Performance analysis and iterative decoding of I-Q trellis space-time codes

S.A. Zummo and W.E. Stark

Abstract: I-Q trellis codes are known to increase the time diversity of coded systems. When I-Q codes are used with multiple transmit antennas, the decoding and performance evaluation requires the construction of the high-complexity super-trellis of the component codes. In the paper, the bit error probability and the design criteria of I-Q space-time (ST) codes are derived using the transfer functions of the component codes. Conditions for the geometrical uniformity of I-Q ST codes are derived from the geometrical uniformity of the component codes. In addition, a low-complexity iterative receiver for I-Q ST codes is presented. The receiver essentially performs iterative detection and decoding. Results show that three iterations of the iterative receiver performs very close to the optimal decoding.

1 Introduction

Information theoretic analysis of multi-antenna systems [1–3] showed that the system capacity improves significantly using space-time (ST) coding. Among different ST coding schemes, trellis codes are suitable for delay-sensitive applications that require low-complexity receivers. Although ST codes were originally proposed for quasi-static fading channels [4,5] where time diversity is not available, they can exist in the downlink of systems that are modelled by a block fading channel model [6]. Examples of such systems include frequency-hopped spread-spectrum (FH-SS) and time-division multiple access (TDMA). If a trellis code is used over a block fading channel with interleaving, the channel can be considered as memoryless provided that the number of fading blocks is several times larger than the code constraint length. From this observation and owing to the difficulty of optimising trellis codes for block fading channels, various ST trellis codes were optimised for fully-interleaved fading channels in [7–9].

The performance of a trellis code is determined by the time diversity of the code, which can be increased using bit-interleaved (BI) coded modulation (BICM) [10] or I-Q encoding [11]. In [12] BI ST coded scheme was shown to outperform trellis ST codes. The time diversity can be further increased by combining BICM and I-Q encoding [13]. In the I-Q encoder shown in Fig. 1, the input stream is encoded using two independent trellis encoders and the output of each encoder is used to determine one dimension of the complex signal constellation, i.e., the I and Q dimensions. This increases the time diversity of the code at the same or a lower decoding complexity compared to conventional trellis codes. When I-Q encoding is used to design ST codes [8] the super-trellis corresponding to the product of the trellises of the component codes is required for decoding and performance evaluation.

The use of the super-trellis has two drawbacks. First, the performance criteria of I-Q ST codes are expressed as functions of the parameters of the super-trellis, which makes it difficult to optimise and design the component codes directly. Secondly, using the super-trellis results in huge decoding complexity. For example, if each component code has 32 states, the super-trellis has $32 \times 32 = 1024$ states. In this paper, the bit error probability in addition to the design criteria of I-Q ST codes are expressed as functions of the parameters of the component codes. The proposed performance analysis applies directly to BI I-Q ST coded systems. Furthermore, an iterative detection and decoding (IDD) receiver for I-Q ST codes is presented.

2 System model

The general I-Q ST transmitter is shown in Fig. 1. During a frame of NT seconds, the transmitter receives an N-length sequence of binary vectors $\{\mathbf{u}_l\}_{l=1}^{N}$ each of k bits, and outputs the n_t-length signal vectors $\{\mathbf{s}_l\}_{l=1}^{N}$, whose components are transmitted over the n_t antennas. Thus the system throughput is k/T bits/s/Hz. The transmitter splits each input vector into an I and Q vectors, i.e., $\mathbf{u}_{I,l}$ and $\mathbf{u}_{Q,l}$. The

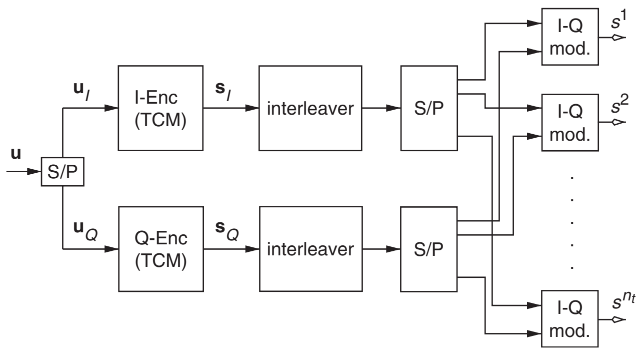

Fig. 1 *The structure of an I-Q encoded ST transmitter*

IEE Proceedings online no. 20045272
doi:10.1049/ip-com:20045272
Paper first received 1st November 2004 and in final revised form 1st June 2005

