

# A Bandwidth Efficient Cognitive Radio with Two-Path Amplify-and-Forward Relaying

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**Abstract**—This letter introduces a new cognitive radio (CR) system employing the two-path amplify-and-forward relaying scheme. In the proposed system, the primary user (PU) transmitter cooperates with the secondary user (SU) transmitter and receiver to relay PU data to the PU destination. The proposed algorithm makes use of the inter-relay interference (IRI) between the two relay nodes to transmit SU data and minimize their IRI effect on the PU destination. Two optimization problems are formulated to find optimal power allocation between SU transmission and relaying amplifying factors: one to minimize the probability of error and the other one to maximize the average achievable rate. Simulation results show that the proposed algorithm outperforms the single data transmission and existing two-path relaying scheme. In addition, the PU network achieves diversity order of 3 when maximum likelihood decoder (MLD) is used, whereas SU network achieves diversity order of 2.

## I. INTRODUCTION

Two-path relaying has been recently considered as an attractive wireless communication scheme to improve the spectral efficiency and performance of half-duplex cooperative networks. The two-path relaying scheme consists of a source node  $S$ , a destination  $D$  and two relay nodes  $R_A$  and  $R_B$ . Transmission time slots are divided between the two relays, i.e., while one relay receives the source data the other relay forwards the previous data received during the previous time slot [1, 2]. The two-path relaying scheme needs  $(N + 1)$  time slots to transmit  $N$  data symbols from  $S$  to  $D$ . In order to increase bandwidth efficiency of  $N/(N + 1)$ ,  $N$  should be sufficiently large. In [3, 4], the two-path relaying scheme was used to relay data from a source  $S$  to a destination  $D$  using one of the two famous relaying protocols; namely, amplify-and-forward (AF) or decode-and-forward (DF). Due to the simultaneous transmission from  $S$  to relay nodes  $R_A$  and  $R_B$ , inter-relay interference (IRI) appears and degrades the system performance. Partial interference cancellation (PIC) [3] and full interference cancellation (FIC) [4] were proposed to mitigate the IRI effect at the destination  $D$ .

Cognitive radio (CR) is widely considered as a promising technology to overcome spectrum scarcity. In [5], cooperative relaying was applied in CR, by allowing the SU to operate as a relay node for the PU. Then, the PU rewards the SU by allowing higher interference threshold if the SU operates in an underlay CR mode, or by allocating a time slot to the SU to transmit his data in an overlay CR mode. Power allocation scheme and time division criteria have been developed for this model. The disadvantage of the proposed model in [5] is

that the PU has to wait the SU for two time slots, the first slot is used to relay PU data, and the second slot is used to transmit SU data, resulting in large delay and lower bandwidth efficiency.

In this letter, we present a new cooperative CR model by employing the two-path relaying in a cooperative CR relay network. As shown in Fig. 1, nodes  $S$  and  $D$  represent the PU network, whereas nodes  $R_A$  and  $R_B$  represent the SU network. At the same time,  $R_A$  and  $R_B$  help the PU network using two-path AF relaying as will be explained later. As a reward, the PU system will allow SU system ( $R_A$  and  $R_B$ ) to transmit its data simultaneously through the proposed protocol described in Section II.

The main challenge is to control SU transmission power and amplifying factors of the two relays in order to minimize the probability of error at both PU and SU destinations. To this end, an optimization problem was formulated to minimize the bit error rate (BER) the proposed system in terms of the SU transmission power and the two relays amplifying factors. The Lagrangian multipliers method is used to find the optimal values to minimize the exact probability of error of the PU system under constraints on available power budget. A sub-optimal power allocation is obtained by minimizing the asymptotic probability of error of PU system resulting in lower complexity optimization. Another optimization problem is formulated to maximize the average achievable rate based on the same parameters and constraints used in the minimization problem.

The proposed model needs three time slots to transmit two PU symbols and one SU symbol resulting in achieving a unity bandwidth efficiency. Simulation results show that the proposed model achieves a diversity order of 3 for the PU system and a diversity order of 2 for the SU system.

The rest of this letter is organized as follows: Section II introduces the proposed system model. Section III formulates BER minimization optimization problem. Rate maximization optimization problem is presented in Section IV. Section V discusses the numerical results. Finally, Section VI concludes the work.

