Performance of Switch-and-Examine DF Relay Systems with CCI at the Relays and Destination over Rayleigh Fading Channels

Anas M. Salhab, Student Member, IEEE, and Salam A. Zummo, Senior Member, IEEE

Abstract—In this paper, we evaluate the outage performance of a decode-and-forward (DF) relay system with two low-complexity relay selection schemes and interference at the relays and destination. The schemes are mainly based on the switch-and-examine diversity combining (SEC) and SEC post-examine selection (SECps) techniques in which a relay out of multiple relays is selected to forward the source message to destination. The selection process is performed such that the signal-to-noise ratio (SNR) of second hop of the selected relay satisfies a predetermined switching threshold. In this paper, we first derive the probability density function (PDF) of SNR of the relay selection scheme and the conditional cumulative distribution function (CDF) of end-to-end (e2e) signal-to-interference plus noise ratio (SINR) assuming Rayleigh fading channels. The derived statistics along with the statistics of first hop channels of the relays and the direct link are then used to derive a closed-form expression for the e2e outage probability. Maximal-ratio combining (MRC) is used at the destination to combine the signals from the relay and the direct link. Furthermore, the outage performance is studied at high SNR regime for various scenarios for the outage probability, diversity order, and coding gain are derived and analyzed. Monte-Carlo simulations and some numerical examples are provided to illustrate the validity of the derived results and to show the effect of interference and other parameters on the system performance. Main results illustrate that when the interference power is fixed, the system can still achieve some performance gain when more relays are added; especially, at SNR values that are comparable to the switching threshold. Asymptotic results show that at high SNR, the system with the SEC and SECps relaying schemes achieves a diversity order of 2 and approximately the same coding gain. Furthermore, findings illustrate that the interference at the destination is more severe on the system performance compared to that at the relays. Finally, results show that the interference is severely affecting the gain achieved in system performance when the SECps relaying scheme is used compared to the conventional SEC relaying.

Index Terms—Decode-and-forward, relay network, Rayleigh fading, co-channel interference, switching threshold.

I. INTRODUCTION

Cooperative or relay networks have generally been studied with respect to the relay selection schemes, coding, multi-user communication, multi-antenna, channel estimation errors, and power allocation, mostly, under conditions of additive white Gaussian noise (AWGN) [1]-[4]. However, the co-channel interference (CCI) dominates AWGN in such wireless systems due to the extensive re-use of frequency bands by system users. Moreover, the effect of interference can be more severe on the relay systems where all relays may use the same frequency band and hence, CCI may exist in every link in the relay network. This shows the need for new studies that address the impact of this channel impairment on the performance of such cooperative networks.

Recently, more attention has been given to evaluate the interference effect on the performance of relay networks [5]-[7]. Particularly, in [7], Al-Qhatani et al. derived closed-form expressions for the outage probability and symbol error rate of a dual-hop amplify-and-forward (AF) relay system with interference at the relay node and assuming Nakagami-m fading channels. The interference effect at the destination node over Rayleigh and Nakagami-m fading channels was studied in [8] and [9], respectively. Recently, the outage performance of a dual-hop AF relay system over Rayleigh and Nakagami-m fading channels was studied in [10] and [11], respectively. As can be seen, most of the existing papers on relay networks considered the interference effect assuming the existence of single relay only.

Most recently, a study on the performance of multi-relay DF conventional relaying systems in the presence of interference at the relay and destination nodes assuming Nakagami-m fading channels was evaluated in [12]. Some key papers on relay systems with multiple relays and opportunistic relaying are the ones presented in [13]-[15]. Particularly, in [13], Salhab et al. evaluated the outage and asymptotic outage performance of an opportunistic DF relay system with interference at the relays and destination. All channels were assumed to follow Nakagami-m distribution with the existence of arbitrary number of unequal power interferers. The lack of comprehensive studies that evaluate the performance of multi-relay cooperative systems with interference at the relay and destination nodes and the importance of such cooperative systems motivate us to contribute in this area of research.

Several relay selection schemes were proposed for cooperative networks with multiple relays, among which is the best-relay or opportunistic relaying [16]. In this scheme, only the best relay is always selected among all other relays to forward the source message to destination which makes it optimum in this sense. Compared to the conventional relaying where all relays participate in the cooperation process, the opportunistic relaying enhances the system spectral efficiency and provides the same or even better performance than the
conventional relaying. Another relay selection scheme is the partial relaying [17]. In such scheme, the relay with first hop SNR greater than a predetermined SNR threshold and being the maximum among other relays is chosen as the best relay. This is useful for certain situations in ad-hoc networks where only first hop channels of relays are available to the source. As can be seen, in order for these relay selection schemes to select among relays, large number of channels need to be estimated each transmission time. This increases the power consumption, reduces the relay battery life, and increases the system complexity.

In DF multi-relay systems, the relays who succeeded in decoding the source message in the first communication phase are called active relays. Having all these relays forwarding the source message to destination in the second phase of communication will be on the expense of system spectral efficiency. Alternatively, the opportunistic relaying can be used where only the best relay among all active relays is selected to forward the message to destination. This enhances the system spectral efficiency and at the same time gives the same performance as the conventional relaying. On the other hand, the opportunistic relaying suffers from a heavy load of channel estimations which are required to select among the active relays each transmission time. In [18], the authors proposed a switching threshold-based relay selection scheme for dual-hop AF relay networks. The scheme is based on the switch-and-examine diversity combining (SEC) technique where the first checked relay with e2e SNR greater than a predetermined switching threshold is selected instead of the best relay to forward the source message to destination. In this scheme, once a checked relay satisfies a certain switching threshold, no need for other relays to estimate the channel for the second hop. Thus, the SEC-based relaying scheme eliminates the need for other relays to operate as channel estimators and hence, reduces the required number of channel estimations, saves the power of these relays, and reduces the system complexity.