S.A. Zummo is with the Electrical Engineering Department, King Fahd University of Petroleum and Minerals (KFUPM), Dhahran 31261, Saudi Arabia
W.E. Stark is with the Dept. of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, 48109, USA
E-mail: zummo@kfupm.edu.sa

vector $\mathbf{u}_{I,l}$ is encoded by the I encoder into a signal vector $\mathbf{s}_{I,l}$ of length n_t, whose elements are drawn from an alphabet $\mathcal{A}_I$ which is a 1-D constellation such as M-PAM. The same applies to the Q branch, resulting in the codewords $\mathbf{S}_I = \{\mathbf{s}_{I,l}\}_{l=1}^N$ and $\mathbf{S}_Q = \{\mathbf{s}_{Q,l}\}_{l=1}^N$. Then, the I and Q codewords $\mathbf{S}_I$ and $\mathbf{S}_Q$ are interleaved, and the 2-D signal $s_l^i = s_{I,l}^i + js_{Q,l}^i$ is transmitted over the ith antenna, where $j = \sqrt{-1}$. Thus the transmitted codeword is $\mathbf{S}=\mathbf{S}_I+j\mathbf{S}_Q$.

The received signal is given by

$$y_l = \sqrt{E_s}\sum_{i=1}^{n_t} h_l^i s_l^i + z_l \tag{1}$$

where E_s is the average received energy and z_l is an AWGN sample modelled as a complex Gaussian $\mathcal{CN}(0, N_0)$. Here, h_l^i is the fading gain from the ith transmit antenna at time l, which is modelled as $\mathcal{CN}(0,1)$. It is assumed that the channel gains from different transmit antennas are uncorrelated. Also, the fading process is assumed independent owing to the infinite interleaving assumption. It is assumed that the decoder has perfect channel state information and employs a maximum likelihood (ML) sequence decoding rule, which minimises the metric [4]

$$\mathbf{M}(\mathbf{Y},\mathbf{S}) = \sum_{l=1}^{N}\left| y_l - \sqrt{E_s}\sum_{i=1}^{n_t} h_l^i s_l^i\right|^2 \tag{2}$$

where $\mathbf{Y} = \{y_l\}_{l=1}^N$. In the following, the bit error probability of I-Q ST codes is derived.

3 Performance analysis

Throughout the paper, the subscripts c, u and b are used to denote conditional, unconditional and bit error probabilities, respectively. The conditional pairwise error probability [4] is defined as the probability of decoding a received sequence as a codeword $\hat{\mathbf{S}}$ given that $\mathbf{S}$ was transmitted and conditioned on the fading gains $\mathbf{H} = \{\mathbf{h}_l\}_{l=1}^N$, where $\mathbf{h}_l = \{h_l^i\}_{i=1}^{n_t}$. It can be written as

$$P_c(\mathbf{S},\hat{\mathbf{S}}) = \Pr(\mathbf{M}(\mathbf{Y},\mathbf{S}) > \mathbf{M}(\mathbf{Y},\hat{\mathbf{S}})|\mathbf{S},\mathbf{H}) \tag{3}$$

After some manipulations, the conditional pairwise error probability becomes

$$P_c(\mathbf{S},\hat{\mathbf{S}}) = \Pr(\kappa > d_E^2(\mathbf{S},\hat{\mathbf{S}})|\mathbf{S},\mathbf{H}) \tag{4}$$

where

$$\kappa = 2\sqrt{E_s}\,\mathrm{Re}\left\{\sum_{l=1}^{N} z_l^* \sum_{i=1}^{n_t} h_l^i e_l^i\right\} \tag{5}$$

where $e_l^i = s_l^i - \hat{s}_l^i$ and $(*)$ denotes the complex conjugate. In (5) the random variable κ is Gaussian with $\mathcal{CN}(0, 2N_0 d_E^2(\mathbf{S},\hat{\mathbf{S}}))$ distribution where

$$d_E^2(\mathbf{S},\hat{\mathbf{S}}) = E_s\sum_{l=1}^{N}\left|\sum_{i=1}^{n_t} h_{j,l}^i e_l^i\right|^2 \tag{6}$$

The unconditional pairwise error probability is found by averaging (4) over the channel statistics as

$$P_u(\mathbf{S},\hat{\mathbf{S}}) = \mathbf{E}_{\mathbf{H}}\left[Q\left(\sqrt{\frac{d_E^2(\mathbf{S},\hat{\mathbf{S}})}{2N_0}}\right)\right] \leq \frac{1}{2}\prod_{l=1}^{L}\left(\frac{1}{1+\frac{d_l}{4N_0}}\right) \tag{7}$$

where the Chernoff bound was used in (7), $d_l = E_s\sum_{i=1}^{n_t}|e_l^i|^2$ and $L = \min|\{l : \mathbf{s}_l \neq \hat{\mathbf{s}}_l\}|$ is the minimum time diversity of the ST code. From (7), the design parameters for ST codes over fully-interleaved fading channels [4] are

1) The diversity gain, L.
2) The coding gain defined by the squared product distance, i.e., $d_p^2 = \prod_{l=1}^{L} d_l$.

