Performance of $N$th-best antenna selection diversity systems with co-channel interference and outdated channel information

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Abstract: In this study, the authors evaluate the performance of space diversity systems with the $N$th-best antenna selection scheme in the presence of interference and outdated channel information (OC1). The $N$th-best antenna selection scheme is efficient in situations where the second or even the $N$th best antenna is mistakenly selected by the destination instead of the first best antenna for data reception. In this study, they first derive the cumulative distribution function (CDF) of the end-to-end (e2e) signal-to-interference plus noise ratio at the selection scheme combiner output. This CDF is then used to derive exact closed-form expressions for the e2e outage probability and symbol error probability (SEP) of the system. In the analysis, the channels of the desired user and the interferers are assumed to follow Rayleigh distribution. Furthermore, to obtain more about system insights, the performance is studied at the high signal-to-noise ratio (SNR) regime where the diversity order and coding gain are derived. Monte Carlo simulations are provided to validate the derived analytical and asymptotic expressions. Main results illustrate that with an interference power that is not scaling with SNR, the system can still achieve diversity gain when more receive antennas are used. Also, findings show that the diversity order of the system is linearly decreasing with increasing the order of the antenna, and linearly increasing with decreasing it. Furthermore, results illustrate that as the higher the correlation coefficient between the SNRs of antennas at the selection and transmission time instances, the better the achieved performance.

1 Introduction

Space or antenna diversity is an efficient technique used to mitigate the multipath fading phenomenon of wireless channels [1, 2]. Among the linear diversity combining techniques are the maximal-ratio combining (MRC), equal-gain combining (EGC), selection combining (SC), and generalised SC (GSC). These schemes have been widely studied and analysed in literature which makes them as the basic diversity combining schemes. Also, the switch-based diversity combining schemes presented in [1] are among the important combining schemes. In the SC scheme, the combiner selects the branch with the largest instantaneous signal-to-noise ratio (SNR) value without the need to know the signal phases on each branch. However, the phase needs to be estimated when implementing MRC or EGC in coherent systems [1].

The performance of the aforementioned diversity combining schemes was widely studied in literature starting with the assumption of noise-limited environments [3–5]. Particularly, in [3], the error rate and outage probabilities of a dual-branch SC diversity system were derived assuming Rayleigh and Nakagami-$m$ fading channels. Recently, Beaulieu et al. studied in [6] the error probability performance of space diversity systems with the MRC, SC and EGC linear diversity combining schemes assuming Nakagami-0.5 fading model. The Nakagami-0.5 fading model is an important special case of the Nakagami distribution as it represents the worst case fading scenario. Most recently, some new results for error probability in space diversity systems with MRC scheme and assuming Nakagami-$m$ fading channels were presented in [7]. It was shown that the symbol error probability (SEP) of MRC diversity systems over Nakagami-$m$ fading channels can be obtained by a simple modification of the diversity order and average SNR values of the Rayleigh fading case. Approximate closed-form expressions for secrecy capacity were derived in [8] for multi-antenna systems with MRC diversity scheme over Rayleigh fading channels. The performance of MRC diversity combining scheme was evaluated in [9] assuming $\eta - \mu$ fading channels. The $\eta - \mu$ fading model is suitable for non-line-of-sight (NLOS) mobile communication channels. Recently, approximate expression for the outage probability of MRC receivers over arbitrarily fading channels was derived in [10]. The authors employed a statistical analysis tool which only requires the knowledge of the moment generating functions of the SNR at the output of each diversity branch.

In [11], the authors derived the channel ergodic capacity for dual-branch MRC and EGC diversity combining schemes in correlated lognormal channels. This correlation could happen because of bulk geometrical and electromagnetic
A main step in the operation of all previously mentioned linear diversity combining schemes is the need of receiver to estimate the channel amplitudes of all diversity branches in order to use them before the combining stage. In the SC scheme, the combiner simply chooses the branch with the best channel instantaneous SNR. The process of channel estimation is the responsibility of the receiver which sometimes falls in error in estimating the channel of the best branch among the available branches. This problem could happen because of channel estimation errors. In this case, the receiver may choose the second or even the Nth best branch instead of the best one. This critical problem motivated us to propose an efficient diversity branch selection scheme which is the Nth-best antenna selection scheme. Also, the inherent existence of interference in wireless systems motivated us to consider its effect on the performance of this selection scheme. The problem of OCI is also an important issue in wireless systems. In such problem, the branch which was the best at the selection time instant $t$ could not be the best at the transmission time instant $t + \tau$. This problem is also considered in our paper.

