Arab J Sci Eng (2014) 39:8979–8987
DOI 10.1007/s13369-014-1458-z

RESEARCH ARTICLE - ELECTRICAL ENGINEERING

Multiuser Cognitive Networks with *N*th-Best User Selection and Imperfect Channel Estimation

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Received: 26 April 2014 / Accepted: 30 September 2014 / Published online: 15 November 2014

Abstract The *N*th-best user selection scheme is efficient in outdated channel information conditions where the user that was the best at the selection time instant could not be the best at the transmission time instant. Also, this scheme is useful when the scheduling unit fails in error in selecting the best user among all the available users. Furthermore, the *N*th-best user selection scheme is efficient in situations where, while the best user is waiting to be scheduled by a certain base station or a scheduling unit, it gets scheduled by other unit. In the proposed scheme, the secondary user with the *N*th-best end-to-end signal-to-noise ratio (SNR) is scheduled the system resources. In our paper, closed-form expressions are derived for the outage probability, average symbol error probability, and ergodic channel capacity assuming imperfect estimation of channel state information and Rayleigh fading channels. Furthermore, to further analyze the system performance, the system is studied at the high SNR regime. The derived analytical and asymptotic expressions are verified by Monte-Carlo simulations. Main results illustrate that the diversity order of the studied multiuser cognitive *N*th-best user selection network is the same as its non-cognitive counterpart. Also, findings show that with perfect channel estimation of secondary users, the diversity order of the system linearly increases with decreasing the order of the scheduled user, and vice versa, whereas a zero diversity gain is achieved by the system and a noise floor appears in the results when the channels of secondary users are imperfectly estimated assuming constant estimation error variance. Finally, results illustrate that the imperfect estimation of the secondary cell-primary cell channel affects only the coding gain of the system without affecting the diversity order.

Keywords User scheduling · Cognitive radio networks · Multiuser diversity · Imperfect channel estimation · Rayleigh fading

الخلاصة

تمت - في هذه الورقة العلمية - دراسة آداء نظام Cognitive Radio متعدد المُستخدمين مع إختيار المُستخدم صاحب أقوى قناة إتصال ، أو ثاني أقوى قناة إتصال ، أو حتى صاحب N^{th}-أقوى قناة إتصال. تعتبر هذه الطريقة في الإختيار بين المُستخدمين فعّالة في العديد من الحالات ، منها: أنّ المُستخدم الذي كان صاحب أفضل قناة إتصال في لحظة الإختيار بين المُستخدمين أصبح لا يمتلك أفضل قناة عند لحظة البدء في إرسال الإشارة، أنْ يتم بشكل خاطىء إختيار مُستخدم آخر غير الذي يمتلك أقوى قناة إتصال من قِبَل المُجَدْوِل.

لقد تمّ حساب صِيَغ جبرية لإحتمال سقوط النظام ، إحتمال حدوث خطأ في إستقبال الإشارة و سِعَة القناة مع فَرَض كافّة قنوات النظام وفقاً لنموذج التلاشي من نوع Rayleigh و وجود تقدير غير مثالي للقنوات. بالإضافة إلى ذلك ، تمّت دراسة الآداء عند القيم العالية جداً لـ SNR ، حيث تمّ حساب صِيَغ جبرية بسيطة وتقريبية لإحتمال سقوط النظام وإحتمال حدوث خطأ في إستقبال الإشارة. أيضاً ، تمّ التأكّد من صحّة النتائج بمقارنتها بالـ Monte-Carlo simulations.

وأظهرت النتائج أنّ الـ diversity order للنظام المدروس بوجود تقنية الـ Cognitive Radio هي نفسها بدون وجود هذه التقنية. كما وبيّنت النتائج أنّه بوجود تقدير مثالي لقنوات المُستخدمين الثانويّين ، الـ diversity order للنظام تزداد بشكل خطّي بتقليل ترتيب المُستخدم المُختار وأنّها تقِلّ بشكل خطّي بزيادة ترتيب المُستخدم المُختار ؛ بينما ، بوجود تقدير غير مثالي للقنوات ، يَصِل النظام إلى diversity order بقيمة صفر. أخيراً ، أظهرت النتائج أنّ التقدير الغير مثالي لقنوات المُستخدمين الإبتدائيّين ، يؤثر فقط على الـ coding gain الخاص بالنظام.

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1 Introduction

Cognitive radio has been proposed to improve the spectrum resource utilization efficiency in wireless networks [1]. Several cognitive radio paradigms have been proposed in [2], among which is the underlay scenario. This paradigm allows users in a secondary cell (secondary or cognitive users) to utilize the frequency bands of users in a primary cell (primary users) only if the interference between them is below a certain threshold. On the other hand, the overlay which is another paradigm allows the secondary users to share the frequency bands of primary users only if the interference between them is zero. Two important objectives exist in designing any cognitive network: protecting the primary user from interference and satisfying adequate level of system performance of secondary users. Between these objectives, the former is of higher priority, making strict regulation of secondary transmit powers necessary.

Among the research areas where the researchers are being attracted is the multiuser cognitive networks. Several user selection schemes were proposed in such systems, among which is the opportunistic scheduling. In this scheme, the user with the best instantaneous channel is assigned the system resources by the scheduler or the central unit. In [3], closed-form expression for the channel capacity was derived in addition to evaluating the multiuser diversity gains where the secondary user with the best end-to-end (e2e) signal-to-noise ratio (SNR) is assigned the spectrum resources. In [4] and [5], the cognitive user with the best channel is allowed by the cognitive base station (BS) to conduct its transmission in both multiple access and broadcast channel spectrum-sharing networks.