The main advantage of I-Q encoding is increasing the diversity gain L of the overall ST code. In Table 1 we compare the diversity gain and decoding complexity of ST trellis codes with two transmit antennas and employing a single encoder and I-Q encoding [8]. Here, the decoding complexity is defined as the number of trellis branches in each transition per input bit [11]. We see that for the same throughput and decoding complexity, I-Q-encoded ST codes have larger diversity gains L than ST codes employing a single encoder.

Table 1: Comparison of diversity order L and complexity of ST codes employing single encoder and I-Q encoding technique

Constellation	ST Encoder	# States	L	Complexity
QPSK (2 bits/s/Hz)	Single	4	2	8
	I-Q	4	3	8
	Single	8	3	16
	I-Q	8	4	16
16-QAM (4 bits/s/Hz)	Single	16	2	64
	I-Q	16	4	32
	I-Q	32	5	64
	Single	32	3	128
	I-Q	64	6	128

If the trellis code is geometrically uniform [14], then the all-zero codeword $\mathbf{S}_0 = \{\mathbf{s}_0\}_{l=1}^N$ can be assumed to be transmitted. In Section 4, it will be shown that an I-Q ST code is uniformly geometric if its component codes are uniformly geometric. In this case the error event probability $P_{e,u}$ is upper bounded as

$$P_{e,u} \leq \sum_{\hat{\mathbf{S}}} P_u(\mathbf{S}_0,\hat{\mathbf{S}}) \tag{8}$$

The transfer function of the I-Q ST code is needed in order to evaluate (8). It is discussed in the following.

3.1 Transfer function

The transfer function of a trellis code enumerates the number of codewords at every input weight and output distance [14]. Denote the distinct squared Euclidean distances from $\mathbf{s}_0$ as $\{\xi_1, \xi_2, \ldots, \xi_m\}$, where $\{\xi : \xi = E_s\|\mathbf{s}-\mathbf{s}_0\|^2, \mathbf{s}\in\mathcal{A}^{n_t}\}$. Each branch in the error state diagram of a trellis code [14] is labelled by $J^u D_1^{v_1}\ldots D_m^{v_m}$, where u is the weight of the input vector of the branch and $v_l = 1$ if the corresponding signal vector has a distance ξ_l from $\mathbf{s}_0$ and zero otherwise. Thus the transfer function of a trellis ST code is

$$T(J,\mathbf{D}) = \sum_{u}\sum_{\mathbf{v}} a(u,\mathbf{v}) J^u D_1^{v_1}\ldots D_m^{v_m} \tag{9}$$

where $\mathbf{D}=\{D_1,\ldots,D_m\}$, $\mathbf{v}=\{v_1,\ldots,v_m\}$ and $a(u,\mathbf{v})$ is the number of codewords with input weight u and v_i error vectors with distance ξ_i from $\mathbf{s}_0$, for $i=1,\ldots,m$. Comparing (8) and (9) and using the integral form of the Q-function [15], the bit error probability is

$$P_b \leq \frac{1}{\pi k}\int_0^{\pi/2} \frac{\partial T(J,\mathbf{D})}{\partial J}\bigg|_{J=1, D_v=\left(1+\frac{\xi_v}{4N_0\sin^2\theta}\right),\ v=1,\ldots,m} d\theta \tag{10}$$

In the following, (10) is expressed as a function of the transfer functions of the component codes rather than that of the their super-trellis.

3.2 Pairwise error probability

Define $\mathcal{E} = \{\mathbf{S}\rightarrow\hat{\mathbf{S}}\}$ to be the event of decoding a received sequence as a codeword $\hat{\mathbf{S}}$ given that $\mathbf{S}$ was transmitted. The conditional probability of $\mathcal{E}$ is denoted by $P_c(\mathcal{E})$. Similarly, the events $\mathcal{I} = \{\mathbf{S}_I\rightarrow\hat{\mathbf{S}}_I\}$ and $\mathcal{Q} = \{\mathbf{S}_Q\rightarrow\hat{\mathbf{S}}_Q\}$ are defined. Since $\mathcal{E}$ occurs if either $\mathcal{I}$ or $\mathcal{Q}$ occurs or both of them, we have

$$P_c(\mathcal{E}) \leq P_c(\mathcal{I}) + P_c(\mathcal{Q}) \tag{11}$$

Our goal is to express (4) and (7) as functions of the parameters of the I and Q codes explicitly. Realising that $e_l^i = e_{I,l}^i + je_{Q,l}^i$, (6) becomes

$$d_E^2(\mathbf{S},\hat{\mathbf{S}}) = E_s\sum_{l=1}^{N}\left(\left|\sum_{i=1}^{n_t} h_l^i e_{I,l}^i\right|^2 + \left|\sum_{i=1}^{n_t} h_l^i e_{Q,l}^i\right|^2\right) \tag{12}$$