To the best of our knowledge, the performance of space diversity systems with Nth-best receive antenna selection scheme in the presence of co-channel interference (CCI) and OCI has not been presented yet. The contributions of our paper are as follows: (i) we propose the Nth-best selection scheme to select between the antennas at the receiver of space diversity systems. Also, we study the effect of the interference and OCI on the behaviour of such systems; (ii) we present exact analysis of the outage and error probability performances where the effect of interference, OCI and other system parameters on the system behaviour is provided; and (iii) furthermore, we study the outage performance at the high SNR regime where approximate expressions for the outage probability, SEP, diversity order and coding gain are derived and analysed. In this paper, we consider the generic non-identically distributed (i.i.d.) case of diversity branches and interferers channels. Also, the independent identically distributed (i.i.d.) case of diversity branches is considered in this paper.

This paper is organised as follows. Section 2 presents the system and channel models. The exact performance is analysed in Section 3. Section 4 provides the asymptotic performance analysis. Some simulation and numerical results are presented and discussed in Section 5. Finally, conclusions are given in Section 6.

### 2 System and channel models

The system under study consists of a source of single antenna and a destination of $L$ antennas. We assume that the signal at the destination is corrupted by interfering signals from $I_d$ co-channel interferers $\{x_i\}^I_d _{i=1}$. The signal at the jth antenna at the receiver side can be written as

$$y_j = x_j + \sum_{i=1}^{I_d} h_{ij} x_i + n_j \tag{1}$$

where $h_j$ is the channel coefficient between the source and the jth antenna, $x_0$ is the transmitted symbol with $E\{|x_0|^2\} = P_0$, $h_{ij}$ is the channel coefficient between the i-th interferer and the jth antenna, $x_i$ is the transmitted symbol from the i-th interferer with $E\{|x_i|^2\} = P_i$, $n_j \sim \mathcal{CN}(0, N_0)$ is an additive white Gaussian noise (AWGN), and $E\{\cdot\}$ denotes the expectation operation. All the channel gains are
assumed to follow Rayleigh distribution. That is, the channel powers denoted by \( |h_j|^2 \) and \( |h_i|^2 \) are exponential distributed random variables (RVs) with average values \( \sigma_j^2 \) and \( \sigma_i^2 \), respectively. Using (1), with the \( n \)th-best antenna selection scheme, the end-to-end (e2e) signal-to-interference plus noise ratio (SINR) at the destination output can be written as

\[
\gamma_n \triangleq \frac{(P_0/N_0)|h_{kn}|^2}{\sum_{l=1}^{l_n} (P_l/N_0) |h_l|^2 + 1}
\]

(2)

where \( h_{kn} \) is the channel coefficient between the source and the \( n \)th best antenna. Let \( \tilde{\gamma}_j = (P_j/N_0)\alpha^2_j \), \( \tilde{\gamma}_l^* = (P_l/N_0)\alpha^2_l \), \( \lambda_j = 1/\tilde{\gamma}_j \), and \( \lambda_l = 1/\tilde{\gamma}_l^* \) denote the average values and parameters of the \( j \)th branch and the \( l \)th interferer, respectively.

3 Exact performance analysis

In this section, we evaluate the outage probability and SEP of the studied system when the CDFs of branches are non-identical.

The outage probability is defined as the probability that the e2e SINR goes below a certain threshold \( \gamma_{out} \) and it is given by

\[
P_{out} = P\left[\log_2 (1 + \gamma_n) < R \right] = P\left[\gamma_n < \gamma_{out}\right]
\]

(3)

Lemma 1: The outage probability for diversity systems with \( n \)th-best antenna selection and interference is given for the case of non-identical branches \( (\lambda_j, j = 1, \ldots, L) \) and non-identical interferers \( (\lambda_l, i = 1, \ldots, l_n) \) by (see (4))

\[
P_{out} = \prod_{j=1}^{L} \prod_{g=1}^{l_j} \frac{\exp(-\lambda_j^* \Delta_2)}{\sum_{l \neq g} (\exp(-\lambda_l \Delta_2) / \lambda_l \Delta_2) - \exp(-\lambda_j \Delta_2) / \lambda_j \Delta_2) + \sum_{q=1}^{L-N} (-1)^q \sum_{s_j < \ldots < s_q} (\exp(-\lambda_j^* \Delta_2) / \lambda_j \Delta_2) - \exp(-\lambda_j \Delta_2) / \lambda_j \Delta_2)}
\]

(4)

where \( \Delta_1 \) and \( \Delta_2 \) are as defined in Appendix 1.