## II. SYSTEM MODEL

Fig. 1 shows the operation of the proposed protocol that enables the transmission of two PU symbols and one SU symbol in three time slots.

The channel gain between  $S$  and  $D$  is denoted by  $h_{SD}$ , and the channel gains between  $S$  and  $R_A$  and  $R_B$  are denoted by

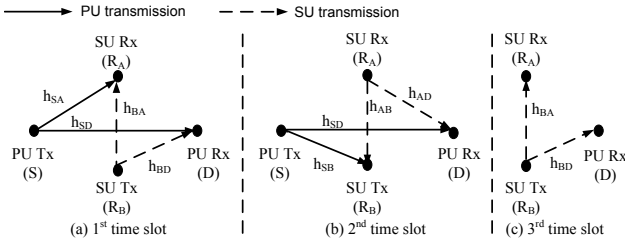


Fig. 1. Cognitive Radio with two path relay scheme.

$h_{SA}$  and  $h_{SB}$  with an average of  $v_S^2$ . The channel gains between  $R_A$  and  $R_B$  are  $h_{AB}$  and  $h_{BA}$  with average channel gain  $v_R^2$ . The two relays nodes  $R_A$  and  $R_B$  have channel gains to the destination node given by  $h_{AD}$  and  $h_{BD}$ , respectively with an average of  $v_D^2$ . For notational simplicity, all the channels are assumed to be independent and identically distributed (i.i.d) flat Rayleigh fading channels. AF protocol is applied by both relays since it is less complex and more flexible in handling IRI than DF protocol [4].

In the first time slot,  $S$  transmits the algebraic subtraction of two successive modulated signals denoted by  $s_1$  and  $s_2$  with a total power of  $P_s$ . At the same time  $R_B$  transmits its data  $b_1$  with power  $P_B$  which interferes with PU data at  $R_A$  and  $D$ . The received signals at  $D$  and  $R_A$  during the first time slot are given by

$$y_D^{(1)} = \sqrt{\frac{P_s}{2}} h_{SD} (s_1 - s_2) + \sqrt{P_B} h_{BD} b_1 + w_D^{(1)} \quad (1)$$

$$y_A^{(1)} = \sqrt{\frac{P_s}{2}} h_{SA} (s_1 - s_2) + \sqrt{P_B} h_{BA} b_1 + w_A^{(1)}, \quad (2)$$

where  $w_D$  and  $w_A$  are AWGN samples with zero-mean and variance  $\sigma^2$ . In the second time slot,  $S$  transmits the second symbol  $s_2$  with a total power of  $P_s$  to  $R_B$  and  $D$ , while  $R_A$  transmits the previous received data after applying AF protocol. The received signals at  $D$  and  $R_B$  during the second time slot are given by

$$y_D^{(2)} = \sqrt{P_s} h_{SD} s_2 + h_{AD} \beta_A y_A^{(1)} + w_D^{(2)}, \quad (3)$$

$$y_B^{(2)} = \sqrt{P_s} h_{SB} s_2 + h_{AB} \beta_A y_A^{(1)} + w_B^{(2)}, \quad (4)$$

where  $w_B$  is AWGN sample with zero-mean and variance  $\sigma^2$ . Assuming  $R_A$  retransmits the data with power  $P_{RA} = \lambda_A P_s$ , then the normalized amplification factor is defined as  $\beta_A^2 = \frac{P_{RA}}{E|y_A^{(1)}|^2} = \frac{\lambda_A P_s}{v_S^2 P_s + v_R^2 P_B + \sigma^2}$ . During the third time slot,  $S$  is idle while  $R_B$  transmits the received signal after removing the interfered SU data  $b_1$  and adding a new fresh version of it but with negative sign, i.e.  $-b_1$  with power  $P_B$ . Under the assumption of knowing CSI by all relay nodes and destinations, the received signal at  $D$  and  $R_A$  during the third time slot are given by

$$y_D^{(3)} = h_{BD} \beta_B (y_B^{(2)} - b_1) - \sqrt{P_B} h_{BD} b_1 + w_D^{(3)} \quad (5)$$