Following the derivation in [4], (12) can be written as

$$\begin{aligned} d_E^2(\mathbf{S},\hat{\mathbf{S}}) &= E_s\sum_{l=1}^{N}|q_l|^2(d_{I,l}+d_{Q,l}) \\ &= d_E^2(\mathbf{S}_I,\hat{\mathbf{S}}_I) + d_E^2(\mathbf{S}_Q,\hat{\mathbf{S}}_Q) \end{aligned} \tag{13}$$

where $d_{I,l} = E_s\sum_{i=1}^{n_t}|e_{I,l}|^2, d_{Q,l} = E_s\sum_{i=1}^{n_t}|e_{Q,l}|^2$ and the variable q_l follows $\mathcal{CN}(0,1)$ distribution [4]. In (13), the squared Euclidean distance is split into two parts: one part owing to the error in the I decoder and another part owing to the error in the Q decoder. Substituting (13) in (4), the conditional pairwise error probability becomes

$$P_c(\mathcal{E}) = \Pr(\kappa > d_E^2(\mathbf{S}_I,\hat{\mathbf{S}}_I) + d_E^2(\mathbf{S}_Q,\hat{\mathbf{S}}_Q)\,|\,\mathbf{S},\mathbf{H}) \tag{14}$$

When no error event occurs in the I code, $d_E^2(\mathbf{S}_I,\hat{\mathbf{S}}_I) = 0$, and hence (14) becomes the probability of an error event in the Q code, i.e., $P_c(\mathcal{Q}) = P_c(\kappa_Q > d_E^2(\mathbf{S}_Q,\hat{\mathbf{S}}_Q)|\mathbf{S}_Q,\mathbf{H})$. Here, κ_Q is the noise affecting the Q direction only which results from κ after removing $e_{I,l}^i$. Similarly, $d_E^2(\mathbf{S}_Q,\hat{\mathbf{S}}_Q) = 0$ when no error event occurs in the Q code and (14) becomes $P_c(\mathcal{I}) = P_c(\kappa_I > d_E^2(\mathbf{S}_I,\hat{\mathbf{S}}_I)|\mathbf{S}_I,\mathbf{H})$. Using the Chernoff bound the pairwise error probability in the I-code is

$$P_u(\mathcal{I}) \leq \frac{1}{2}\prod_{l=1}^{L_I}\frac{1}{1+\dfrac{d_{I,l}}{4N_0}} \tag{15}$$

where $L_I = \min|\{l : \mathbf{s}_{I,l} \neq \hat{\mathbf{s}}_{I,l}\}|$ is the minimum time diversity of the I code. The coding gain for the I code is defined as $d_{P,I}^2 = \prod_{i=1}^{L_I} d_{I,l}$. Similar expressions for $P_u(\mathcal{Q})$, L_Q and $d_{P,Q}^2$ hold for the Q code. From (15) and (11), we conclude that for fully-interleaved Rayleigh fading channels

1) The diversity gain achieved by an I-Q ST code is $L=\min(L_I, L_Q)$.
2) The coding gain achieved by an I-Q ST code is $d_P^2 = \min(d_{P,I}^2, d_{P,Q}^2)$.

Note that $L=L_I=L_Q$ and $d_P^2 = d_{P,I}^2 = d_{P,Q}^2$ in the case of identical component codes. Exact expressions for $P_{b,I}$ and $P_{b,Q}$ are given by (10) by replacing $T(J,\mathbf{D})$ by $T_I(J,\mathbf{D})$ and $T_Q(J,\mathbf{D})$ for the I and Q codes, respectively, and replacing L, d_l and $\{\delta_v\}$ by the corresponding parameters of the I and Q codes. From (11), the bit error probability of the I-Q ST code is given by

$$P_b \leq P_{b,I} + P_{b,Q} \tag{16}$$

The above performance analysis is not limited to trellis codes, and hence it applies to any coded system employing I-Q encoding. Similar to the work in [13], ST systems that combine BI and I-Q encoding is promising to outperform ST systems employing either I-Q encoding or BI. Recently, a tight bound on the performance of BICM with single transmit antenna was derived in [16]. Using the analysis in [16] along with the analysis presented above, the performance of BI I-Q ST coded systems can be analysed. Note that the design and analysis of BI I-Q ST systems is beyond the scope of this paper.

4 Geometrical uniformity

In the following, we prove that an I-Q ST code is geometrically uniform if its components codes are geometrically uniform. A trellis code is said to be geometrically uniform [17] if the distance spectrum of the code relative to any codeword is the same as that taken relative to the all-zero codeword. In [14], Biglieri *et al.* derived sufficient conditions for geometrical uniformity of trellis codes, which are stated as follows.