Yang et al. [6] evaluated in the performance of multiuser cognitive networks with multiple antennas. The secondary transmitters and receivers were assumed to have multiple antennas where the opportunistic scheduling was used in selecting among secondary users. Opportunistic scheduling was also adopted in [7] in selecting among users where some scheduling fairness and power control schemes were considered. In [8], the exact outage and error probabilities were evaluated for multiuser cognitive networks with opportunistic scheduling and Nakagami-*m* fading channels. The opportunistic scheduling was used to select among secondary users in spectrum-sharing networks with adaptive modulation in [9]. In [10], Ratnarajah et al. derived some performance measures including the multiuser diversity gain and bit error rate for multiple access channels, broadcast channels, and parallel access channels in spectrum-sharing networks. Opportunistic scheduling was used to select among secondary users in addition to considering the interference from primary users on the performance of secondary network. It is worthwhile to mention here that there exist two main issues to take care of in multiuser wireless networks: sum-rate capacity and fairness among users. Maximum-rate or conventional scheduling maximizes the sum capacity at the expense of unfairness among users, whereas proportional fair user selection scheme satisfies fairness among users at the expense of system sum-rate [11,12]. Therefore, the selection of the scheduling scheme depends on the system requirements and the nature of the system. As an example on the suitability of the scheme that is to be used, although the proportional fair scheduling could be helpful for users of weak channels, the loss happens in the throughput when this scheduling scheme is used can be large in situations where users are scattered across the cell [13]. In summary, the opportunistic and evenly the *N*th-best user selection schemes are suitable to be implemented when the system overall sum-rate capacity or the overall performance is the main requirement of the system, whereas the proportional fair scheduling is more desirable in systems where the fairness among users is the first priority.

Several situations can be seen in practical wireless systems where the opportunistic scheduling becomes inefficient, among which are the following: (1) in the presence of imperfect channel state information where the scheduling unit could fail in error in selecting the best user among the available users; (2) in the presence of outdated channel information (OCI) where the user that was the best at the selection time instant could not be the best at the transmission time instant; and (3) sometimes, while the best user is waiting to be scheduled by its BS, it gets scheduled by other one. In such situation, the first unit can assign the system resources to the second best or any other user in its area. This could happen in handoff conditions. An efficient scheme that can deal with such conditions is the *N*th-best user scheduling scheme [14]. In this scheme and instead of the first best user, the user with the second or even the *N*th-best channel is assigned the system resources by the scheduling unit. This scheme was firstly proposed in literature as an antenna selection scheme [15]; then, it was presented as a way to select among relays in relay cooperative networks [16]. A practical example where the *N*th-best user selection scheme can be implemented is in networks that operate in time-slotted manner like in long-term evolution (LTE). In such networks and while a specific number of users is being scheduled by one BS in one time slot, the user with the best channel may be busy in relaying or load balancing duties as such in LTE advanced heterogeneous networks [17]. In this case, the same previous limited number of users will only be considered for scheduling again. This means the second best user will never be considered as the best user will be still allowed to be scheduled in future. The importance of the *N*th-best user selection scheme motivated us to conduct this research. Also, the importance of the channel estimation process in cognitive networks and its role in determining the interference constraint on secondary sources transmit power is another motivation behind presenting this work. Furthermore, the *N*th-best user selection scheme and the accompanied performance analysis are a generalization of the common opportunistic scheduling or user selection scheme.

In this paper, we propose the use of the *N*th-best user selection scheme for multiuser cognitive networks in addition to analyzing its performance in the presence of imperfect channel estimation (ICE). In this selection scheme, the user with the second or even the *N*th-best instantaneous channel is assigned the spectrum resources. Closed-form expressions are derived for the outage probability, average symbol error probability (ASEP), and ergodic channel capacity assuming Rayleigh fading channels. Furthermore, approximate expressions are derived for the outage probability, ASEP, diversity order, and coding gain of the system. First, the probability density function (PDF) of the e2e SNR conditioned on the statistics of the secondary cell-to-primary cell channel is derived. Then, this PDF is used to obtain the cumulative distribution function (CDF) of the SNR at the selection scheme combiner output, which is then used to evaluate the various performance measures. Both independent non-identically distributed (i.n.i.d.) and independent identically distributed (i.i.d.) cases of secondary users' channels are considered in the analysis.

2 System and Channel Models

The studied system consists of one secondary user (SU) source S, K SU destinations D_k ($k = 1, \ldots, K$), and one primary user (PU) receiver P. All nodes are assumed to be equipped with single antenna. The SU source sends its message x to K SU destinations with a transmit power constraint which guarantees that the interference with the PU receiver P does not exceed an interference temperature $\mathcal{I}_{\mathsf{p}}$. As a result, the SU source S must transmit at a power given by $P_{\mathsf{s}} = \mathcal{I}_{\mathsf{p}}/|h_{\mathsf{s,p}}|^2$, where $h_{\mathsf{s,p}}$ is the channel coefficient of the $\mathsf{S} \rightarrow \mathsf{P}$ link. Therefore, the message at the k^{th} destination D_k from the source S is given by $y_{\mathsf{s},k} = \sqrt{P_{\mathsf{s}}} h_{\mathsf{s},k} x + n_{\mathsf{s},k}$, where $h_{\mathsf{s},k}$ is the channel coefficient of the $\mathsf{S} \rightarrow \mathsf{D}_k$ link and $n_{\mathsf{s},k}$ represents the additive white Gaussian noise (AWGN) term at D_k with a power of N_0. We assume no interference is introduced from the primary user on the secondary receivers.[1] All channel coefficients are assumed to be Rayleigh distributed, so the channel gains $|h_{\mathsf{s,p}}|^2$ and $|h_{\mathsf{s},k}|^2$ follow exponential distribution with mean powers $\Omega_{h_{\mathsf{s,p}}}$ and $\Omega_{h_{\mathsf{s},k}}$, respectively.

The channel coefficient of the $\mathsf{S} \rightarrow \mathsf{D}_k$ channel can be written as [19–21]

$$h_{\mathsf{s},k} = \hat{h}_{\mathsf{s},k} + e_{h_{\mathsf{s},k}}, \tag{1}$$

where $\hat{h}_{\mathsf{s},k}$ is the estimate of the $\mathsf{S} \rightarrow \mathsf{D}_k$ link and $e_{h_{\mathsf{s},k}}$ is the channel estimation error, which is assumed to be complex Gaussian with zero mean and variance $\sigma^2_{e_{h_{\mathsf{s},k}}} = \Omega_{h_{\mathsf{s},k}} - \mathbb{E}[|\hat{h}_{\mathsf{s},k}|^2]$, with $\mathbb{E}[.]$ denoting the expectation operator. Also, $\hat{h}_{\mathsf{s},k}$ is also complex Gaussian with zero mean and variance $\Omega_{\hat{h}_{\mathsf{s},k}} = \Omega_{h_{\mathsf{s},k}} + \sigma^2_{e_{h_{\mathsf{s},k}}}$. The above definition also applies to the $\mathsf{S} \rightarrow \mathsf{P}$ channel, i.e., $\hat{h}_{\mathsf{s,p}} \sim \mathcal{CN}\left(0, \Omega_{\hat{h}_{\mathsf{s,p}}} = \Omega_{h_{\mathsf{s,p}}} + \sigma^2_{e_{h_{\mathsf{s,p}}}}\right)$.