$$y_A^{(3)} = h_{BA} \beta_B (y_B^{(2)} - b_1) - \sqrt{P_B} h_{BA} b_1 + w_A^{(3)}, \quad (6)$$

where  $b_1'$  is the modified image of SU data  $b_1$  such that  $b_1' = \beta_A h_{AB} h_{BA} b_1$ . Assuming that  $R_B$  transmits the received signal by power  $P_{RB} = \lambda_B P_s$ , then the normalized amplifying factor is defined as  $\beta_B^2 = \frac{P_{RB}}{E|y_B^{(2)}|^2} = \frac{\lambda_B P_s}{v_S^2 P_s + \lambda_A v_R^2 P_s + \sigma^2}$ . From the above equations and the presence of two receivers in this

model, the matrix model for the 3-slot system at  $D$  can be written as

$$\mathbf{y}_D = \mathbf{H}_D \mathbf{x}_s + \mathbf{w}'_D, \quad (7)$$

where  $\mathbf{y}_D = [y_D^{(1)}, y_D^{(2)}, y_D^{(3)}]^T$ ,  $\mathbf{x}_s = [\sqrt{P_s} s_1, \sqrt{P_s} s_2, \sqrt{P_B} b_1]^T$ ,

$$\mathbf{H}_D = \begin{bmatrix} \sqrt{\frac{1}{2}} h_{SD} & -\sqrt{\frac{1}{2}} h_{SD} & h_{BD} \\ \sqrt{\frac{1}{2}} \alpha_A & h_{SD} - \sqrt{\frac{1}{2}} \alpha_A & \beta_A h_{AD} h_{BA} \\ \sqrt{\frac{1}{2}} \beta_B h_{BD} \alpha'_A & \beta_B h_{BD} (h_{SD} - \sqrt{\frac{1}{2}} \alpha'_A) & -h_{BD} \end{bmatrix}, \quad (8)$$

and the noise vector at  $D$  is given by

$$\mathbf{w}'_D = \begin{bmatrix} w_D^{(1)} \\ w_D^{(2)} + \beta_A h_{AD} w_A^{(1)} \\ w_D^{(3)} + h_{BD} \beta_B (w_B^{(2)} + h_{AB} \beta_A w_A^{(1)}) \end{bmatrix}, \quad (9)$$

where  $\alpha_A = \beta_A h_{AD} h_{SA}$  and  $\alpha'_A = \beta_A h_{AB} h_{SA}$ . For the SU system, the received signals of the receiver  $R_A$  can be expressed as

$$\mathbf{y}_A = \mathbf{H}_A \mathbf{x}_s + \mathbf{w}'_A, \quad (10)$$

where  $\mathbf{y}_A = [y_A^{(1)}, y_A^{(3)}]^T$ ,  $\mathbf{x}_s = [\sqrt{P_s} s_1, \sqrt{P_s} s_2, \sqrt{P_B} b_1]^T$ ,

$$\mathbf{H}_A = \begin{bmatrix} \sqrt{\frac{1}{2}} h_{SA} & -\sqrt{\frac{1}{2}} h_{SA} & h_{BA} \\ \sqrt{\frac{1}{2}} \beta_B h_{BA} \alpha'_A & \beta_B h_{BA} (h_{SD} - \sqrt{\frac{1}{2}} \alpha'_A) & -h_{BA} \end{bmatrix}, \quad (11)$$

and the noise vector at  $R_A$  is given by

$$\mathbf{w}'_A = [w_A^{(1)}, w_A^{(2)} + h_{BA} \beta_B (w_B^{(2)} + h_{AB} \beta_A w_A^{(1)})]^T. \quad (12)$$

In case there is no direct link between  $S$  and  $D$  or the direct link is too weak, the same equations and expressions are valid with setting  $h_{SD} = 0$ .