Consider a trellis code whose output signal s is given by a mapping of a binary code vector $\mathbf{c}$ onto a signal constellation point, $s=f(\mathbf{c})$. Assume that the code space $\mathbf{C}$ is partitioned into subsets $\mathcal{C}$ and $\tilde{\mathcal{C}}$. Moreover, codewords from $\mathcal{C}$ are permitted at a subset of trellis states $\mathcal{S}$, where the other state subset $\tilde{\mathcal{S}}$ permits codewords from $\tilde{\mathcal{C}}$. Define the partition of signal space corresponding to $\mathcal{C}$ as $\mathcal{A} = \{s : s = f(\mathbf{c}),\ \forall \mathbf{c}\in\mathcal{C}\}$. Similarly, $\tilde{\mathcal{A}}$ is defined resulting in the signal space being partitioned into $\mathcal{A}$ and $\tilde{\mathcal{A}}$. In [14], it was shown that a trellis code is geometrically uniform if:

1) The subset $\tilde{\mathcal{C}}$ is a coset of $\mathcal{C}$, i.e., $\tilde{\mathcal{C}} = \mathcal{C}+\tilde{\mathbf{c}}$, where $\tilde{\mathbf{c}}$ is the coset representative of $\tilde{\mathcal{C}}$ in bits and addition is performed bitwise for each codeword in $\mathcal{C}$.
2) The signal partitions $\mathcal{A}$ and $\tilde{\mathcal{A}}$ are isometrics, i.e., they have the same distance spectrum

$$d_E^2[f(\mathbf{c}), f(\mathbf{c}+\mathbf{e})] = d_E^2[f(\mathbf{c}+\tilde{\mathbf{c}}), f(\mathbf{c}+\tilde{\mathbf{c}}+\mathbf{e})] \tag{17}$$

for all $\mathbf{c}\in\mathcal{C}, \mathbf{e}\in\mathbf{C}$, where d_E^2 represents the squared Euclidean distance between two signal points.

Proposition 1: An I-Q ST code is geometrically uniform if its component I and Q codes are geometrically uniform.

Proof: Consider an I-Q ST code with geometrically uniform component codes, i.e., the code spaces of the I and Q encoders are partitioned into $\mathcal{C}_I, \tilde{\mathcal{C}}_I = \mathcal{C}_I + \tilde{c}_I$ and $\mathcal{C}_Q$,

$\tilde{\mathcal{C}}_Q = \mathcal{C}_Q + \tilde{c}_Q$, respectively. The output signal vector of an I-Q ST encoder is given by a mapping $\mathbf{s} = f(\mathbf{c})$, where $\mathbf{c} = (\mathbf{c}_I, \mathbf{c}_Q)$ is the concatenation of the I and Q code vectors. Therefore, the code space of the I-Q ST encoder is partitioned into four sets $(\mathcal{C}_I, \mathcal{C}_Q)$, $(\tilde{\mathcal{C}}_I, \mathcal{C}_Q)$, $(\mathcal{C}_I, \tilde{\mathcal{C}}_Q)$ and $(\tilde{\mathcal{C}}_I, \tilde{\mathcal{C}}_Q)$, resulting in partitioning the complex signal into $(\mathcal{A}_I, \mathcal{A}_Q)$,$(\tilde{\mathcal{A}}_I,\mathcal{A}_Q)$, $(\mathcal{A}_I, \tilde{\mathcal{A}}_Q)$ and $(\tilde{\mathcal{A}}_I, \tilde{\mathcal{A}}_Q)$, where $\mathcal{A}_I$ and $\mathcal{A}_Q$ are the signal partitions corresponding to $\mathcal{C}_I$ and $\mathcal{C}_Q$, respectively. Now we have:

1) The code sets $(\mathcal{C}_I, \mathcal{C}_Q)$, $(\tilde{\mathcal{C}}_I, \mathcal{C}_Q)$, $(\mathcal{C}_I, \tilde{\mathcal{C}}_Q)$ and $(\tilde{\mathcal{C}}_I, \tilde{\mathcal{C}}_Q)$ are cosets of each other since their I and Q elements are cosets.
2) The complex signal partitions $(\mathcal{A}_I, \mathcal{A}_Q)$, $(\tilde{\mathcal{A}}_I, \mathcal{A}_Q)$, $(\mathcal{A}_I, \tilde{\mathcal{A}}_Q)$ and $(\tilde{\mathcal{A}}_I, \tilde{\mathcal{A}}_Q)$ are isometrics since their I and Q components are isometrics.