Upon using the values $h_{\mathsf{s},k} = \hat{h}_{\mathsf{s},k} + e_{h_{\mathsf{s},k}}$ and $h_{\mathsf{s,p}} = \hat{h}_{\mathsf{s,p}} + e_{h_{\mathsf{s,p}}}$, the signal at the k^{th} user can be rewritten as

$$\begin{aligned} y_{\mathsf{s},k} = &\sqrt{\frac{\mathcal{I}_{\mathsf{p}}}{|\hat{h}_{\mathsf{s,p}}|^2 + \sigma^2_{e_{h_{\mathsf{s,p}}}}}}\hat{h}_{\mathsf{s},k} x \\ &+ \sqrt{\frac{\mathcal{I}_{\mathsf{p}}}{|\hat{h}_{\mathsf{s,p}}|^2 + \sigma^2_{e_{h_{\mathsf{s,p}}}}}} e_{h_{\mathsf{s},k}} x + n_{\mathsf{s},k}. \end{aligned} \tag{2}$$

From (2), the SNR of the $\mathsf{S} \rightarrow \mathsf{D}_k$ link can be easily obtained after simple manipulations as

$$\gamma_{\mathsf{S}-\mathsf{D}_k} = \frac{\bar{\gamma}|\hat{h}_{\mathsf{s},k}|^2}{|\hat{h}_{\mathsf{s,p}}|^2 + \sigma^2_{e_{h_{\mathsf{s,p}}}} + \bar{\gamma}\sigma^2_{e_{h_{\mathsf{s},k}}}} = \gamma_k, \tag{3}$$

where $\bar{\gamma} = \mathcal{I}_{\mathsf{p}}/N_0$. The *N*th-user scheduling is performed by choosing the destination with the *N*th-best e2e SNR γ_k. It is worthwhile to mention here that the estimation error variance can be made small by transmitting large number of pilots at medium to high SNRs [19].[2]

3 Exact Performance Analysis

Here, we evaluate the exact performance of the studied system.

3.1 Outage Probability

Here, we derive the outage probability for i.n.i.d. and i.i.d. users' channels. The outage probability is defined as the probability that the SNR at the selected destination γ_{Sel} goes below a predetermined outage threshold γ_{out}, i.e., $P_{\text{out}} = \Pr\left[\gamma_{\text{Sel}} \leq \gamma_{\text{out}}\right]$, where $\Pr[.]$ denotes the probability operation.

In order to derive the outage probability for i.n.i.d. users' channels, the CDF of γ_{Sel} is required to be obtained first. The CDF of the SNR in (3) conditioned on $\hat{h}_{\mathsf{s,p}}$ can be easily obtained as

[1] This assumption is valid when the primary transmitter is in a location far from the secondary receiver [18].

[2] The variance of the estimation error can be also assumed to be inversely proportional to SNR as 1/SNR.

$$F_{\gamma_k}\left(\gamma|\hat{h}_{\mathsf{s,p}}\right) = 1-\exp\left(-\lambda_{\mathsf{s},k}\gamma|\hat{h}_{\mathsf{s,p}}|^2\right), \qquad (4)$$

where $\lambda_{\mathsf{s},k} = \left(\sigma^2_{e_{h_{\mathsf{s,p}}}} + \sigma^2_{e_{h_{\mathsf{s},k}}}\bar{\gamma}+1\right)\Big/\left(\Omega_{\hat{h}_{\mathsf{s},k}}\bar{\gamma}\right)$.

The conditional PDF of the selected Nth-best user is given by [22]

$$f_{\gamma_{\mathsf{Sel}}}(\gamma|\hat{h}_{\mathsf{s,p}}) = \sum_{l=1}^{K} f_{\gamma_l}(\gamma|\hat{h}_{\mathsf{s,p}})\sum_{\mathcal{P}}\prod_{j=1}^{K-N} F_{\gamma_{i_j}}(\gamma|\hat{h}_{\mathsf{s,p}}) \times \prod_{w=K-N+1}^{K-1}\left(1-F_{\gamma_{i_w}}(\gamma|\hat{h}_{\mathsf{s,p}})\right), \qquad (5)$$

where $\sum_{\mathcal{P}}$ denotes the summation over all $n!$ permutations $(i_1, i_2, \ldots, i_K)$ of $(1, 2, \ldots, K)$ and N is the order of the selected user. Upon substituting (4) in (5) and using the binomial rule and applying the identity

$$\prod_{j=1}^{K-N}\left(1-t_j\right) = 1+\sum_{j=1}^{K-N}(-1)^j\sum_{s_1<\cdots<s_j}\prod_{n=1}^{j} t_{s_n}, \qquad (6)$$

with $\sum_{s_1<\cdots<s_j}$ being a short hand-notation for $\sum_{s_1=1}^{K-N-j+1}\sum_{s_2=s_1+1}^{K-N-j+2}\cdots\sum_{s_j=s_{j-1}+1}^{K-N}$, (5) can be rewritten as

$$f_{\gamma_{\mathsf{Sel}}}(\gamma|\hat{h}_{\mathsf{s,p}}) = \sum_{l=1}^{K}\lambda_{\mathsf{s},l}|\hat{h}_{\mathsf{s,p}}|^2\sum_{\mathcal{P}}\left[\exp\left(-\Delta_1|\hat{h}_{\mathsf{s,p}}|^2\gamma\right) + \sum_{j=1}^{K-N}(-1)^j \times \sum_{s_1<\cdots<s_j}\exp\left(-\Delta_2|\hat{h}_{\mathsf{s,p}}|^2\gamma\right)\right], \qquad (7)$$