Proof: Please see Appendix 1.

The SEP can be written as

\[
P_s = \int_0^\infty aQ(\sqrt{2b\tau}) f_{\gamma_n}(\gamma) d\gamma
\]

\[
= \alpha\sqrt{\pi} \int_0^\infty e^{-b\tau} F_{\gamma_n}(\gamma) d\gamma
\]

(5)

where \( Q(\cdot) \) is the Gaussian \( Q \)-function, \( a \) and \( b \) are the modulation-specific constants which, for example, given for BPSK modulation scheme as 0.5 and 1, respectively.

Upon substituting \( \gamma_{out} = \gamma \) in (4) and then substituting it in (5), and with the help of [27, Eq. (3.381.4)] and [27, Eq. (3.383.10)], the SEP for the studied system with non-identical diversity branches \( (\lambda_j, j = 1, \ldots, L) \) and non-identical interferers \( (\lambda_l, i = 1, \ldots, l_n) \) can be obtained in a closed-form expression as (see (6))

3.2 i.i.d. diversity branches

In this section, we evaluate the outage probability and SEP of the studied system when the CDFs of branches are identical.

Lemma 2: The outage probability for diversity systems with \( n \)th-best antenna selection and interference is given for the case of identical branches \( \lambda_j = \lambda_{pn}, j = 1, \ldots, L \) and non-identical interferers \( (\lambda_l, i = 1, \ldots, l_n) \) by (see (7))

Proof: In deriving (7), the e2e SINR is first written as \( Y_2/Z_1 \), where \( Z_1 \) is as defined in Appendix 1 and \( Y_2 \) is now having a

\[
P_{out} = \prod_{j=1}^{L} \prod_{g=1}^{l_j} \frac{\exp(-\lambda_j^* \Delta_2)}{\sum_{l \neq g} (\exp(-\lambda_l \Delta_2) / \lambda_l \Delta_2) - \exp(-\lambda_j \Delta_2) / \lambda_j \Delta_2) + \sum_{q=1}^{L-N} (-1)^q \sum_{s_j < \ldots < s_q} (\exp(-\lambda_j^* \Delta_2) / \lambda_j \Delta_2) - \exp(-\lambda_j \Delta_2) / \lambda_j \Delta_2)}
\]

(4)

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P_s = \alpha\sqrt{\pi} \int_0^\infty e^{-b\tau} F_{\gamma_n}(\gamma) d\gamma
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Upon substituting \( \gamma_{out} = \gamma \) in (4) and then substituting it in (5), and with the help of [27, Eq. (3.381.4)] and [27, Eq. (3.383.10)], the SEP for the studied system with non-identical diversity branches \( (\lambda_j, j = 1, \ldots, L) \) and non-identical interferers \( (\lambda_l, i = 1, \ldots, l_n) \) can be obtained in a closed-form expression as (see (6))

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Proof: In deriving (7), the e2e SINR is first written as \( Y_2/Z_1 \), where \( Z_1 \) is as defined in Appendix 1 and \( Y_2 \) is now having a
probability density function (PDF) given by

\[ f_{Y_i}(y) = \left( \frac{L - 1}{N - 1} \right) L f_{\gamma_i}(y) \left( F_{\gamma_i}(y) \right)^{L-N} \left( 1 - F_{\gamma_i}(y) \right)^{N-1} \]  

(8)

Upon substituting (8) and (27) in (26), and after some algebraic manipulations, the result in (7) can be obtained.