### III. POWER ALLOCATION FOR BER MINIMIZATION

In this section, optimal and sub-optimal power allocation problems are presented to minimize the probability of error. Since different images of the data symbols are sent during different time slots creating a virtual MIMO network, maximum likelihood detector (MLD) can be used by the PU and SU systems to detect their data. MLD is the optimal detector in terms of minimizing the probability of error [6]. The MLD estimates the symbol vector  $\hat{\mathbf{x}}_s$  that gives the minimum Euclidean distance metric at  $D$  and  $R_A$ , independently. The Euclidean distance metric can be expressed [7] for  $D$  and  $R_A$ , respectively as

$$\mu_D = \|\mathbf{y}_D - \mathbf{H}_D \mathbf{x}_s\|^2 = \sum_{l=1}^{L=3} |y_D^{(l)} - \mathbf{h}_D^{(l)} \mathbf{x}_s|^2, \quad (13)$$

$$\mu_A = \|\mathbf{y}_A - \mathbf{H}_A \mathbf{x}_s\|^2 = \sum_{l=1}^{L=2} |y_A^{(l)} - \mathbf{h}_A^{(l)} \mathbf{x}_s|^2, \quad (14)$$

where  $\mathbf{h}_D^{(l)}$  and  $\mathbf{h}_A^{(l)}$  denote the  $l$ -th row of  $\mathbf{H}_D$  and  $\mathbf{H}_A$ , respectively. The MLD computational complexity depends on the number of points in the signal constellation and the number of transmitters which are three nodes in this system; namely  $S$ ,  $R_A$  and  $R_B$ . The pairwise-error probability is defined as the probability that the MLD chooses the erroneous data

vector  $c_i = (c_{i1}, c_{i2}, c_{i3})$  instead of the transmitted data vector  $c_j = (c_{j1}, c_{j2}, c_{j3})$ , where the data symbols  $c_{im}$  and  $c_{jm}$  are for the  $m$ -th user. Based on the derivations presented in [7–9], the union bound of the probability of error for  $m$ -th user is given by

$$P_{sm} \leq \sum_i \prod_{l=1}^L \frac{1}{(1 + r_{sm,ijl})}, \quad (15)$$

where  $i$  includes all the indexes of vectors in  $c_i$  that differ in their  $m$ -th position from the transmitted vector  $c_j$ ,  $m = 1, 2$  and  $3$ . The number of independent paths  $L$  takes the value of 3 for PU system and  $L = 2$  for SU system. The term  $r_{sm,ijl}$  is given by [7, eq:(9)]

$$r_{sm,ijl} = a_{sm,ijl} \Gamma_{sm,jl} \sqrt{(a_{sm,ijl} \Gamma_{sm,jl})^2 + 2(a_{sm,ijl} \Gamma_{sm,jl}) + 1}, \quad (16)$$

where  $a_{sm,ijl} = \|d_i - d_j\|^2 / 2E_{sl}$ ,  $E_{sl}$  symbol energy per branch and  $\Gamma_{sm,jl} = E_{sl}/N_0$  is the average symbol SNR per diversity branch as shown in [7, eq:(10)].

#### A. Optimal Power Allocation

In this part, the power allocation optimization problem was formulated to minimize the probability of error for the proposed system by controlling the SU transmission power  $P_B$  and the two relays amplifying factors  $\beta_A$  and  $\beta_B$ . The goal is to find the values of those parameters that minimize the overall BER. The BER is a function of the SNR and then the BER for a given channel state may be expressed as [10]  $P_b(e) = f(P_B, \lambda_A, \lambda_B)$ , where  $f(\cdot)$  is a function determined by a specific modulation scheme and detection method. In this problem,  $f(\cdot)$  equals the probability of error given in (15). Then, an optimization problem has been formulated in which the target function can be minimizing the PU BER only or minimizing the total sum BER of the PU and SU. Such that

$$\begin{aligned} & \text{minimize } f(P_B, \lambda_A, \lambda_B) \\ & \text{subject to: } P_B + \lambda_B P_s \leq \bar{P}_B, \\ & \quad 2P_B + \lambda_A P_s + \lambda_B P_s \leq \bar{P}_{\text{total}}. \end{aligned} \quad (17)$$

To find the optimal values for  $P_B$ ,  $\lambda_A$  and  $\lambda_B$ , Lagrangian multipliers method [11] with the two power constraints in (17) is used. The Lagrangian function  $\mathcal{J}(\cdot)$  can be expressed as

$$\begin{aligned} \mathcal{J}(P_B, \lambda_A, \lambda_B) = & f(P_B, \lambda_A, \lambda_B) + \Lambda_1 (P_B + \lambda_B P_s - \bar{P}_B) \\ & + \Lambda_2 (2P_B + \lambda_A P_s + \lambda_B P_s - \bar{P}_{\text{total}}), \end{aligned} \quad (18)$$

where  $\Lambda_1$  and  $\Lambda_2$  denote the Lagrangian multipliers. Since finding a closed-form solution for the BER function in (15) is difficult, steepest decent algorithm [12] is employed to adaptively find the optimal power allocation in an iterative manner.