where $\Delta_1 = \sum_{w=K-N+1}^{K-1}\lambda_{\mathsf{s},i_w}$ and $\Delta_2 = \Delta_1+\sum_{n=1}^{j}\lambda_{\mathsf{s},s_n}+\lambda_{\mathsf{s},l}$. Up to now, the PDF of γ_{Sel} can be obtained using $\int_0^\infty f_{\gamma_{\mathsf{Sel}}}(\gamma|\hat{h}_{\mathsf{s,p}})f_{|\hat{h}_{\mathsf{s,p}}|^2}(y)dy$ as follows

$$f_{\gamma_{\mathsf{Sel}}}(\gamma) = \lambda_{\mathsf{s,p}}\sum_{l=1}^{K}\lambda_{\mathsf{s},l} \sum_{\mathcal{P}}\left[(\Delta_1\gamma+\lambda_{\mathsf{s,p}})^{-2}+\sum_{j=1}^{K-N}(-1)^j\sum_{s_1<\cdots<s_j}(\Delta_2\gamma+\lambda_{\mathsf{s,p}})^{-2}\right], \qquad (8)$$

where [23, Eq. (3.381.4)] has been used in getting (8).

The outage probability can be obtained by integrating (8) using $\int_0^{\gamma_{\mathsf{out}}} f_{\gamma_{\mathsf{Sel}}}(z)dz$ as follows

$$P_{\mathsf{out}} = \lambda_{\mathsf{s,p}}\sum_{l=1}^{K}\lambda_{\mathsf{s},l}\sum_{\mathcal{P}}\left[\frac{\left\{(\lambda_{\mathsf{s,p}})^{-1}-(\Delta_1\gamma_{\mathsf{out}}+\lambda_{\mathsf{s,p}})^{-1}\right\}}{\Delta_1} + \sum_{j=1}^{K-N}(-1)^j\times\sum_{s_1<\cdots<s_j}\frac{\left\{(\lambda_{\mathsf{s,p}})^{-1}-(\Delta_2\gamma_{\mathsf{out}}+\lambda_{\mathsf{s,p}})^{-1}\right\}}{\Delta_2}\right]. \qquad (9)$$

For i.i.d. users' channels ($\lambda_{\mathsf{s},1} = \cdots = \lambda_{\mathsf{s},K} = \lambda_{\mathsf{s,d}} = \left(\sigma^2_{e_{h_{\mathsf{s,p}}}}+\sigma^2_{e_{h_{\mathsf{s,d}}}}\bar{\gamma}+1\right)/\bar{\gamma}\Omega_{\hat{h}_{\mathsf{s,d}}}$), the outage probability in (9) simplifies to

$$P_{\mathsf{out}} = K\binom{K-1}{N-1}\lambda_{\mathsf{s,d}}\lambda_{\mathsf{s,p}}\sum_{k=0}^{K-N}\binom{K-N}{k}(-1)^k \times\frac{\left\{(\lambda_{\mathsf{s,p}})^{-1}-((N+k)\lambda_{\mathsf{s,d}}\gamma_{\mathsf{out}}+\lambda_{\mathsf{s,p}})^{-1}\right\}}{(N+k)\lambda_{\mathsf{s,d}}}. \qquad (10)$$

To evaluate (10), the PDF of the selected Nth-best user needs to be obtained first. It is given by

$$f_{\gamma_{\mathsf{Sel}}}(\gamma|\hat{h}_{\mathsf{s,p}}) \approx \binom{K-1}{N-1}Kf_{\gamma_{\mathsf{d}}}(\gamma|\hat{h}_{\mathsf{s,p}})\left(F_{\gamma_{\mathsf{d}}}(\gamma|\hat{h}_{\mathsf{s,p}})\right)^{K-N} \left(1-F_{\gamma_{\mathsf{d}}}(\gamma|\hat{h}_{\mathsf{s,p}})\right)^{N-1}. \qquad (11)$$

Upon substituting the statistics $F_{\gamma_{\mathsf{d}}}(\gamma|\hat{h}_{\mathsf{s,p}})$, $f_{\gamma_{\mathsf{d}}}(\gamma|\hat{h}_{\mathsf{s,p}})$ in (11), and after some simple manipulations, we get (10).

3.2 Average Symbol Error Probability

Here, we derive the ASEP for i.n.i.d. and i.i.d. users' channels. The ASEP can be written in terms of the CDF of γ_{Sel}, $F_{\gamma_{\mathsf{Sel}}}(\gamma) = P_{\mathsf{out}}(\gamma_{\mathsf{out}} = \gamma)$ as

$$\mathrm{ASEP} = \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^\infty\frac{\exp(-b\gamma)}{\sqrt{\gamma}}F_{\gamma_{\mathsf{Sel}}}(\gamma)d\gamma, \qquad (12)$$

where a and b are some constants to specify the modulation scheme which for example in case of BPSK modulation scheme is equal to $a = 1/2$, $b = 1$.

By replacing γ_{out} with γ in (9), and with the help of [23, Eq. (3.381.4)] and [23, Eq. (3.383.10)], the ASEP can be obtained as

$$\mathrm{ASEP} = \frac{a\sqrt{b}}{2}\lambda_{\mathsf{s,p}}\sum_{l=1}^{K}\lambda_{\mathsf{s},l}\sum_{\mathcal{P}}\left[\frac{\left\{\frac{(\lambda_{\mathsf{s,p}})^{-1}}{b^{1/2}}-(\Delta_1\lambda_{\mathsf{s,p}})^{-1/2}\exp\left(\frac{b\lambda_{\mathsf{s,p}}}{\Delta_1}\right)\Gamma\left(1/2,\frac{b\lambda_{\mathsf{s,p}}}{\Delta_1}\right)\right\}}{\Delta_1} + \sum_{j=1}^{K-N}(-1)^j\sum_{s_1<\cdots<s_j}\frac{\left\{\frac{(\lambda_{\mathsf{s,p}})^{-1}}{b^{1/2}}-(\Delta_2\lambda_{\mathsf{s,p}})^{-1/2}\exp\left(\frac{b\lambda_{\mathsf{s,p}}}{\Delta_2}\right)\Gamma\left(1/2,\frac{b\lambda_{\mathsf{s,p}}}{\Delta_2}\right)\right\}}{\Delta_2}\right], \qquad (13)$$

where $\Gamma(.,.)$ is the incomplete Gamma function defined in [23, Eq. (8.350.2)].