Thus I-Q ST codes that employ geometrically uniform component codes are also geometrically uniform. ■

5 Iterative detection and decoding (IDD)

The ML rule at the I and Q decoders requires the computation of the likelihood functions $p(\mathbf{Y}|\mathbf{S}_I, \mathbf{H})$ and $p(\mathbf{Y}|\mathbf{S}_Q, \mathbf{H})$, respectively. However, $\mathbf{Y}$ is a function of $\mathbf{S}_I$ and $\mathbf{S}_Q$ and thus the super-trellis is needed for optimal decoding. In this paper we propose the IDD receiver [12, 18] to solve this problem. The block diagram of the IDD receiver is shown in Fig. 2. It consists of a detection stage and two soft-input soft-output (SISO) modules for the I and Q codes. The detection stage computes for each codeword the probabilities for $l = 1, \ldots, N$

$$p(y_l|\mathbf{s}_{I,l}, \mathbf{h}_l) = Kp(\mathbf{s}_{I,l}) \sum_{\forall \mathbf{s}_Q} p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \mathbf{h}_l)\, p(\mathbf{s}_{Q,l}) \quad (18)$$

$$p(y_l|\mathbf{s}_{Q,l}, \mathbf{h}_l) = K(\mathbf{s}_{Q,l}) \sum_{\forall \mathbf{s}_I} p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \mathbf{h}_l)\, p(\mathbf{s}_{I,l}) \quad (19)$$

where K is a normalisation constant and $p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \mathbf{h}_l)$ is the channel transition probability. As in turbo decoding [19], only extrinsic information is passed to the I and Q decoders, which is defined as the probabilities in (18) and (19) after removing the *a priori* information.

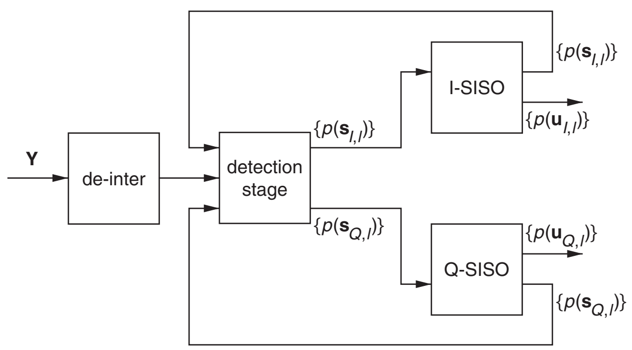

Fig. 2 *The structure of the IDD receiver*

The I and Q SISO modules are maximum *a posteriori* decoders that accept soft information about signal vectors and update them using the BCJR algorithm in [20]. The I-SISO decoder uses $\{p(\mathbf{s}_{I,l})\}_{l=1}^{N}$ as its observation and computes for $l = 1, \ldots, N$

$$p(\mathbf{s}_{I,l}|\mathbf{Y}, \mathbf{H}) = K \sum_{(m,m'):\mathbf{s}_{I,l}} \gamma_l(m, m')\alpha_{l-1}(m')\beta_l(m) \quad (20)$$

where $\gamma_l(m, m') = p(y_l|\mathbf{s}_{I,l}, \mathbf{h}_l)$ is the branch metric for a transition in the I code from state m at time l to state m' at time $l+1$, which is computed in the detection stage. The variables α_l and β_l are the forward and backward recursions in the BCJR algorithm [20]. A similar expression for the Q-SISO module exist by replacing I with Q. The detection stage and the SISO modules keep exchanging extrinsic information about signal vectors in the codeword and decision is made in the last iteration. Clearly, the complexity of the I and Q SISO modules is linear in the number of states of the component codes, which is a significant reduction in complexity compared to the super-trellis.

6 Numerical results

In this paper, we consider I-Q ST systems with one receive and two transmit antennas. As illustrative examples, I-Q ST codes employing QPSK and 16-QAM signal constellations [8] are used. The QPSK I-Q ST code is a 4-state trellis code with a throughput of 2 bit/s/Hz, whereas we use 16-QAM codes with either a 4-state or a 32-state encoders. The results are shown against the signal-to-noise ratio (SNR) per information bit $\gamma_b = E_s/kN_0$.

The performance of the QPSK and the 4-state 16-QAM I-Q codes are shown in Figs. 3 and 4, respectively. In both cases the optimal decoder uses a 16-state super-trellis. The

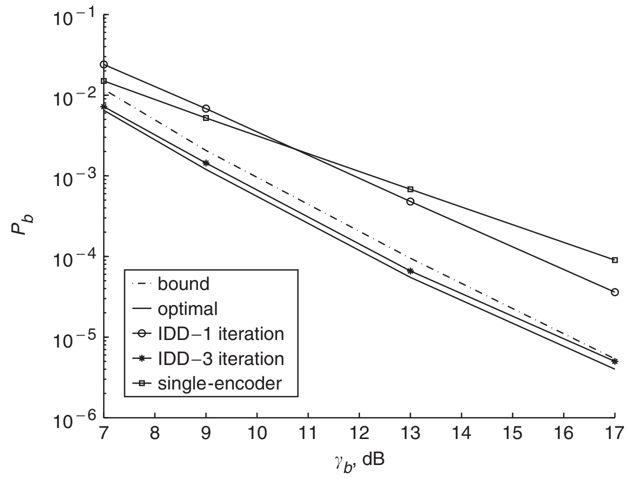

Fig. 3 *Bit error probability of the I-Q ST QPSK code using IDD and optimal super-trellis decoding*