Upon substituting \( \gamma_{out} = \gamma \) in (7) and then substituting it in (5), and with the help of [27, Eq. (3.381.4)] and [27, Eq. (3.383.10)], the SEP for the studied system with identical diversity branches \((3.3 \text{ to } 3.10)\] can be rewritten as

\[ P_{out} = \frac{\sum_{i=1}^{I_d} (P_i/N_0) |\hat{h}_i|^2}{\sum_{i=1}^{I_d} (P_i/N_0) |\hat{h}_i|^2} \times \frac{\exp(-\lambda_p \gamma_{out} + \lambda_p \gamma)}{(k + N)\lambda_p \gamma_{out} + \lambda_p \gamma} \]  

(7)

\[ P_x = \frac{a\sqrt{b}}{2} \frac{L}{N-1} \sum_{m=1}^{I_d} \sum_{g=1}^{I_d} \left( \frac{L - 1}{N - 1} \right) \sum_{k=0}^{L-N} \left( \frac{L - N}{k} \right) (-1)^k \frac{\exp(-\lambda_p \gamma_{out} + \lambda_p \gamma)}{(k + N)\lambda_p \gamma_{out} + \lambda_p \gamma} \]  

(9)

Assuming i.i.d. branches \((\lambda_j = \lambda_p, j = 1, \ldots, L)\) and non-identical interferers \((\lambda'_j, i = 1, \ldots, I_d)\), the e2e SNR in (10) can be written as \( \gamma_{\gamma}/Z_1 \), where \( Z_1 \) is as defined in Appendix 1 and \( \gamma_{\gamma} \) is as defined in (8) in Lemma 2 but now for an outdated SNR and with a PDF given by

\[ f_{\gamma_{\gamma}}(\gamma) = \left( \frac{L - 1}{N - 1} \right) L f_{\gamma_i}(y) \left( F_{\gamma_i}(y) \right)^{L-N} \left( 1 - F_{\gamma_i}(y) \right)^{N-1} \]  

(11)

with the help of the binomial rule, (11) can be simplified as

\[ f_{\gamma_{\gamma}}(\gamma) = L\lambda_p \left( \frac{L - 1}{N - 1} \right) \sum_{k=0}^{L-N} \left( \frac{L - N}{k} \right) (-1)^k \]  

\[ \times \exp(-(k + N)\lambda_p \gamma) \]  

(12)

where we have assumed in (12) that the channels are slowly varying and hence, the average fading power remains constant over the time delay \( \tau \) [1]. In other words, we have assumed that \( \gamma_{\gamma} \) and \( \hat{\gamma}_{\gamma} \) or equivalently, \( \lambda_p \) and \( \lambda_{\gamma} \) are equal.

The PDF of actual \( \gamma_{\gamma} \) at the data transmission instant can be obtained from

\[ f_{\gamma_{\gamma}}(\gamma) = \int_0^\infty f_{\gamma_{\gamma}/\gamma_0}(\gamma|x) f_{\gamma_0}(x) dx \]  

(13)

The conditional PDF \( f_{\gamma_{\gamma}/\gamma_0}(\gamma|x) \) can be expressed as [29]

\[ f_{\gamma_{\gamma}/\gamma_0}(\gamma|x) = \frac{\exp(-(\lambda_p \gamma + \lambda_p \gamma_0)/(1 - \rho)\gamma_0)}{(1 - \rho)\gamma_0} I_0 \left( \frac{2\sqrt{\rho\gamma}}{(1 - \rho)\gamma_0} \right) \]  

(14)

where \( I_0(.) \) denotes the modified Bessel function of the first kind and order zero.

Now, upon substituting (12) and (14) in (13), and with the help of [27, Eq. (6.614.3)] and [27, Eq. (9.215.1)], and after
some algebraic manipulations we obtain

\[
f_\gamma(y) = \frac{L}{\gamma_p(1-\rho)} \sum_{k=0}^{L-N} \binom{L-N}{k} (-1)^k \exp(-\Delta_3 y)\tag{15}
\]

where

\[
\Delta_3 = \left( \frac{1}{(1-\rho) \gamma_p} - \frac{\rho \gamma_p}{(k + N + (\rho/(1-\rho)))} \right)
\]

Upon substituting (15) and (27) in (26), and after some algebraic manipulations we obtain (see (16))