For i.i.d. users' channels, the ASEP in (13) simplifies to

$$\begin{aligned}\text{ASEP} = &\frac{Ka\sqrt{b}}{2}\binom{K-1}{N-1}\lambda_{\mathsf{s,d}}\lambda_{\mathsf{s,p}}\sum_{k=0}^{K-N}\binom{K-N}{k}(-1)^k \\ &\times((N+k)\lambda_{\mathsf{s,d}})^{-1}\left[\frac{(\lambda_{\mathsf{s,p}})^{-1}}{b^{1/2}} - ((N+k)\lambda_{\mathsf{s,d}}\lambda_{\mathsf{s,p}})^{-1/2}\right. \\ &\left.\times\exp\left(\frac{b\lambda_{\mathsf{s,p}}}{(N+k)\lambda_{\mathsf{s,d}}}\right)\Gamma\left(1/2, \frac{b\lambda_{\mathsf{s,p}}}{(N+k)\lambda_{\mathsf{s,d}}}\right)\right]. \quad (14)\end{aligned}$$

By replacing γ_{out} with γ in (10), and after some simple manipulations, we get (14).

3.3 Ergodic Channel Capacity

Here, we derive the ergodic channel capacity for i.n.i.d. and i.i.d. users' channel. The ergodic capacity can be written in terms of the PDF of γ_{Sel} as

$$C = \frac{1}{\ln(2)}\int_0^\infty \ln(1+\gamma) f_{\gamma_{\mathsf{Sel}}}(\gamma)d\gamma. \quad (15)$$

Upon substituting (8) in (15), and with the help of [23, Eq. (4.291.15)], the ergodic channel capacity can be obtained as

$$\begin{aligned}C = &\frac{\lambda_{\mathsf{s,p}}}{\ln(2)}\sum_{l=1}^{K}\lambda_{\mathsf{s},l} \\ &\times\sum_{\mathcal{P}}\left[\frac{\ln\left(\frac{\Delta_1}{\lambda_{\mathsf{s,p}}}\right)}{\Delta_1(\Delta_1-\lambda_{\mathsf{s,p}})} + \sum_{j=1}^{K-N}(-1)^j\sum_{s_1<\cdots<s_j}\frac{\ln\left(\frac{\Delta_2}{\lambda_{\mathsf{s,p}}}\right)}{\Delta_2(\Delta_2-\lambda_{\mathsf{s,p}})}\right]. \quad (16)\end{aligned}$$

For i.i.d. users' channels, the channel capacity in (16) simplifies to

$$\begin{aligned}C = &\frac{K}{\ln(2)}\binom{K-1}{N-1}\lambda_{\mathsf{s,p}}\sum_{k=0}^{K-N}\binom{K-N}{k}(-1)^k \\ &\times\frac{\ln\left(\frac{(N+k)\lambda_{\mathsf{s,d}}}{\lambda_{\mathsf{s,p}}}\right)}{(N+k)((N+k)\lambda_{\mathsf{s,d}}-\lambda_{\mathsf{s,p}})}. \quad (17)\end{aligned}$$

Upon substituting (11) in $\int_0^\infty f_{\gamma_{\mathsf{Sel}}}(\gamma|\hat{h}_{\mathsf{s,p}}) f_{|\hat{h}_{\mathsf{s,p}}|^2}(y)dy$ and then substituting the result in (15), and with the help of [23, Eq. (4.291.15)], we get (17).

4 Asymptotic Performance Analysis

To get more details about the system behavior, we study here the performance at the high SNR values.

4.1 Outage Probability

The outage probability can be expressed at the high SNR regime as $P_{\mathsf{out}} \approx (G_{\mathsf{c}}\text{SNR})^{-G_{\mathsf{d}}}$, where G_{c} and G_{d} denote the coding gain and the diversity order of the system, respectively [24]. Obviously, G_{c} represents the horizontal shift in the outage probability performance relative to the benchmark curve $(\text{SNR})^{-G_{\mathsf{d}}}$, and G_{d} refers to the increase in the slope of the outage probability versus SNR curve [24, Ch. 14]. The parameters on which the diversity order depends will affect the slope of the outage probability curves, and the parameters on which the coding gain depends will affect the position of the curves. In the upcoming analysis, the users are assumed to have identical channels, that is, $(\lambda_{\mathsf{s},1} = \lambda_{\mathsf{s},2} = \cdots = \lambda_{\mathsf{s,d}} = (\sigma^2_{e_{h_{\mathsf{s,p}}}} + \sigma^2_{e_{h_{\mathsf{s,d}}}}\bar{\gamma} + 1)/\bar{\gamma}\Omega_{\hat{h}_{\mathsf{s,d}}})$.

As $\bar{\gamma} \to \infty$, the CDF in (4) simplifies to $F_{\gamma_{\mathsf{d}}}\left(\gamma|\hat{h}_{\mathsf{s,p}}\right) \approx \lambda_{\mathsf{s,d}}|\hat{h}_{\mathsf{s,p}}|^2\gamma$ and, accordingly, the PDF simplifies to $f_{\gamma_{\mathsf{d}}}\left(\gamma|\hat{h}_{\mathsf{s,p}}\right) \approx \lambda_{\mathsf{s,d}}|\hat{h}_{\mathsf{s,p}}|^2$. Upon substituting the approximated CDF and PDF in (11) and following the same procedure as in Sect. 3.1, the outage probability at high SNR values can be evaluated with the help of [23, Eq. (3.353.1)] as

$$\begin{aligned}P_{\mathsf{out}}^{\infty} = &K\binom{K-1}{N-1}\left(\frac{\lambda_{\mathsf{s,d}}}{\lambda_{\mathsf{s,p}}}\right)^{K-N+1}\sum_{k=0}^{N-1}\binom{N-1}{k}(-1)^k \\ &\times\left(\frac{\lambda_{\mathsf{s,d}}}{\lambda_{\mathsf{s,p}}}\right)^k (k+K-N)! \times (\gamma_{\mathsf{out}})^{k+K-N+1}. \quad (18)\end{aligned}$$