To the best of our knowledge, the performance of dual-hop DF relay systems with the SEC-based relaying scheme and interference at the relays and destination over Rayleigh fading channels has not been presented yet. The contributions of our paper over the existing studies can be summarized in the following points: i) we present the SEC and SECps relaying schemes to be used for DF relay systems with interference at the relays and destination; ii) in the SEC and SECps relaying schemes and in contrast to the relaying scheme presented in [14], the first checked relay whose second hop SNR exceeds a predetermined switching threshold is selected instead of the best relay to forward the source message to destination. This reduces the required number of channel estimations, saves the power of relays, and reduces the system complexity; iii) we present a full evaluation for the system outage performance where the effect of interference and some system parameters on the system performance is provided. Furthermore, to get more about system insights, we study the outage performance at high SNR regime where approximate expressions for the outage probability, diversity order, and coding gain are derived and analyzed. In this paper, we derive exact closed-form expressions for the outage probability for the generic independent non-identically distributed (i.i.d.) case of relay second hop channels for the SEC relaying scheme and for the independent identically distributed (i.i.d.) case for the SECps relaying scheme. Firstly, the PDF of SNR at the selection scheme combiner output, the CDF of e2e SNR conditioned on the decoding set of relays, the CDF of SINR of the relays first hop channels, and the CDF of the direct link SINR are derived. Then, these statistics are used to derive a closed-form expression for the system outage probability. Further analysis is conducted following the same procedure to evaluate the asymptotic system behavior. The switching threshold is selected to optimize the e2e outage probability and is numerically calculated.

This paper is organized as follows. Section II presents the system and channel models. The exact outage performance is analyzed in Section III. Section IV provides the asymptotic outage performance. Some simulation and numerical results are presented and discussed in Section V. Finally, conclusions are given in Section VI.

II. SYSTEM AND CHANNEL MODELS

Figure 1 shows the relay system under consideration. It consists of one source, one destination, \( K \) relay nodes, and arbitrary number of interferers at both the relays and destination. The entire communication takes place in two phases. In the first phase, the source \( S \) transmits its message to the destination \( D \) and the \( K \) relays. In the second phase, the relay which satisfies a predetermined switching threshold among all other relays who succeeded to decode the source message in the first phase is selected to forward a re-encoded version of it to \( D \). Compared to the opportunistic relaying, instead of estimating the second hop channels of all active relays each transmission time, this amount of channel estimations is noticeably reduced by using the SEC-based relaying scheme.

With referring to the flowchart in Figure 2, the SEC-based relay selection scheme works as follows: at the guard period of each transmission time, the source sends a ready-to-send (RTS) packet to relays and destination. This packet allows each relay to estimate its first hop channel. To reduce the overall overhead in communication, a method based on time is selected: as soon
as the RTS packet is received, each relay who successfully received the source message starts a timer based on its first hop instantaneous channel estimation. The relay whose timer is expired first sends a RTS packet to destination through a one bit feedback. This process continues till a suitable relay is found. If the checked relay is found unacceptable, it will be negatively acknowledged by the destination where it will keep silent. In this case, the timer of other relay expires and the same process is repeated. This process continues until a suitable relay is found or the last relay is reached. If the last relay is reached and found acceptable, it will be positively acknowledged by the destination to start its transmission. If not, the destination will negatively acknowledge it and wait for a certain time interval \( \Delta \), if it does not receive another RTS packet from other active relays and within this time period, it will ask the last checked relay to start its transmission. In the case of SECs which is an enhanced version of the conventional SEC relaying and in the case where the last relay is reached and found unacceptable, the destination asks the best relay among all checked relays to conduct its transmission.

We assume that the signal at the \( k \)th relay is corrupted by interfering signals from \( I_k \) co-channel interferers \( \{ x_i \} \). The received signal at the \( k \)th relay can be expressed as

\[
y_{r_k} = h_{s,k} x_0 + \sum_{i=1}^{I_k} h_{i,k}^L x_{i,k}^L + n_{s,k},
\]

where \( h_{s,k} \) is the channel coefficient between \( S \) and the \( k \)th relay, \( x_0 \) is the transmitted symbol with \( \mathbb{E}\{ |x_0|^2 \} = P_0 \), \( h_{i,k}^L \) is the channel coefficient between the \( i \)th interferer and the \( k \)th relay, \( x_{i,k}^L \) is the transmitted symbol from the \( i \)th interferer with \( \mathbb{E}\{ |x_{i,k}^L|^2 \} = P_{i,k}^L \), \( n_{s,k} \sim \mathcal{CN}(0, N_0) \) is an additive white Gaussian noise (AWGN), and \( \mathbb{E}\{ \cdot \} \) denotes the expectation operation. Let us define \( h_{d,k}, h_{s,k}, \) and \( h_{i,k}^L \) as the channel coefficients between \( S \) and \( D \), the \( k \)th relay and \( D \), the \( i \)th interferer and \( D \), respectively. All channel coefficients are assumed to follow the Rayleigh distribution, that is, the channel powers denoted by \( |h_{s,d}|^2, |h_{s,k}|^2, |h_{k,d}|^2, |h_{i,k}^L|^2 \), and \( |h_{i,d}^L|^2 \) are exponentially distributed random variables (RVs) with parameters \( \sigma_{s,d}^2, \sigma_{s,k}^2, \sigma_{k,d}^2, \sigma_{i,k}^2, \) and \( \sigma_{i,d}^2 \), respectively. Using (1), the signal-to-interference plus noise ratio (SINR) at the \( k \)th relay can be written as

\[
\gamma_{s,k} = \frac{P_{s,k}|h_{s,k}|^2}{\sum_{i=1}^{I_k} P_{i,k}^L |h_{i,k}^L|^2 + 1}.
\]

Let \( C_L \) denote a decoding set defined by the set of active relays that could have correctly decoded the source message in first phase of communication. It is defined as [14]

\[
C_L \triangleq \bigg\{ k \in S_r : \frac{1}{2} \log_2 (1 + \gamma_{s,k}) \geq R \bigg\} = \bigg\{ k \in S_r : \gamma_{s,k} \geq 2^{2^R} - 1 \bigg\},
\]

where \( S_r \) is a set of \( L \) relays and \( R \) denotes a fixed spectral efficiency threshold.