Upon substituting \( \gamma_{\text{out}} = \gamma \) in (16) and then substituting it in (5), and with the help of [27, Eq. (3.381.4)] and [27, Eq. (3.83.10)], the SEP for the studied system with identical diversity branches \((\lambda_j = \lambda_p, j = 1, \ldots, L)\) and non-identical interferers \((\lambda_i, i = 1, \ldots, I_d)\) can be obtained in a closed-form expression as (see (17))

### 4 Asymptotic outage performance

Owing to complexity of the achieved expressions, it is hard to obtain more details on the system performance. To simplify these expressions and to obtain more insights about the system performance, we see it is important to study the system performance at the high SNR regime where the outage probability can be expressed as \( P_{\text{out}} \sim (G_d, G_c)\), where \( G_c \) denotes the coding gain of the system and \( G_d \) is the diversity order of the system [1]. Obviously, \( G_c \) represents the horizontal shift in the outage probability performance relative to the benchmark curve (SNR)\(^{-1}\) and \( G_d \) refers to the increase in the slope of the outage probability against SNR curve. The parameters on which the diversity order depends will affect the slope of the outage probability curves and the parameters on which the coding gain depends will affect the position of the curves.

In the upcoming analysis, the diversity branches are assumed to be identical and the interferers are also assumed to be identical. Furthermore, the number of interferers \( I_d \) and the interferers power \( \gamma_p \) are assumed to be constant.

\[
P_{\text{out}} = \prod_{j=1}^{I_d} \prod_{g=1}^{I_d} \sum_{\lambda_j} \frac{L}{\gamma_p(1-\rho)} \sum_{\lambda_g} \frac{L}{\gamma_p(1-\rho)} \exp(-\Delta_3 y) \tag{16}
\]

\[
P_{\gamma} = \frac{\gamma\sqrt{b}}{2} \frac{L}{\gamma_p(1-\rho)} \frac{L}{\gamma_p(1-\rho)} \sum_{\lambda_g} \frac{L}{\gamma_p(1-\rho)} (-1)^k \left[ \begin{array}{c}
\binom{L}{k} \frac{L - N}{k} \left[ \exp(-\Delta_3 y) \gamma_p + \left( \Delta_3 y + \gamma_p \right) \right] \\
\Delta_3 (k + N + (\rho/(1-\rho)))
\end{array} \right] \tag{17}
\]

In deriving the asymptotic outage probability for this case, the e2e SINR is first written as \( \gamma_2/\gamma_0 \), where \( \gamma_2 \) has a PDF as defined in (8) and \( \gamma_0 \) is now consisting of i.i.d. exponential RVs \( \gamma_0 = \sum_{i=1}^{I_d} (P_d/\lambda_i)^{1/2} + 1 = \gamma_2 + 1 \) with a PDF of \( \gamma_0 \) given by

\[
f_\gamma(x) = \frac{(\lambda_i y)^{1/2}}{(I_d - 1)!} x^{I_d - 1} \exp(-\lambda_i y) \tag{18}
\]

Using the transformation of RVs and then binomial rule, the PDF of \( \gamma_0 \) can be obtained as

\[
f_{\gamma_0}(z) = -\frac{(\lambda_i y)^{1/2}}{(I_d - 1)!} \exp(-\lambda_i y) \sum_{g=0}^{I_d - 1} \binom{I_d - 1}{g} (I_d - 1 - g) \times (-1)^g \gamma_0^{I_d - g + 1} \tag{19}
\]

At high SNR values, the exponential CDF and PDF can be, respectively, approximated by \( F_y(y) \approx (y/\gamma_0) \) and \( f_y(y) \approx (1/\gamma_0) \). Upon using these statistics, the PDF of \( \gamma_2 \) in (8) simplifies to

\[
f_{\gamma_2}(y) \approx \frac{L}{(N - 1)} (\lambda_p)^{L-N+1} \sum_{k=0}^{N-1} \binom{N - 1}{k} \left[ \begin{array}{c}
\exp(-\Delta_3 y) \gamma_2 + \left( \Delta_3 y + \gamma_2 \right) \\
\Delta_3 (k + N + (\rho/(1-\rho)))
\end{array} \right] \tag{20}
\]