The result in (18) is still dominant for the first term of the summation $k = 0$. With $\lambda_{\mathsf{s,d}} = \left(\sigma^2_{e_{h_{\mathsf{s,p}}}} + \sigma^2_{e_{h_{\mathsf{s,d}}}}\bar{\gamma} + 1\right)/\bar{\gamma}\Omega_{\hat{h}_{\mathsf{s,d}}}$, (18) can be written as three main cases:

Case 1 $\sigma^2_{e_{h_{\mathsf{s,p}}}} = \sigma^2_{e_{h_{\mathsf{s,d}}}} = 0$ (Perfect channel estimation)
Here, the outage probability can be simplified as

$$P_{\mathsf{out}}^{\infty} = \{G_{\mathsf{c}1}\bar{\gamma}\}^{-G_{\mathsf{d}1}}, \quad (19)$$

where $G_{\mathsf{c}1} = \left(K(K-N)!\binom{K-1}{N-1}(\lambda_{\mathsf{s,p}}\Omega_{\hat{h}_{\mathsf{s,d}}})^{-(K-N+1)}(\gamma_{\mathsf{out}})^{K-N+1}\right)^{\frac{-1}{(K-N+1)}}$ is the coding gain of the system and $G_{\mathsf{d}1} = K - N + 1$ is the diversity order.

Case 2 $\sigma^2_{e_{h_{\mathsf{s,d}}}} \neq 0$ (Imperfect channel estimation)
In this case, the numerator of $\lambda_{\mathsf{s,d}}$ can be approximated by $(\sigma^2_{e_{h_{\mathsf{s,d}}}}\bar{\gamma})$, and hence, $\lambda_{\mathsf{s,d}}$ simplifies to $\sigma^2_{e_{h_{\mathsf{s,d}}}}/\Omega_{\hat{h}_{\mathsf{s,d}}}$. As a result, the outage probability can be simplified as

$$P_{\mathsf{out}}^{\infty} = G_{\mathsf{c}2}(\bar{\gamma})^{-G_{\mathsf{d}2}}, \quad (20)$$

where $G_{c2} = K(K-N)!\binom{K-1}{N-1}\left(\frac{\lambda_{s,p}\Omega_{\hat{h}_{s,d}}}{\sigma^2_{e_{h_{s,d}}}}\right)^{-(K-N+1)}$ $(\gamma_{out})^{K-N+1}$ and $G_{d2} = 0$.

Case 3 $\sigma^2_{e_{h_{s,p}}} = \sigma^2_{e_{h_{s,d}}} = 1/\mathrm{SNR} = 1/\bar{\gamma}$ (Imperfect channel estimation)
Here, $\lambda_{s,d}$ simplifies to $1/\bar{\gamma}\Omega_{\hat{h}_{s,d}}$. As a result, the outage probability can be simplified as obtained in (19) with the same coding gain and diversity order.

4.2 Average Symbol Error Probability

The asymptotic ASEP for the studied system can be obtained by replacing γ_{out} by γ in (18) and then substituting the result in (12). Upon doing this, and with the help of [23, Eq. (3.381.4)], we can easily get the following two cases:

Case 1 $\sigma^2_{e_{h_{s,p}}} = \sigma^2_{e_{h_{s,d}}} = 0$ (Perfect channel estimation)
Here, the asymptotic ASEP can be obtained as

$$\mathrm{ASEP}^{\infty} = \left\{\left(\chi\left(\lambda_{s,p}\Omega_{\hat{h}_{s,d}}\right)^{-(K-N+1)} \frac{\Gamma(K-N+3/2)}{(b)^{K-N+3/2}}\right)^{\frac{-1}{(K-N+1)}}\bar{\gamma}\right\}^{-(K-N+1)}, \quad (21)$$

where $\chi = K(K-N)!\binom{K-1}{N-1}$.

Case 2 $\sigma^2_{e_{h_{s,d}}} \neq 0$ (Imperfect channel estimation)
In this case, the asymptotic ASEP can be obtained as

$$\mathrm{ASEP}^{\infty} = \chi\left(\frac{\lambda_{s,p}\Omega_{\hat{h}_{s,d}}}{\sigma^2_{e_{h_{s,d}}}}\right)^{-(K-N+1)} \frac{\Gamma(K-N+3/2)}{(b)^{K-N+3/2}}(\bar{\gamma})^0. \quad (22)$$

Case 3 $\sigma^2_{e_{h_{s,p}}} = \sigma^2_{e_{h_{s,d}}} = 1/\mathrm{SNR} = 1/\bar{\gamma}$ (Imperfect channel estimation)
Here, the asymptotic ASEP can be obtained to be similar to that found in (21).

It is clear from (19) and (21) that the multiuser cognitive network with the Nth-best user selection scheme has a coding gain that is affected by several parameters such as K, N, $\Omega_{\hat{h}_{s,d}}$, $\lambda_{s,p}$, and γ_{out}, while the diversity order is constant at $K-N+1$. This is valid for the case where the channels are perfectly estimated. Also, this applies when the estimation errors are inversely proportional to SNR as shown in Case 3. On the other hand, when the channels are imperfectly estimated with constant estimation errors, it is obvious from (20) and (22) that the system has a zero diversity order and a coding gain that is affected by the same previous parameters but now with the effect of the channel estimation error $\sigma^2_{e_{h_{s,d}}}$.