#### B. Suboptimal Power Allocation

A less sophisticated approach for power allocation optimization is to minimize the asymptotic union bound of the probability of error instead of the exact one in (15). This results in a less complex optimization problem and yields an approximate power allocation that works well in high SNR

regions. In high SNR regions, the expression in (15) can be reduced to [7]

$$P_{sm, \text{asym}} \leq \sum_i \prod_{l=1}^L r_{sm,ijl}^{-1}. \quad (19)$$

Applying the same discussion in Section III-A provides the suboptimal power allocation.

#### IV. POWER ALLOCATION FOR RATE MAXIMIZATION

In this section, the average achievable rate of the proposed system is discussed. The average achievable rate for the proposed model can be obtained by

$$R_D = \frac{1}{3} E \left\{ \log \left[ \det \left( \mathbf{I} + \frac{\mathbf{H}_D \mathbf{H}_D^*}{E[\mathbf{w}'_D \mathbf{w}_D^*]} \right) \right] \right\}. \quad (20)$$

It is clear that the average achievable rate is a function of  $P_B$ ,  $\lambda_A$  and  $\lambda_B$ . The goal is to find the optimal values which maximize the average achievable rate. In this case,  $g(\cdot)$  equals the average achievable rate given by (20). Then, an optimization problem has been formulated such that:

$$\begin{aligned} & \text{maximize } g(P_B, \lambda_A, \lambda_B) \\ & \text{subject to: } P_B + \lambda_B P_s \leq \bar{P}_B, \\ & \quad 2P_B + \lambda_A P_s + \lambda_B P_s \leq \bar{P}_{\text{total}}. \end{aligned} \quad (21)$$

Following the same steps in Section III-A in solving (17), the optimal solution for rate maximization can be obtained.

#### V. NUMERICAL RESULTS

Numerical examples are presented to verify the performance of proposed scheme. Since the proposed scheme transmits 3 data symbols in 3 time slots with bandwidth efficiency equals 1, the simulation of FIC algorithm [4] were generated with 64 symbols per frame to result in almost a unity bandwidth efficiency. For a fair comparison with the FIC model in [4], the total power budget is set to be the same as that used in [4] for three successive transmissions, i.e.,  $4P_s$ . Since the proposed model has no control on the PU source power, then  $2P_s$  is excluded from the total power budget such as  $\bar{P}_{\text{total}} = \lambda_A P_s + \lambda_B P_s + 2P_B = 2P_s$ . The power budget for  $R_B$  ( $\bar{P}_B$ ) is defined as the maximum power allowed at  $R_B$  for both data relaying and SU data transmission during a single transmission. Then  $\bar{P}_B \geq \lambda_B P_s + P_B$ . The steepest decent algorithm was employed to find the solution in an iterative manner with a step size given by  $\mu(i) = \rho \min_{k:(P_B(i+1), \lambda_A(i+1), \lambda_B(i+1)) \leq 0} \tilde{\mu}_k(i)$ , where  $\rho$  is a positive scaling factor smaller than 1, and  $\tilde{\mu}_k(i)$  is the updated step-size with  $k = 1, 2$  and  $3$  for  $P_B$ ,  $\lambda_A$  and  $\lambda_B$ , respectively.