Upon substituting (20) and (19) in (26), and after some algebraic manipulations, the outage probability of the studied system can be obtained at high SNR as

\[
P_{\gamma} = \frac{\gamma\sqrt{b}}{2} \left[ \begin{array}{c}
\prod_{j=1}^{I_d} \prod_{g=1}^{I_d} \sum_{\lambda_j} \frac{L}{\gamma_p(1-\rho)} \sum_{\lambda_g} \frac{L}{\gamma_p(1-\rho)} \left[ \begin{array}{c}
\binom{L}{k} \frac{L - N}{k} \left[ \exp(-\Delta_3 y) \gamma_p + \left( \Delta_3 y + \gamma_p \right) \right] \\
\Delta_3 (k + N + (\rho/(1-\rho)))
\end{array} \right] \\
\prod_{j=1}^{I_d} \prod_{g=1}^{I_d} \sum_{\lambda_j} \frac{L}{\gamma_p(1-\rho)} \sum_{\lambda_g} \frac{L}{\gamma_p(1-\rho)} \left[ \begin{array}{c}
\binom{L}{k} \frac{L - N}{k} \left[ \exp(-\Delta_3 y) \gamma_p + \left( \Delta_3 y + \gamma_p \right) \right] \\
\Delta_3 (k + N + (\rho/(1-\rho)))
\end{array} \right]
\end{array} \right] \tag{21}
\]

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*IET Commun.*, 2014, Vol. 8, Iss. 10, pp. 1674–1683
After applying the result in (21) in Maple, we noticed that it is still dominant for the first term of the second summation \( k = 0 \) and hence, it simplifies to

\[
P_{\text{out}} \simeq -\frac{(\lambda')^L L(L - 1)(\lambda_p)^{L-N+1}(-1)^{L-1}}{(L - N + 1)(I_d - 1)!} \exp(-\lambda') \times \sum_{g=0}^{L-1} \binom{I_d - 1}{g} (-1)^g \frac{\Gamma(L + g - N + 2, \lambda')}{(\lambda')^{L + g - N + 2}} (\gamma_{\text{out}})^{-N+1} \tag{22}\]

With noticing that \( \text{SNR} = 1/\lambda_p \), the outage probability in (22) can be written in a more compact form as (see (23))

\[
\mu = \frac{(\lambda')^L L(L - 1) \exp(\lambda')(-1)^{L}}{(L - N + 1)(I_d - 1)!} \tag{23}
\]

Upon substituting \( \gamma_{\text{out}} = \gamma \) in (22) and then substituting it in (5), and with the help of [27, Eq. (3.351.3)], the SEP for the studied system can be obtained at high SNR as (see (24))

With noticing that \( \text{SNR} = 1/\lambda_p \), the SEP in (24) can be written in a more compact form as (see (25))

\[
\beta = \frac{a\sqrt{b}(\lambda')^L L(L - 1) \exp(\lambda')(-1)^{L}}{2\sqrt{\pi}(L - N + 1)(I_d - 1)!b^{L-N+(3/2)}} \tag{24}
\]

As can be seen from (23) and (25), the coding gain of the system is affected by several parameters such as \( \lambda' \), \( I_d \) and \( \gamma_{\text{out}} \); while the diversity order is fixed and equal to \( L - N + 1 \). Note that having more antennas provides extra diversity order. Also, one can see that the diversity order of the system is linearly proportional with the order of the selected antenna where increasing the order of the selected antenna decreases the diversity order and decreasing the order of the selected antenna increases the diversity order of the system.

5 Simulation and numerical results

In this section, we illustrate the validity of the achieved analytical results via a comparison with Monte Carlo simulations [In generating the simulation curves, 60 000 samples/each SNR value have been used.]. Also, we give some numerical examples to show the effect of interference, OCI and other parameters such as number of diversity branches on the system performance.

Fig. 1 studies the effect of order of antenna on the outage performance. It is clear from this figure that the analytical and asymptotic results perfectly fit with Monte Carlo simulations. Also, we can noted from this figure that as the order of antenna \( N \) increases, the diversity order decreases and the system performance is more degraded. On the other hand, as \( N \) decreases, the diversity order increases and hence, better the achieved performance. Another important result of this figure