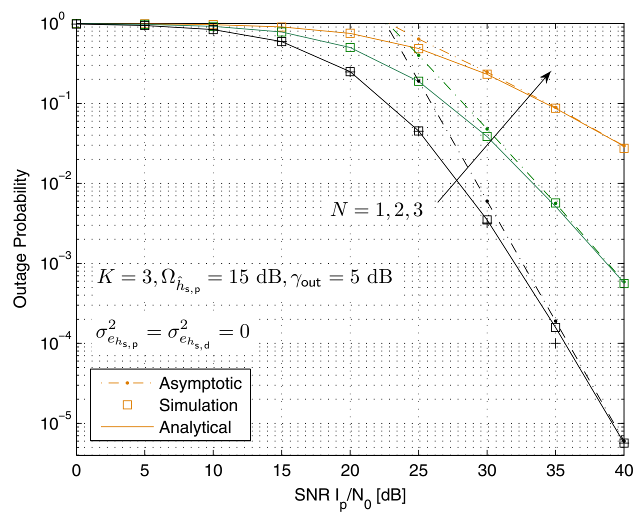

Fig. 1 P_{out} versus SNR of multiuser cognitive networks with Nth-best user selection for different values of N and $\Omega_{\hat{h}_{s,1}} = \cdots = \Omega_{\hat{h}_{s,3}} = 1$

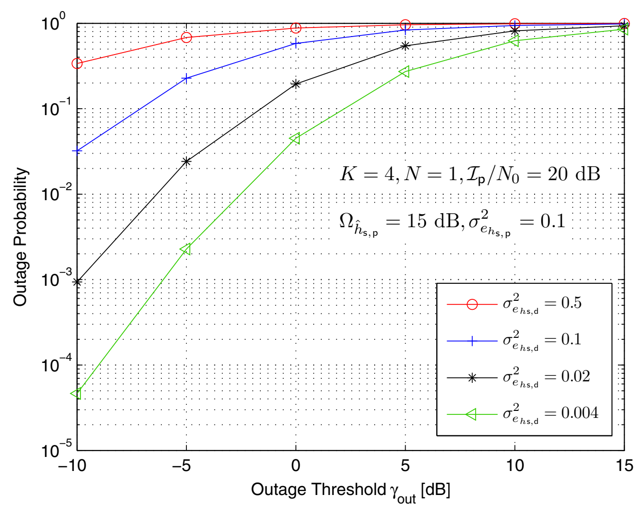

Fig. 2 P_{out} versus outage threshold of multiuser cognitive networks with Nth-best user selection for different values of $\sigma^2_{e_{h_{s,d}}}$ and $\Omega_{\hat{h}_{s,1}} = \cdots = \Omega_{\hat{h}_{s,4}} = 1$

5 Simulation and Numerical Results

We see from Fig. 1 that the asymptotic and analytical results perfectly fit with Monte-Carlo simulations. Also, we can notice that as the order of the selected user N increases, the diversity order of the system decreases, and the system performance is more degraded. On the other hand, as N decreases, the diversity order increases and hence the achieved performance is better.

Figure 2 illustrates the effect of ICE of secondary users' channels represented by $\sigma^2_{e_{h_{s,d}}}$ on the system performance. We see that as the outage threshold γ_{out} increases, the achieved performance is worse, as expected. Also, it is obvi-

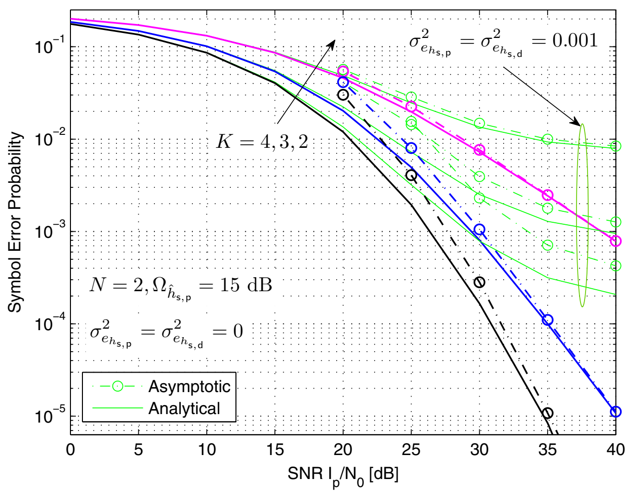

Fig. 3 ASEP versus SNR of multiuser cognitive networks with Nth-best user selection for different values of K and $\Omega_{\hat{h}_{s,1}} = \cdots = \Omega_{\hat{h}_{s,4}} = 1$

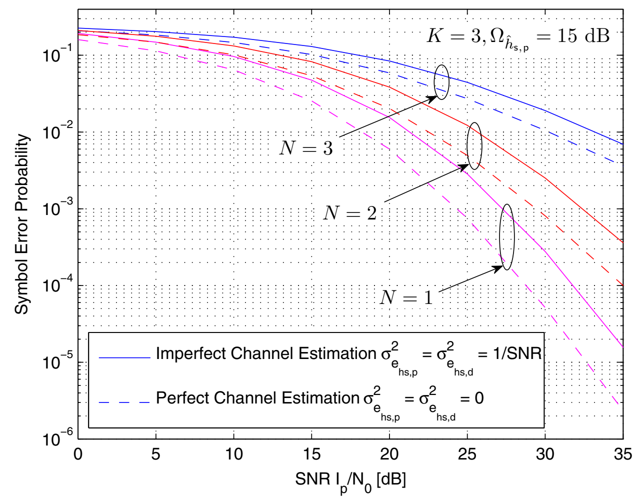

Fig. 4 ASEP versus SNR of multiuser cognitive networks with Nth-best user selection for different values of N and $\Omega_{\hat{h}_{s,1}} = \cdots = \Omega_{\hat{h}_{s,3}} = 1$

ous that as the power of channel estimation error $\sigma^2_{e_{h_{s,d}}}$ increases, the system behavior is more degraded.

Figure 3 portrays the error probability performance for different number of users. The figure is plotted for two cases: perfect channel estimation and ICE with constant estimation error variance. Again, it is clear that the asymptotic and analytical results perfectly fit with Monte-Carlo simulations. Also, we can see from this figure that for the case of perfect channel estimation $\sigma^2_{e_{h_{s,p}}} = \sigma^2_{e_{h_{s,d}}} = 0$, as the number of secondary users K increases, the diversity order of the system increases and the system performance is more enhanced. Also, it is clear that as K decreases, the diversity order decreases and hence the achieved performance is worse. On the other hand, in the presence of channel estimation error $\sigma^2_{e_{h_{s,p}}} = \sigma^2_{e_{h_{s,d}}} = 0.001$, a zero diversity gain is achieved by the system and a noise floor appears in the results due to the effect of ICE on the system behavior. This can be easily concluded from the asymptotic results where the diversity order of the system becomes zero when $\sigma^2_{e_{h_{s,d}}} \neq 0$. In such case, any further increase in the SNR will add no enhancement to the system behavior.