In the second phase and after decoding the received message, the first checked relay in \( C_L \) whose second hop channel SNR is greater than the predetermined switching threshold forwards the re-encoded message to the destination. The selected relay is chosen according to the SEC selection scheme and it is the first checked relay in \( C_L \) whose \( \gamma_{i,d} \) is greater than a predetermined switching threshold. It can be written as

\[
\gamma_{i,d} = \frac{P_{i,d} |h_{i,d}|^2}{\sum_{j=d+1}^{I_d} P_{j,d} |h_{j,d}|^2 + 1},
\]

where \( P_{i,d} \), \( P_{i,d} \), \( N_0 \) and \( I_d \) are the transmit power of the \( i \)th active relay, the transmit power of the \( i \)th interferer, and the AWGN power at the destination, respectively, and \( I_d \) is the number of interferers at the destination node. Equivalently, the relay with the second hop channel SNR \( \gamma_{i,d} \) greater than a predetermined switching threshold is selected to forward the source signal to destination since the denominator is common to the SINRs from all relays belonging to \( C_L \).

The destination finally combines the signals from the source and the selected relay using maximal-ratio combining (MRC). The end-to-end (e2e) SINR at the destination output can be

\[\text{SINR}_{\text{e2e}}\]

We are assuming that the channels of the second hop transmission do not change while a decision on which relay is selected is made.
where the internal summation is taken over all of $u$ and SECps relay selection schemes.

For the outage probability of the studied system with the SEC communications.

where we have assumed in the considered system that the interferers’ activities are unchanged over the two phases of communications.

III. EXACT OUTAGE PERFORMANCE

In this section, we evaluate exact closed-form expressions for the outage probability of the studied system with the SEC and SECps relay selection schemes.

Let $C_L$ be a decoding subset with a number of $L$ active relays (i.e., cardinality $|C_L| = L$), then

$$P_r [C_L] = \prod_{l \in C_L} P_r [\gamma_{s,l} \geq u] \prod_{m \notin C_L} P_r [\gamma_{s,m} < u],$$

where $u = (2^{2R} - 1)$. The outage probability for the studied system is given by [14]

$$P_{out} \triangleq P_r \left[ \frac{1}{2} \log_2 (1 + \gamma_d) < R \right] = \sum_{L=0}^{K} \sum_{C_L} P_r [\gamma_d < u|C_L] P_r [C_L],$$

where the internal summation is taken over all of $\binom{K}{L}$ possible subsets of size $L$ from the set with the $K$ relays. To evaluate (8), we need first to derive $P_r [\gamma_d < u|C_L]$ and $P_r [C_L]$ for the case of i.n.d. second hops $\{\lambda_{s,i} \}_{i=1}^{K}$ and interferers of unequal powers $\{\lambda_{i,n}^l\}_{i=n=1}^{K}$ summarized in the following two lemmas, respectively.

**Lemma 1:** The term $P_r [\gamma_d < u|C_L]$ in (8) is given for $L \geq 1$ by (5) on the top of this page.

**Proof:** Please see Appendix A.

**Lemma 2:** The CDF $P_r [\gamma_{s,k} < u]$ which is a part of the term $P_r [C_L]$ in (7) is given by

$$P_r [\gamma_{s,k} < u] = \prod_{i=k}^{I_k} \lambda_{s,i}^l \prod_{g=1}^{I_g} \exp \left( \frac{\lambda_{g,k}^l \Xi_1 - \Xi_2}{\Lambda_{s,g}^l} \right),$$

where $\Xi_1 = \frac{\Gamma(1, \lambda_{u,t})_{s,d}}{\lambda_{u,t}^l}, \Xi_2 = \frac{\Gamma(1, \lambda_{u,t}^l + \lambda_{g,k}^l)}{\lambda_{u,t}^l + \lambda_{g,k}^l}$, and $\Xi_3 = \frac{\Gamma(1, \lambda_{u,t}^l + \lambda_{g,k}^l)}{\lambda_{u,t}^l + \lambda_{g,k}^l}.$

A. SEC-Based Relay Selection

In this section, we evaluate the outage probability of the SEC relaying when the second hops of relays are non-identical and the interferers have unequal average powers.

Let $\rho \triangleq P_0/N_0 = P_t/N_0$ and $\rho_t \triangleq P_{t,d}/N_0$. Then, $\rho|h_{s,d}^2|, \rho|h_{s,k}^2|, \rho|l_{s,k}^t|^2, \rho|h_{i,d}^2|$, and $\rho|l_{i,d}^t|^2$ are exponential distributed with parameters $\lambda_{s,d} = 1/\rho h_{s,d}^2, \lambda_{s,k} = 1/\rho h_{s,k}^2, \lambda_{i,d} = 1/\rho h_{i,d}^2$, and $\lambda_{i,d}^t = 1/\rho|l_{i,d}^t|^2.$ For the i.n.d. case, we have $\lambda_{i,n}^l \neq \lambda_{j,n}^l$, when $i \neq j, n \in S_{r} \cup \{d\}$. The results of the terms $P_r [\gamma_d < u|C_L]$ and $P_r [C_L]$ are summarized in the following two corollaries, respectively.