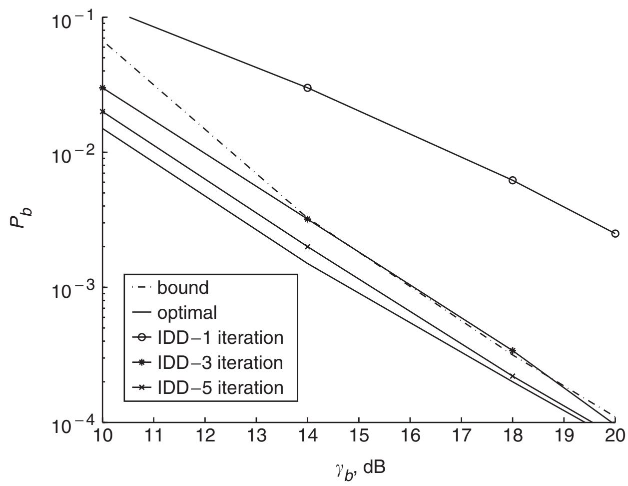

Fig. 4 *Bit error probability of the 4-state I-Q ST QAM code using IDD and optimal super-trellis decoding*

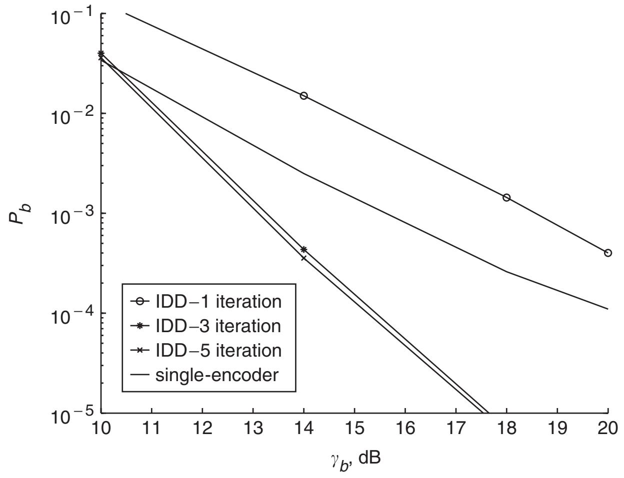

Fig. 5 *Bit error probability of the 32-state I-Q ST QAM code using IDD*

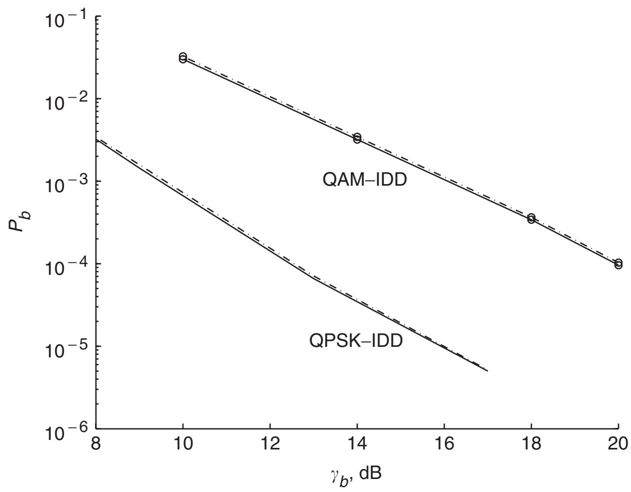

Fig. 6 *Bit error probability of the I-Q QPSK and 16-QAM codes using IDD with three iterations for different packet sizes*
Solid: $N = 500$, dashed: $N = 200$

codeword size is $N = 500$. We observe that the bound is tight to the optimal decoder simulations. Furthermore, using IDD with three iterations performs very close to the optimal decoding. Also shown in Fig. 3 is the performance of a single-encoder QPSK ST code optimised for fully-interleaved fading channels [8]. This code uses a 4-state single encoder and has double the complexity of the corresponding I-Q ST code. From the Figure, the I-Q code with three iterations is 5 dB better than the single-encoder QPSK code at $P_b = 10^{-4}$.