Figure 4 shows the error probability for different orders of selected user. The figure includes two cases: perfect channel estimation and ICE with an estimation error variance that is inversely proportional to SNR. The effect of channel estimation error on the system performance is obvious in this figure where worse behavior is achieved compared with the case where the channels are perfectly estimated. More importantly, for the case of ICE and as the variance of channel estimation error is assumed to be inversely proportional to SNR, the system can still achieve full diversity order when better user is selected for data reception or when N decreases.

Figure 5 illustrates the ergodic channel capacity for different values of $\sigma^2_{e_{h_{s,p}}}$ and $\sigma^2_{e_{h_{s,d}}}$. Two cases are shown in this

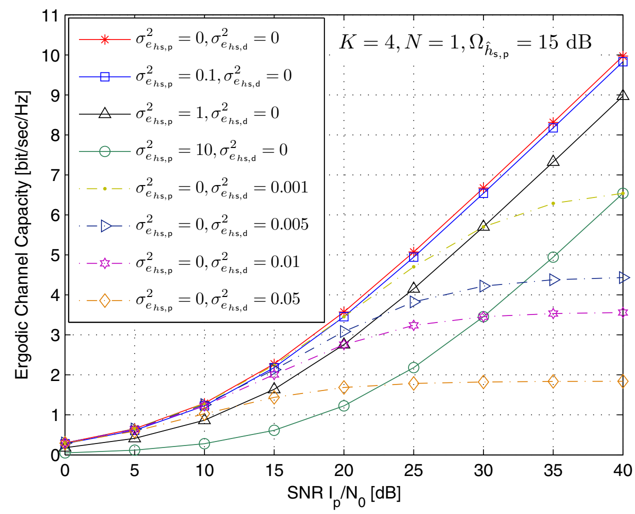

Fig. 5 Channel capacity versus SNR of multiuser cognitive networks with Nth-best user selection for different values of $\sigma^2_{e_{h_{s,p}}}$, $\sigma^2_{e_{h_{s,d}}}$ and $\Omega_{\hat{h}_{s,1}} = \cdots = \Omega_{\hat{h}_{s,4}} = 1$

figure: ICE of the $\mathsf{S} \rightarrow \mathsf{P}$ link with perfect channel estimation of the $\mathsf{S} \rightarrow \mathsf{D}$ link and perfect channel estimation of the $\mathsf{S} \rightarrow \mathsf{P}$ link with ICE of the $\mathsf{S} \rightarrow \mathsf{D}$ link. For the first case, where $\sigma^2_{e_{h_{s,p}}}$ is taking different values and $\sigma^2_{e_{h_{s,d}}} = 0$, the system capacity or performance keeps enhancing as SNR increases. On the other hand, when $\sigma^2_{e_{h_{s,p}}} = 0$ and $\sigma^2_{e_{h_{s,d}}}$ is taking different values, a noise floor appears in all results of this case. This behavior of the system is expected as in the first case, the power of the channel estimation error of the $\mathsf{S} \rightarrow \mathsf{P}$ link $\sigma^2_{e_{h_{s,p}}}$ is not affecting the SNR as clear from the asymptotic results, whereas the power of the channel estimation error of the $\mathsf{S} \rightarrow \mathsf{D}$ link $\sigma^2_{e_{h_{s,d}}}$ is a multiplied factor by the SNR in the second case.

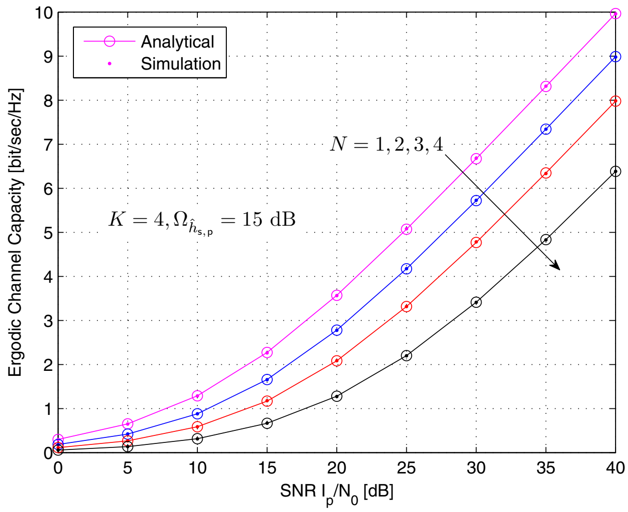

Fig. 6 Channel capacity versus SNR of multiuser cognitive networks with Nth-best user selection for different values of N and $\Omega_{\hat{h}_{s,1}} = \cdots = \Omega_{\hat{h}_{s,4}} = 1$

Figure 6 portrays the ergodic channel capacity for different orders of the selected user N. It is clear that as N increases, the achieved capacity is less, as expected.

6 Conclusion

We proposed and studied the performance of multiuser cognitive networks with the Nth-best user scheduling and imperfect channel estimation. Exact expressions for the outage probability, ASEP, and system capacity were derived. Furthermore, the system performance was evaluated at high SNR values. Findings illustrated that the diversity order of the system is the same as that of its non-cognitive counterpart and is independent of primary user. Also, results showed that with perfect channel estimation, the diversity order of the system linearly increases with decreasing the order of the selected user and vice versa. Furthermore, findings illustrated that a zero diversity gain is achieved by the system and that a noise floor appears in the results when the channels of the secondary cell are imperfectly estimated assuming constant estimation error variance. Finally, results showed that the imperfect estimation of the secondary cell-primary cell channel affects only the coding gain of the system without affecting the diversity order.

Acknowledgments This work is supported by King Fahd University of Petroleum and Minerals (KFUPM) through project of Grant Number FT131009.

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