**Corollary 1:** The term $P_r [\gamma_d < u|C_L]$ in (8) is given for $L \geq 1$ by (10) on the top of next page.

In evaluating the term $P_r [\gamma_d < u|C_L]$, the e2e SINR $\gamma_d$ can be written as $Y_1/Z_2$, where $Y_1$ as defined in Appendix A with a PDF as derived in (30) and $Z_2$ is now constituting of a summation of i.i.d. RVs as $Z_2 = \sum_{i=1}^{I_2} \rho_t |l_{i,d}^t|^2 + 1 = \Lambda_{s} + 1$ with a PDF of $X_2$ given by

$$f_{X_2}(x) = \frac{(\lambda_{t}^{d})_{s,d}}{(I_d-1)!} x^{I_d-1} \exp (-\lambda_{t}^{d} x).$$

Using the transformation of RVs and then the Binomial rule,
\[ P_r[\gamma_d < u|C_L] = -\frac{(\lambda_d^L)^I_d}{(I_d - 1)!} \exp(\lambda_d^L) (-1)^I_d \sum_{g=0}^{I_d-1} \binom{I_d - 1}{g} (-1)^g \left( \sum_{i=0}^{L-1} \pi_i \prod_{k=0}^{L-1} (1 - \exp(-\lambda_{k,d} \gamma_T)) \right) \times \left[ (\lambda_1 - \lambda_2) \left( 1 - \frac{\lambda_d}{\lambda_{d,a}} \right) \right. \]

\[ + \left. \left( \frac{\lambda_1 - \lambda_3}{1 - \frac{\lambda_d}{\lambda_{d,a}}} \right) + \exp(-\lambda_{i,d} \gamma_T) \left( \frac{\lambda_1 - \exp(\lambda_{d,a} \gamma_T) \lambda_2}{1 - \frac{\lambda_d}{\lambda_{d,a}}} \right) + \left( \lambda_1 - \exp(\lambda_{d,a} \gamma_T) \lambda_3 \right) \right] \]
where $\Lambda_1$, $\Lambda_2$ are as defined before, and $\Lambda'_3 = \Lambda_3$ with replacing $i$ by $R$.

Upon substituting (12) and (16) in (33), and with the help of [20, Eq. (3.351.2)] and after some algebraic manipulations, we get the result in (17).

### B. SECps-Based Relay Selection

In this section, we evaluate the outage probability of the SECps relaying when the second hops of relays are identical and the interferers have unequal average powers. For this case, the term $P_r[C_L]$ is as derived in Lemma 2 and the term $P_r[\gamma_d < u|C_L]$ is given in the following Lemma.

**Lemma 4:** The term $P_r[\gamma_d < u|C_L]$ in (8) is given for $L \geq 1$ by (18) on the top of next page.

**Proof:** In evaluating $P_r[\gamma_d < u|C_L]$, the e2e SINR can be written as $Y_3/Z_1$, where $Z_1$ is as defined in Appendix A with a PDF as derived in (29). The CDF of $\rho h_{SECPsd}^2[\gamma]$ which is a part of the term $Y_3$ can be written as [19]

\[
F_{\rho h_{SECPsd}^2}(\gamma) = \begin{cases} 
1 - \sum_{j=0}^{L-1} [F_{\rho h_{Rsd}^2}(\gamma_T)]^j [1 - F_{\rho h_{Rsd}^2}(\gamma)], & \gamma < \gamma_T; \\
F_{\rho h_{Rsd}^2}(\gamma), & \gamma \geq \gamma_T.
\end{cases}
\]

Using the CDF in (19) and following the same procedure as in Appendix A, the PDF of $Y_3$ can be obtained as

\[
f_{Y_3}(\gamma) = \left[1 - (1 - \exp(-\lambda R_d \gamma_T))^L\right] \left\{ \frac{\exp(-\lambda R_d \gamma_T)}{\lambda R_d} \right\} \left( \frac{\exp((\gamma - \gamma_T) / \Lambda_{sd} \gamma_T)}{\Lambda_{sd} \gamma_T} \right) \times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

\[
+ \left[ (1 - \exp(-\lambda R_d \gamma_T)) \right]^L \left( \frac{\exp((\gamma - \gamma_T) / \Lambda_{sd} \gamma_T)}{\Lambda_{sd} \gamma_T} \right) \times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

\[
+ \left[ \exp(\lambda R_d \gamma_T) \right] \left( \frac{\exp((\gamma - \gamma_T) / \Lambda_{sd} \gamma_T)}{\Lambda_{sd} \gamma_T} \right) \times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

\[
- \exp((-i+1) \lambda R_d \gamma_T) \left( \frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right) \left( \frac{\exp((\gamma - \gamma_T) / \Lambda_{sd} \gamma_T)}{\Lambda_{sd} \gamma_T} \right) \times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

\[
+ \left[ \exp(\lambda R_d \gamma_T) \right] \left( \frac{\exp((\gamma - \gamma_T) / \Lambda_{sd} \gamma_T)}{\Lambda_{sd} \gamma_T} \right) \times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

\[
+ \left[ \exp(\lambda R_d \gamma_T) \right] \left( \frac{\exp((\gamma - \gamma_T) / \Lambda_{sd} \gamma_T)}{\Lambda_{sd} \gamma_T} \right) \times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

\[
\times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

\[
f_{Y_3}(\gamma) = \left[1 - (1 - \exp(-\lambda R_d \gamma_T))^L\right] \left( \frac{\exp(-\lambda R_d \gamma_T)}{\lambda R_d} \right) \left( \frac{\exp((\gamma - \gamma_T) / \Lambda_{sd} \gamma_T)}{\Lambda_{sd} \gamma_T} \right) \times \left(\frac{\exp((-i+1) \lambda R_d \gamma_T)}{\lambda R_d} \right)
\]

Upon substituting (29) and (20) in (33), and with the help of [20, Eq. (3.351.2)] and after some algebraic manipulations, we get the result in (18).