Fig. 2 shows a comparison between different transmission schemes that the PU can use to transmit its data over a Rayleigh fading channel. The modulation scheme used is QPSK. It is clear that the proposed scheme with MLD detector outperforms the two-path relaying with FIC in [4]. The proposed scheme with ICD detector yields a poor performance in comparison with MLD as it depends on linearity operations with low computational complexity. Although the matrix  $\mathbf{H}_D$  is a full rank matrix of 3 resulting in a full PU diversity

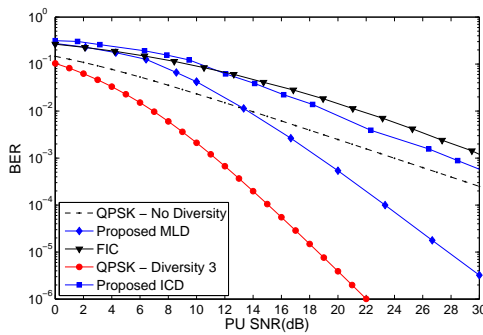


Fig. 2. BER performance comparison between FIC algorithm in [4] and the proposed algorithm using (ICD/MLD) in Rayleigh fading channel.

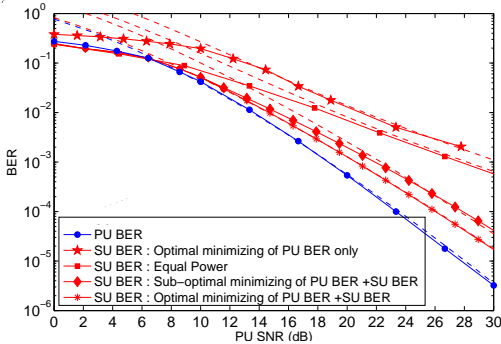


Fig. 3. BER performance of PU and SU employing different power allocation criteria (solid: simulation, dashed: analytical).

order of 3. Fig. 2 shows that the MLD performance is slightly less than 3. Because of the channel model matrix of the proposed system at  $D$  (i.e.,  $\mathbf{H}_D$ ) has some repeated entries such as  $h_{SD}$  and  $h_{BD}$ . In addition, the products of two channel coefficients in  $\mathbf{H}_D$  lead to different fading distribution. Thus, the differences in the channel model causes the loss of diversity compared to classical MIMO.

Fig. 3 introduces the effect of different optimization target functions where the PU performance does not change due to the presence of a direct link while there is a huge improvement in the SU performance depending on the target function. Fig. 3 compares the SU performance under optimal power allocation ( $P_B = 0.812$ ,  $\lambda_A = 1$ ,  $\lambda_B = 0.188$ ) and suboptimal power allocation ( $P_B = 0.612$ ,  $\lambda_A = 1$ ,  $\lambda_B = 0.336$ ) versus equal power allocation schemes.

A comparison between the proposed model and the FIC model [4] in terms of average achievable rate is presented in Fig. 4. In the presence of a direct link between  $S$  and  $D$ , results show that the proposed model achieves higher data rate than FIC until on SNR value of 20dB both models tend to achieve the same average rate. While in the absence of direct link scenario, the proposed model outperforms the FIC algorithm in terms of average achievable rate. The proposed model provides higher data rate depending on the joint detection of PU and SU data.

## VI. CONCLUSION

In this letter, the two relay nodes in the two-path AF relaying scheme are allowed to act as a complete secondary system as well as serving as relay nodes for the PU system. The goal is to fully utilize the channel bandwidth by using

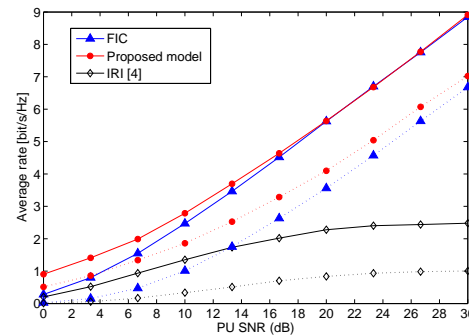


Fig. 4. Comparison in terms of achievable average rate between the proposed model and the two-path FIC scheme in [4] for different scenarios.(solid: with direct link, dashed: without direct link)

the IRI between the two relays to transmit the SU data. Two optimization problems are formulated to minimize the proposed system BER and to maximize the average achievable rate in terms of SU transmission power and the two amplifying factors of the relays. Results show that the PU and SU systems achieve diversity orders of 3 and 2, respectively with no additional complexity at the receivers. It was shown that employing different power allocation schemes do not change the performance of the PU system, but rather have a great impact on the SU system performance. Finally, the proposed model can achieve higher data rate than the two-path relay model proposed in [4].

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