In Fig. 5 the performance of the 32-state 16-QAM I-Q ST code is compared with a 16-QAM ST code employing a 16-state single encoder [8] having the same complexity. We observe that the I-Q code with three iterations is 5 dB better than the single-encoder code. Note that the optimal decoder of the 32-state I-Q ST code needs a 1024-state super-trellis, which has a high complexity compared to the IDD receiver. In Fig. 6, it is shown via simulation that the effect of the codeword size N on the performance of the IDD receiver is negligible, which applies similarly for BI coded modulation [10]. This insures that I-Q ST codes are suitable for delay-sensitive applications.

7 Conclusions

In this paper, the bit error probability of I-Q ST codes was derived as a function of the transfer functions of the component codes. Moreover, design parameters for I-Q ST codes over fully-interleaved fading channels were derived in terms of the parameters of the component codes. A low-complexity iterative receiver for I-Q ST codes was derived and tested. Results show that the IDD receiver with three iterations results in a very close performance to the optimal decoder employing the super-trellis of the component codes.

8 Acknowledgment

This work was supported in part by King Fahd University of Petroleum and Minerals (KFUPM) and the Office of Naval Research under grant N00014-03-1-0232.

9 References

1 Telatar, E.: 'Capacity of multi-antenna Gaussian channels', AT&T Bell Laboratories Internal Technical Memo., 1995
2 Foschini, G., and Gans, M.: 'On The limits of wireless communications in a fading environment when using multiple antennas', *Wirel. Pers. Commun.*, 1998, **6**, (3), pp. 311–335
3 Biglieri, E., Caire, G., and Taricco, G.: 'Limiting performance of block-fading channels with multiple antennas', *IEEE Trans. Inf. Theory*, 2001, **47**, pp. 1273–1289
4 Tarokh, V., Seshadri, N., and Calderbank, A.: 'Space-time codes for high data rate wireless communication: performance criterion and code construction', *IEEE Trans. Inf. Theory*, 1998, **44**, pp. 744–765
5 Foschini, G.: 'Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas', *Bell Labs Tech. J.*, 1996, **1**, (2), pp. 41–59
6 McEliece, R.J., and Stark, W.E.: 'Channels with block interference', *IEEE Trans. Inf. Theory*, 1984, **30**, pp. 44–53
7 Firmanto, W., Vucetic, B., and Yuan, J.: 'Space-time TCM with improved performance on fast fading channels', *IEEE Commun. Lett.*, 2001, **5**, (4), pp. 154–156
8 Zummo, S., and Al-Semari, S.: 'Design of space-time trellis codes for rapid rayleigh fading channels', *J. Commun. Netw*, 2003, **5**, (2), pp. 174–183
9 Zummo, S., and Al-Semari, S.: 'Space-time coded 8-PSK schemes for rapid rayleigh fading channels', *EURASIP J. Appl. Signal Process.*, 2002, **5**, pp. 482–486
10 Caire, G., Taricco, G., and Biglieri, E.: 'Bit-interleaved coded modulation', *IEEE Trans. Inf. Theory*, 1998, **44**, pp. 927–946
11 Al-Semari, S., and Fuja, T.: 'I-Q TCM reliable communication over rayleigh fading channel close to the cutoff rate', *IEEE Trans. Inf. Theory*, 1997, **43**, pp. 250–262
12 Tonello, A.: 'Space-time bit-interleaved coded modulation with iterative decoding strategy'. IEEE Vehicular Technology Conference, VTC/Fall, 2000, pp. 473–478
13 Al-Semari, S., and Fuja, T.: 'Bit interleaved I-Q TCM'. International Symposium of Information Theory and Applications (ISITA), 1996, pp. 17–20
14 Biglieri, E., Divsalar, D., McLane, P., and Simon, M.: 'Inroduction to trellis-coded modulation with applications' (Macmillan Publishing, New York, 1991)
15 Simon, M., and Divsalar, D.: 'Some new twists to problems involving the Gaussian probability integral', *IEEE Trans. Commun.*, 1998, **46**, pp. 200–210
16 Yeh, P., Zummo, S., and Stark, W.: 'Error probability of bit-interleaved coded modulation (BICM) in wireless environments', *IEEE Trans. Veh. Technol.*, Accepted for publication 2005
17 Forney, G.: 'Geometrically uniform codes', *IEEE Trans. Inf. Theory*, 1991, **5**, (37), pp. 1241–1260
18 Bauch, G.: 'Concatenation of space-time block codes and "turbo"-TCM'. IEEE Int. Conf. on Communication ICC, 1999, pp. 1202–1206
19 Berrou, C., and Glavieux, A.: 'Near optimum error correcting coding and decoding: turbo codes', *IEEE Trans. Commun.*, 1996, **44**, pp. 1261–1271
20 Bahl, L., Cocke, J., Jelinek, F., and Raviv, J.: 'Optimal decoding of linear codes for minimizing symbol error rate', *IEEE Trans. Inf. Theory*, 1974, **IT-20**, pp. 284–287