For the case of identical second hops and interferers of equal powers at the relays and destination, the term $P_r[C_L]$ is as derived in Corollary 2 and the term $P_r[\gamma_d < u|C_L]$ is given in the following Corollary.

**Corollary 4:** The term $P_r[\gamma_d < u|C_L]$ in (8) is given for $L \geq 1$ by (21) on the top of next page.

Upon substituting (12) and (20) in (33), and with the help of [20, Eq. (3.351.2)] and after some algebraic manipulations, we get the result in (21).

### IV. ASYMPTOTIC OUTAGE PERFORMANCE

In this section, we evaluate the asymptotic (high SNR) outage performance of the studied system with the SEC and SECps relaying schemes. At high SNR, the outage probability can be expressed as $P_{out}\approx (G_{SR}SNR)^{-G_d}$, where $G_c$ and $G_d$ denote the coding gain and the diversity order of the system, respectively. In the upcoming analysis, $I_d$ and $P_t$ are assumed to be constant. Also, the second hops of relays are assumed to be identical and the interferers at the relays and destination are assumed to have equal powers.

#### A. SEC-Based Relay Selection

In this section, we derive the asymptotic outage probability for the studied system with the SEC relaying scheme. At high SNR regime, the exponential CDF and PDF can be respectively approximated by $F_{\gamma}(\gamma) \approx \frac{\gamma}{\gamma_T}$ and $f_{\gamma}(\gamma) \approx \frac{1}{\gamma_T}$. Upon substituting these statistics in (15) and following the same procedure as in Appendix A, the term $P_r[\gamma_d < u|C_L]$ can be obtained at high SNR for $L \geq 1$ as

\[
P_r[\gamma_d < u|C_L] \approx \frac{(\lambda_3')^L}{(I_d - 1)!} \exp (\lambda_3') (-1)^L \sum_{g=0}^{L-1} (I_d - 1)^g 
\]

\[
\times (-1)^g \left( \frac{\lambda_3' I_d}{(I_d - 1)!} \right)^L \sum_{j=0}^{L-1} \left( \frac{I_d - 1}{I_d} \right)^j \lambda_3' \lambda_3' \left[ \frac{\gamma T \chi_1 u - \gamma (\gamma T)^2 \chi_2}{2} \right] 
\]

\[
+ \sum_{j=0}^{L-1} \left( \frac{I_d - 1}{I_d} \right)^j \lambda_3' \lambda_3' \left[ \frac{\chi_3 u^2 - \gamma (\gamma T)^2 \chi_2}{2} \right] \right),
\]

\[
(22)
\]

where $\chi_1 = \frac{\gamma T \chi_1 u}{(\lambda_3')^2}$, $\chi_2 = \frac{\gamma (\gamma T)^2 \chi_2}{(\lambda_3')^2}$, and $\chi_3 = \frac{\gamma (\gamma T)^2 \chi_2}{(\lambda_3')^2}$.

Now, the CDF $P_r[\gamma_{k,h} < u]$ which is a part of the term $P_r[C_L]$
\[ P_r[\gamma_d < u|C_L] = \prod_{i_d=1}^{I_d} \lambda_{i_d,d}^{I_d} \sum_{g=0}^{I_d} \exp\left(\lambda^T_{g,d}\right) \left\{ \frac{1 - (1 - \exp(-\lambda_{R,d} \gamma_T))}{(\lambda_{R,d} - \frac{1}{\lambda_{R,d}})} \right\}^{L} \exp\left(-\lambda_{R,d} \gamma_T\right) \frac{\exp\left(-\lambda_{s,d} \gamma_T\right)}{\lambda_{s,d}} \\
- \exp\left(\lambda_{s,d} \gamma_T\right) \frac{\exp\left(-\lambda_{s,d} \gamma_T\right)}{\lambda_{s,d}} \right] + L \sum_{i=0}^{L-1} \frac{(L-1)(-1)^i}{i} (\xi_1 - \xi_2) - \exp\left(-\lambda_{s,d} \gamma_T\right) \frac{\exp\left(-\lambda_{s,d} \gamma_T\right)}{\lambda_{s,d}}] \right\}, \]

where \( \xi_1, \xi_2, \xi_3 \) are as defined before, and \( \xi_4 = \frac{\Gamma(1,(i+1)\lambda_{R,d} u + \lambda^T_{s,d})}{(i+1)(\lambda_{R,d} u + \lambda^T_{s,d})} \).

\[ P_r[\gamma_s < u|C_L] = \frac{\lambda_{s,k}^{I_d}}{(I_d - 1)!} \exp\left(\lambda^T_{s,k}\right) \sum_{g=0}^{I_d} \frac{(I_d - 1)}{g} \\left\{ (-1)^g \lambda^T_{s,k}^{(g+1)-1} \Gamma (g + 2, \lambda^T_{s,k}) u \right\}. \]

Upon substituting (23) in (7) and then substituting (7) and (22) in (8), the asymptotic outage probability can be evaluated. While plotting the outage probability in some common mathematical tools like Maple, it was noticed that the term \( E_{\gamma_d < u|C_L} \) in (22) is the one who dominates the final result of the outage probability when compared with the term \( P_r[\gamma_s < u|C_L] \) in (23). Also, it was noticed that the first part in (22) is dominated by the second part. Furthermore, this term can be further simplified due to the fact that it is still dominant when \( j = 0 \). Therefore, the result in (22) can be simplified as

\[ P_r[\gamma_d < u|C_L] \approx \frac{(\lambda^T_{s,k} I_d)}{(I_d - 1)!} \exp\left(\lambda^T_{s,k}\right) \sum_{g=0}^{I_d} \frac{(I_d - 1)}{g} \\left\{ (-1)^g \lambda^T_{s,k}^{(g+1)-1} \Gamma (g + 2, \lambda^T_{s,k}) u \right\}. \]

