18}$$

Substituting (17) and (18) in (7) and then in (4) and (5) afterward results in the union bounds of the BICM over the Rician and Rayleigh fading channels. In Fig. 3, our union bound is compared to the EX bound of [6] on the standard rate 1/2 (133, 171) convolutional coded BICM systems over Rayleigh fading channels. We observe that the new bound is tighter than the EX bound in all cases. Note that the EX bound becomes looser as the size of the signal constellation increases due to the use of the log–sum approximation in the derivation of the EX bound. On the other hand, such a trend is not observed in our union bound. The analytical and simulation results for Rician fading channels with different κ values are shown in Fig. 4 for Gray-labeled 64-QAM signaling. The BICM considered here is a rate 1/2 (1, 5/7) recursive systematic convolutional (RSC) coded BICM with an interleaver size of $N = 2052$ bits. From the figure, it is observed that the bound curves are tight to the simulation results for a wide range of the specular-to-diffuse energy ratio κ.

C. Nakagami Fading

The Nakagami distribution was shown to fit a large variety of channel measurements [3]. The pdf of a Nakagami distributed random variable [2] is given by

$$f_a(a) = \frac{2\mu^{\mu}}{\Gamma(\mu)A^{\mu}}a^{2\mu-1}\exp\left(-\frac{\mu a^2}{A}\right), \qquad a \geq 0,\ \mu \geq 0.5 \tag{19}$$

where $A = \mathrm{E}[a^2] = 1$, $\mu = A^2/\mathrm{Var}[a]$ is the fading parameter, and $\Gamma(\cdot)$ is the Gamma function. As the fading parameter μ increases, the fading becomes less severe and reaches the nonfading case when $\mu \to \infty$. The Nakagami distribution covers a wide range of fading scenarios including the Rayleigh fading when $\mu = 1$ and the single-sided Gaussian distribution when $\mu = 0.5$.

The pairwise error probability for the BICM over the Nakagami fading channels with perfect channel side information is obtained by averaging (12) over the fading distribution in (19) as

$$P_{\mathrm{u}}(d|\mathbf{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w}\left[\sum_{i=1}^{k_v} p_{v,i}\cdot\left(\frac{1}{1+\frac{\xi_{v,i}\beta(\theta)}{\mu}}\right)^{\mu}\right]^{j_v} d\theta. \tag{20}$$

Substituting (20) in (7) and then (4) and (5) afterward results in the union bounds for BICM systems over the Nakagami fading channels. Fig. 5 shows the analytical and simulation results for the rate 1/2 (1, 5/7) RSC-coded BICM with Gray-labeled 64-QAM signaling over Nakagami fading channels with different fading parameters μ. We observe that the union bound provides a satisfactory performance evaluation of the BICM for a wide range of the Nakagami fading parameter μ.

D. Turbo Coded BICM

In [8], an efficient weight-spectrum estimation algorithm was proposed to estimate the weight spectrum for the dominant code weights of a turbo code. Using the weight-spectrum estimation algorithm, we are able to estimate $w_d = \sum_j w_{j,d}$ of the dominant turbo code weights. Substitute the result in (5) and we can compute an approximation to the packet error probability of turbo coded BICM systems. In Figs. 6 and 7, we show the packet error probability approximation of a rate 1/3 (1, 33/37) turbo coded BICM system with a block length of 3000 over different Rician and Nakagami fading channels. Due to the extremely long simulation time, a fixed S-random interleaver is used through the whole simulation with $S = 18$. Note that some simulation points are above the bound curves because the bound was computed from the weight-spectrum estimation of the ten smallest turbo code weights only. Nevertheless, the analysis and simulation results are tight for all channel distributions.

V. Conclusion

In this paper, we derived union bounds on the bit and packet error probabilities of the BICM method over the AWGN, Rayleigh, Rician, and Nakagami fading channels. The derivation is based on the uniform interleaving of coded bits prior to mapping them onto the signal constellation. The bounds are functions of the distance spectrum of the channel code and the signal constellation used in the BICM system. Results show that the proposed bounds are tight to simulation curves in medium-to-high SNR regions for various coding schemes and signal constellations. By combining the analysis with the weight-spectrum-estimation algorithm for turbo codes, we were able to accurately analyze the performance of turbo coded BICM systems over various fading channels.

References

[1] S. Rice, "Statistical properties of a sine wave plus random noise," *Bell Syst. Tech. J.*, vol. 27, no. 1, pp. 109–157, Jan. 1948.

[2] M. Nakagami, "The m-distribution-a general formula of intensity distribution of fading," in *Statistical Methods in Radio Wave Propagation*. London, U.K.: Pergamon, 1960.

[3] E. Al-Hussaini and A. Al-Bassiouni, "Performance of MRC diversity systems for the detection of signals with Nakagami fading," *IEEE Trans. Commun.*, vol. COMM-33, no. 12, pp. 1315–1319, Dec. 1985.

[4] G. Ungerboeck, "Channel coding with multilevel/phase signals," *IEEE Trans. Inf. Theory*, vol. IT-28, no. 1, pp. 55–67, Jan. 1982.

[5] E. Zehavi, "8-PSK trellis codes for a Rayleigh channel," *IEEE Trans. Commun.*, vol. 40, no. 5, pp. 873–884, May 1992.

[6] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inf. Theory*, vol. 44, no. 3, pp. 927–946, May 1998.

[7] A. Chindapol and J. Ritcey, "Design, analysis, and performance evaluation for BICM-ID with square QAM constellations in Rayleigh fading channels," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 5, pp. 944–957, May 2001.

[8] P. C. Yeh, A. Özgür Yilmaz, and W. E. Stark, "On the error floor analysis of turbo codes: Weight spectrum estimation (WSE) scheme," in *Proc. IEEE Int. Symp. Information Theory*, Yokohama, Japan, Jul. 2003, p. 439.

[9] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inf. Theory*, vol. IT-20, no. 2, pp. 284–287, Mar. 1974.

[10] X. Li and J. Ritcey, "Trellis-coded modulation with bit interleaving and iterative decoding," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 4, pp. 715–724, Apr. 1999.

[11] A. Viterbi, "Convolutional codes and their performance in communication systems," *IEEE Trans. Commun.*, vol. COMM-19, no. 5, pp. 751–772, Oct. 1971.

[12] M. Simon and D. Divsalar, "Some new twists to problems involving the Gaussian probability integral," *IEEE Trans. Commun.*, vol. 46, no. 2, pp. 200–210, Feb. 1998.

[13] D. Divsalar and F. Pollara, "Multiple turbo codes for deep-space communications," JPL, Pasadena, CA, TDA Progress Rep. 42-121, May 1995.

[14] G. L. Stuber, *Principles of Mobile Communication*, 2nd ed. Boston, MA: Kluwer, 2001.