![Outage probability against SNR for diversity systems with Nth-best antenna selection and CCI for different values of N](image)

\[
P_{\text{out}} \simeq \left\{-\mu \sum_{g=0}^{L-1} \binom{I_d - 1}{g} (-1)^g \frac{\Gamma(L + g - N + 2, \lambda')}{(\lambda')^{L + g - N + 2}} (\gamma_{\text{out}})^{-N+1} \right\}^{(\text{SNR})^{-1}} \tag{23}
\]

\[
P_{\text{out}} \simeq -\frac{a\sqrt{b}(\lambda')^L L(L - 1)(\lambda_p)^{L-N+1}(-1)^{L-1}}{(L - N + 1)(I_d - 1)!} \exp(-\lambda') \times \sum_{g=0}^{L-1} \binom{I_d - 1}{g} (-1)^g \frac{\Gamma(L + g - N + 2, \lambda')}{(\lambda')^{L + g - N + 2}} (\gamma_{\text{out}})^{-N+1} \tag{24}
\]

\[
P_{\text{out}} \simeq \left\{-\beta \sum_{g=0}^{L-1} \binom{I_d - 1}{g} (-1)^g \frac{\Gamma(L + g - N + 2, \lambda')}{(\lambda')^{L + g - N + 2}} (\gamma_{\text{out}})^{-N+1} \right\}^{(\text{SNR})^{-1}} \tag{25}
\]
It is that with the interference power $\gamma^f$ being fixed and not scaling with SNR, the system can still achieve performance gain when better antenna is selected. This is clear also from the diversity order of the system which is $L - N + 1$.

Fig. 2 studies the effect of number of antennas on the outage performance. Again, it is clear from this figure that the analytical and asymptotic results perfectly fit with Monte Carlo simulations. Also, we can noted from this figure that as the number of antennas $L$ increases, the diversity order increases and the system performance is more enhanced. On the other hand, as $L$ decreases, the diversity order decreases and hence, worse the achieved performance, as expected. Another important result of this figure is that with the interference power $\gamma^f$ being fixed and not scaling with SNR, the system can still achieve performance gain when more antennas are used. This is clear from the diversity order of the system which is $L - N + 1$.

The outage probability is plotted against the outage threshold $\gamma_{\text{out}}$ in Fig. 3 for different number of interferers $I_d$. It is clear from this figure that as $\gamma_{\text{out}}$ increases, the system performance is more degraded. Also, we can see from this figure that as $I_d$ increases, the system performance is more degraded with the worst performance being achieved at the highest value of $I_d$. Finally, one can noted from this figure that increasing $I_d$ degrades the system performance through affecting its coding gain without affecting the diversity order.

Fig. 4 studies the effect of the correlation coefficient $\rho$ on the system performance. It is clear from this figure that as $\rho$ increases or equivalently, as $g_B$ and $\hat{g}_B$ are more correlated, better the achieved performance, as expected. On the other hand, the outage performance is severely degraded when $\rho$ deviates from 1. Also, we can see from this figure that the effect of OCI on the system performance obtains larger as the order of the selected antenna decreases or equivalently, as the quality of the selected antenna enhances. In other words, increasing $\rho$ is more beneficial for system performance as the quality of the selected antenna is made better. Finally, as the interference power is assumed to be scaling with SNR in this figure, a noise floor appears and a zero diversity gain is achieved by the
system because of the effect of interference on the system behaviour.

The outage performance is portrayed against number of antennas \( L \) in Fig. 5 for different values of interference power \( \hat{g} \) and outage threshold \( \gamma_{\text{out}} \). It is clear from this figure that when the interference power is not scaling with SNR, the system can still achieve performance gain as more antennas are used. Also, we can noted from this figure that the best performance is achieved at the smallest value of \( \gamma_{\text{out}} \), as expected.

Fig. 6 portrays the SEP of the system against the order of antenna \( N \). As expected, as \( N \) increases, the system performance is more degraded. Another important notice of this figure is that the gain achieved in system performance because of having more receiving antennas becomes larger as SNR increases. This is clear from the slope of the curves which increases as SNR increases.

The error performance is studied in Fig. 7 for different values of interference power \( \hat{g} \). It can be seen from this figure that as \( \hat{g} \) increases, worse the achieved performance, as expected. Also, it is obvious from this figure that the interference power is degrading the system performance via reducing the coding gain.