By noticing that \( \lambda_{R,d} = \lambda_{s,d} = (\text{SNR})^{-1} \), the result in (24) which as mentioned before represents the asymptotic outage probability can be rewritten at \( u = \gamma_{\text{out}} \) as

\[ P_{\text{out}} \approx \left\{ \frac{(\lambda^T_{s,k} I_d)}{(I_d - 1)!} \exp\left(\lambda^T_{s,k}\right) \sum_{g=0}^{I_d} \frac{(I_d - 1)}{g} \right\} \times \left[ \frac{\text{SNR}}{(\frac{\gamma_{\text{out}}}{2})^2} \right]^{-\frac{1}{2}}. \]

B. SECps-Based Relay Selection

In this section, we derive the asymptotic outage probability for the studied system with the SECps relaying scheme. Upon substituting the approximate statistics of the exponential distribution in (19) and following the same procedure as in Appendix A, the term \( P_r[\gamma_d < u|C_L] \) can be obtained at high SNR for \( L \geq 1 \) as in (26) on the top of next page.

The CDF \( P_{\text{out}}[\gamma_s < u|C_L] \) which is a part of the term \( P_r[\gamma_s < u|C_L] \) in (23) is similar to that obtained in (23). Upon substituting (23) in (7) and then substituting (7) and (26) in (8), the asymptotic outage probability can be evaluated. While plotting the result in Maple, it was noticed that the term \( P_r[\gamma_s < u|C_L] \) in (26) is the one who dominates the final result of the outage probability when compared with the term \( P_r[\gamma_s < u|C_L] \) in (23). Also, it was noticed that the second part in (26) is dominated by the first part. Furthermore, this term is still dominant when \( i = 0 \).
Therefore, the result in (26) can be simplified as

\[
P_r [\gamma_d < u | C_L] \approx -\frac{(\lambda_d^I)^k_j}{(I_d - 1)!} \exp \left(\frac{\lambda_d^I}{(I_d - 1)!} \sum_{g=0}^{I_d-1} \frac{(I_d - 1)^g}{g!} \right) (-1)^g \left\{ \sum_{i=0}^{L-1} (\lambda_{R,d} \gamma_T)^i \lambda_{R,d} \chi_1 i u + \frac{(\gamma_T)^2 \chi_2}{2} \right\},
\]

where \(\chi_2\) as defined before.

Regarding the coding gain of the system and based on the asymptotic results of the two relaying schemes, one can notice that various system parameters affect the system performance through affecting its coding gain. Such parameters are: \(\lambda_d^I\), \(I_d\), \(\gamma_T\), and \(\gamma_{out}\). This is clear from the numerical results of next section where the outage threshold \(\gamma_{out}\) and the number of interferers \(I_d\) affect the position of the curves and hence, affecting the coding gain of the system and its performance.

V. Simulation and Numerical Results

In this section, we illustrate the validity of the achieved analytical and asymptotic expressions. We also provide some numerical examples to show the effect of the interference and some system parameters like number of relays, and switching threshold on the system performance.

Figure 3 portrays the outage probability versus average SNR for the studied system with the SEC-based relaying scheme for different numbers of relays \(K\). It is clear from this figure that the achieved analytical and asymptotic results perfectly fit with Monte-Carlo simulations. Also, it can be seen from this figure that the SEC relaying scheme has nearly the same performance as the best-relay selection scheme for very low SNR region; whereas, as we go further in increasing SNR, the best relay selection scheme is clearly outperforming the SEC relaying, as expected. In addition, we can see from this figure that for the SEC relaying as \(K\) increases, the system performance becomes more enhanced; especially, at the range of SNR values that are comparable to \(\gamma_T\). More importantly, for \(K = 2, 3,\) and \(4\), it is obvious that at both low and high SNR values, all curves asymptotically converge to the same behavior and no gain is achieved in system performance with adding more relays. This is expected since when \(\gamma_T\) takes values much smaller or larger than the average SNR, the system asymptotically converges to the case of two relays and hence, adding more relays will not help in enhancing the system performance. The curves in this figure greatly match the results achieved from the asymptotic analysis where the diversity order was shown to be constant and equal 2. Another important result in this figure is that the interference power is assumed not scaling with SNR, the system still can achieve more gain in performance due to adding more relays and this is clear in the range of SNR values that are comparable to the value of \(\gamma_T\). Finally, it is clear from this figure that for the case of 4 relays, the system with a randomly selected relay behaves similar to the SEC and SECps relaying at the very low values of SNR. As we go further in increasing SNR, the SEC and

\[
\text{where } \chi_2 \text{ as defined before.}
\]
Fig. 3: $P_{\text{out}}$ vs. SNR for SEC with various values of $K$, $\sigma_{s,d}^2 = 1$, $\sigma_{s,1}^2 = 0.2$, $\sigma_{s,2}^2 = 0.4$, $\sigma_{s,3}^2 = 0.6$, $\sigma_{s,4}^2 = 0.8$, $\sigma_{d,k}^2 = 0.4$ and $(\sigma_k^2)^2 = 0.01$ for $k = 1, \ldots, 4$, and $(\sigma_d^2)^2 = 0.01$.

Fig. 4: $P_{\text{out}}$ vs. SNR for SECps with various values of $\sigma_{s,d}^2$, $\sigma_{s,k}^2 = 0.2$ and $\sigma_{d,k}^2 = 0.4$ and $(\sigma_k^2)^2 = 0.1$ for $k = 1, 2$, and $(\sigma_d^2)^2 = 0.1$.

Fig. 5: $P_{\text{out}}$ vs. SNR for SEC with various values of $K$ and i.i.d. hops and $\sigma_{s,d}^2 = 1$, $(\sigma_k^2)^2 = 0.01$ for $k = 1, \ldots, 3$, and $(\sigma_d^2)^2 = 0.01$.

Fig. 6: $P_{\text{out}}$ vs. outage threshold for SEC and SECps schemes with various values of $\sigma_{s,d}^2$, $\sigma_{s,k}^2 = 0.1$, $\sigma_{d,k}^2 = 0.2$, $\sigma_{s,2}^2 = 0.4$, $\sigma_{s,3}^2 = 0.6$, $\sigma_{s,4}^2 = 0.4$ and $(\sigma_k^2)^2 = 0.01$ for $k = 1, \ldots, 3$, and $(\sigma_d^2)^2 = 0.01$.

SECps schemes outperform the random selection scheme and this is because at high SNR, the probability of finding better relays in the SEC and SECps schemes as well as the random selection increases. The increase in this probability beside the increase in the SNR is more beneficial for the SEC and SECps schemes compared to the random selection of relays. This is because the selection of relays in SEC and SECps relaying depends on comparing the SNR of relays with a switching threshold.

Figure 4 illustrates the outage performance versus average SNR for the SECps relaying scheme for different values of $\sigma_{s,d}^2$ with and without interference. It can be seen from this figure that as $\sigma_{s,d}^2$ increases, better the achieved performance. This is valid for both cases; the system with interference and with no interference. Also, one can notice from this figure that for the case where the interference power scales with SNR, a noise floor appears and zero diversity gain is achieved in all curves of this case due to the effect of interference on the system performance. On the other hand, for the case where the interference power is not scaling with SNR, the system outage performance keeps enhancing as we increase SNR.

Figure 5 studies the outage performance versus average SNR for the SEC relaying scheme for different values of $K$ for the i.i.d. and i.n.d. cases of relay hops. As expected, as $K$ increases, better the achieved performance, especially, in the region where the average SNR values are comparable to $\gamma_T$. The figure also shows that this behavior extends to the case of i.n.d. relay hops. Also, we can see from this figure that the gain achieved in system performance becomes smaller as we go further in increasing $K$.

Figure 6 portrays the outage performance versus outage threshold $\gamma_{\text{out}}$ for the SECps relaying scheme for different
The optimum switching threshold for this figure was found to be: 3.106078593, 3.247839189, 3.503769880, 3.877223182, 4.167763991, 4.304530201, 4.354690471, 4.371361248, 4.37509590, and 4.378452258.

Figure 10 shows the outage performance versus average SNR for the SEC relaying scheme for different values of $\gamma_T$. As expected, as $\gamma_T$ increases and hence, the probability of outage, the worse the achieved performance. It is clear from this figure that $\gamma_T$ degrades the system performance by reducing the coding gain of the system without affecting the diversity order. Also, the effect of interference on the system performance is clear in this figure where when the interference power is assumed to scale with SNR, a noise floor appears in all curves of this case and hence, zero diversity gain is achieved by the system. On the other hand, in the case where there is no interference, increasing the SNR keeps decreasing the outage probability using the Maple software.

values of SNR. As expected, as the average SNR takes larger values and hence, enhancing the quality of the direct link and relay paths, better the achieved performance. In addition, due to the effect of interference, the gain achieved in system performance when the SECps relaying scheme is used is very small compared to the case where the SEC scheme is used.

Figure 7 studies the outage performance versus average SNR for the SEC and SECps-based relaying schemes for different values of $\sigma_{s,d}^2$. It is clear that the gain achieved in system performance when the SECps relaying is used is very small compared to the SEC scheme. As the value of the average SNR becomes much larger or smaller than the average SNR, this small gain in system behavior vanishes and both schemes behave the same. More importantly, this gain in system performance is negligible compared to what we showed in our results in [18] where there was no interference. This is expected as the effectiveness of the SECps relaying scheme over the conventional SEC relaying is reduced due to the existence of the interference.

Figure 8 shows the outage performance versus average SNR for the SECps relaying scheme for different values of $\rho_1$. A perfect fitting between the analytical and the asymptotic results is obvious in this figure. Also, the effect of interference power on the system performance is clear in this figure where as $\rho_1$ increases, the system behavior becomes more degraded, as expected. This degradation in system performance is due to the reduction in coding gain caused by the interference.

Figure 9 illustrates the outage performance versus average SNR for the SEC relaying scheme for different values of $\gamma_T$. It is clear from this figure that the best performance is achieved when the optimum switching threshold $\gamma_T$ is used, as expected. Due to the complexity of the analytical expression of the outage probability, deriving a closed-form expression for the optimum switching threshold is very hard if not impossible. Alternatively, it is numerically calculated to optimize the outage probability using the Maple software.

The optimum switching threshold for this figure was found to be: 3.106078593, 3.247839189, 3.503769880, 3.877223182, 4.167763991, 4.304530201, 4.354690471, 4.371361248, 4.37509590, and 4.378452258.
average SNR and relays $K$ for the SEC relaying scheme for different numbers of relays $K$ when they are not equal and $\sigma^2_{s,d} = 0.1, \sigma^2_{s,1} = 0.2, \sigma^2_{s,2} = 0.4, \sigma^2_{k,d} = 0.4$ and $(\sigma^2_{k})^2 = 0.01$ for $k = 1, \ldots, 3$, and $(\sigma^2_{d})^2 = 0.01$.

The outage probability where no noise floors can be seen.

Figure 11 studies the outage performance versus average SNR for the SEC relaying scheme for different numbers of relays $K$ when they are not equal. As can be seen, the interference at the destination node affects the system performance more severely than the interference at the relay. This result is expected as the interference at the destination affects the signal on the direct link and that through the relay; whereas, the interference at the relay affects only the signal through the relay. Finally, the worst performance is achieved when the interference simultaneously increases at the relay and the destination nodes, as expected.