6 Conclusion

In this paper, exact and asymptotic outage and SEPs were derived for space diversity systems with the \( N \)th-best antenna selection in the presence of interference and OCI. Monte Carlo simulations proved the accuracy of the achieved analytical and asymptotic results. Findings illustrated that for fixed interference power, the system can still achieve diversity gain when more antennas are used. Also, results showed that the diversity order of the system is linearly increasing with decreasing the order of the antenna, and linearly decreasing with increasing it. Furthermore, findings illustrated that the diversity order is linearly increasing with increasing the number of antennas, and linearly decreasing with decreasing it. Also, results showed that when the interference power scales with SNR, a noise floor appears in the results and a zero diversity gain is achieved by the system because of the effect of interference on the system performance. Finally, findings illustrated that the effect of OCI on the system performance obtains larger as the order of the selected antenna decreases or equivalently, as the quality of the selected antenna enhances.

7 Acknowledgment

This work is supported by King Fahd University of Petroleum & Minerals (KFUPM) through project of Grant no. FT121002.

8 References

Appendix 9

9.1 Appendix 1 Proof of Lemma 1

In this appendix, we derive the outage probability of the studied system for the case of non-identical branches \((\lambda_j, j = 1, \ldots, L)\) and non-identical interferers \((\lambda_i^j, i = 1, \ldots, I)\).

The e2e SINR given in (2) can be written as a ratio of two RVs \(\gamma_d = Y_d/Z_1\). The CDF of \(\gamma_d\) is given by Salhab et al. [30]

\[
P_r[\gamma_d < \gamma_{\text{out}}] = \int_0^{\gamma_{\text{out}}} f_{\gamma_d}(y) dy dz
\]

First, we evaluate the PDF of \(Z_1\)

\[
Z_1 = \sum_{i=1}^{I} \frac{P}{N_0} |h_{i}^{\text{ref}}|^2 + 1 = X_1 + 1
\]

The PDF of \(X_1\) is given by

\[
f_{X_1}(x) = \prod_{i=1}^{I} \lambda_i^j \sum_{g=1}^{N} \prod_{m=1}^{I} (\lambda_m^j - \lambda_g^j)
\]

Using the transformation of RVs, we obtain

\[
f_{Z_1}(z) = \prod_{i=1}^{I} \lambda_i^j \exp(\lambda_i^j) \sum_{g=1}^{N} \prod_{m=1}^{I} (\lambda_m^j - \lambda_g^j)
\]

Now, we evaluate the PDF of \(Y_1 = (P_0/N_0) h_{\text{ref}}^2\) which is given by Vaughan and Venables [31]

\[
f_{Y_1}(y) = \sum_{l=1}^{L} f_{\gamma_l}(y) \prod_{j=1}^{L} F_{\gamma_j}(y) \prod_{w=L-N+1}^{L} (1 - F_{\gamma_w}(y))
\]

where \(\sum_{p}\) denotes the summation over all \(N!\) permutations \((i_1, i_2, \ldots, i_L)\) of \((1, 2, \ldots, L)\).

For Rayleigh fading, the PDF \(f_{\gamma_l}(y)\) and the CDF \(F_{\gamma_l}(y)\) are, respectively, given by \(\lambda_l \exp(-\lambda_l y)\) and \(1 - \exp(-\lambda_l y)\). Upon substituting these statistics in (28) and after some algebraic manipulations, the final result can be obtained as (see (29))

\[
f_{Y_1}(y) = \sum_{l=1}^{L} \frac{\lambda_l^{\gamma_{\text{out}}}}{\gamma_{\text{out}}} \sum_{p} \left[ \exp(-\Delta y) + \sum_{y_l=1}^{L-N} (-1)^y \sum_{x_l=0}^{\gamma_{\text{out}}} \exp(-\Delta y_l) \right]
\]

where
\[ \sum_{s_1, \ldots, s_q} = \sum_{s_1=1}^{L-N-q+1} \sum_{s_2=s_1+1}^{L-N-q+2} \sum_{s_3=s_2+1}^{L-N} \sum_{s_q=s_{q-1}+1}^{L-N}, \quad \Delta_1 = \sum_{w=L-N+1}^{L-1} \lambda_w \cdots \]

and \[ \Delta_2 = \Delta_1 + \sum_{n=1}^{q} \lambda_{s_n} + \lambda_j \]

Upon substituting (29) and (27) in (26), and after some algebraic manipulations, the result in (4) can be obtained.