Figure 12 shows the outage performance versus number of relays $K$ for the SEC relaying scheme for different values of average SNR and $\sigma^2_{s,d}$. It can be seen from this figure that the considered relay system still achieves performance gain and the outage probability decreases when the number of relays $K$ increases, but the slope depends on the SNR values. Also, the achieved gain in system performance due to increasing the power of the direct link is clear in this figure.

Figure 13 illustrates the outage performance versus interference power for the SEC relaying scheme for different values of $K$. It is clear from this figure that adding more relays is more beneficial for system performance at the low values of interference power $\rho_I$. As we go further in increasing $\rho_I$, the gain achieved in system performance due to adding more relays becomes smaller, as expected.

Figure 14 shows the average number of channel estimations versus switching threshold $\gamma_T$ for the SEC and SECPs-based relaying schemes in comparison with the best-relay selection scheme for the case of 4 active relays. We can see from this figure that as the quality of all relay second hop channels
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VI. CONCLUSION

In this paper, we evaluated the performance of a dual-hop DF relay system with the low-complexity SEC and SECPs relaying schemes and interference at the relays and destination. The e2e outage probability was derived for the generic i.n.d. case of second hops of the SEC relaying scheme and for the i.i.d. case for the SECPs scheme. Furthermore, the system outage performance was evaluated at high SNR values where the diversity order and coding gain were derived. Monte-Carlo simulations proved the accuracy of the achieved analytical and asymptotic results. Findings illustrated that for fixed number of interferers of fixed power or equivalently, when the interference power does not scale with SNR, the system can still achieve diversity gain; especially, in the range of SNR values that are comparable to the switching threshold. Also, asymptotic results showed that the system achieves the same diversity order which is 2 and approximately the same coding gain in the cases of SEC and SECPs relaying schemes. Furthermore, results illustrated the effectiveness of the proposed relaying schemes in reducing the channel estimation load compared to the opportunistic relaying.

APPENDIX A

PROOF OF LEMMA 1

In this Appendix, we evaluate the first term $P_r[\gamma_d < u|C_L]$ in (8). The e2e SINR can be written as a ratio of two RVs $\gamma_d = Y_1/Z_1$. The RV $Z_1$ can be written as $Z_1 = \sum_{i=1}^{L} \rho_I|h_{i,d}|^2 + 1 = X_1 + 1$ with a PDF of $X_1$ given by $f_{X_1}(x) = \Pi_{i=1}^{L} \lambda_{i,d} \exp(-\lambda_{i,d}x)$.

Using the transformation of RVs for $Z_1 = X_1 + 1$, the PDF of $Z_1$ can be obtained as

$$f_{Z_1}(z) = \prod_{i=1}^{L} \lambda_{i,d} \sum_{g=1}^{L} \exp(-\lambda_{g,d}z) \sum_{m \neq g} \prod_{i=1}^{L} \lambda_{i,m} - \lambda_{g,d}.$$ (29)

**Proposition 1:** The PDF of $Y_1 = \rho|h_{s,d}|^2 + \rho|h_{SEC,d}|^2$ with $|C_L| = L, L \geq 1$ is given by

$$f_{Y_1}(\gamma) = \sum_{i=0}^{L-1} \prod_{k=0}^{L-1} (1 - \exp(-\lambda_{k,d}\gamma T)) \left[ \exp\left(-\frac{\lambda_{i,d}\gamma}{\lambda_{s,d}}\right) - \exp\left(-\frac{\lambda_{i,d}\gamma}{\lambda_{d,s}}\right) \right]$$

$$+ \sum_{i=0}^{L-1} \prod_{k=0}^{L-1} (1 - \exp(-\lambda_{i-j+k,d}\gamma T)) \left[ \exp\left(-\frac{\gamma T}{\lambda_{i,d}}\right) \right],$$ (30)

where $\gamma T = \exp(-\lambda_{s,d}(\gamma - \gamma T))/\left(\frac{1}{\lambda_{s,d}} - \frac{1}{\lambda_{d,s}}\right), \gamma T = \exp(-\lambda_{i,d}(\gamma - \gamma T))/\left(\frac{1}{\lambda_{i,d}} - \frac{1}{\lambda_{s,d}}\right)$, and $U(.)$ is the unit step function.

**Proof:** In finding this PDF, we use the moment generating function (MGF) approach. The CDF of $\rho|h_{SEC,d}|^2$ can be written as [19]

$$F_{\rho|h_{SEC,d}|^2}(\gamma) =$$

$$\left\{ \begin{array}{ll}
\sum_{i=0}^{L-1} \prod_{k=0}^{L-1} F_{\rho|h_{i,k,d}|^2}(\gamma T), & \gamma < \gamma T;
\sum_{i=0}^{L-1} \frac{1}{\lambda_{i,d}} \prod_{k=1}^{L} F_{\rho|h_{i,k,d}|^2}(\gamma T)
\end{array} \right.$$ (31)

where $L$ is the number of active relays and $\gamma T$ is a predetermined switching threshold, $\pi_i, i = 0,\ldots, L - 1$ are the stationary distribution of a $L$-state Markov chain and it is the probability that the $i$th relay is chosen as given in [19], and $(i - j), i \neq j$ denotes $i - j$ modulo $L$. For Rayleigh fading channels, the CDF and the PDF of the $i$th relay path are respectively given by $F_{\rho|h_{i,d}|^2}(\gamma) = 1 - \exp(-\lambda_{i,d}\gamma)$ and $f_{\rho|h_{i,d}|^2}(\gamma) = \lambda_{i,d} \exp(-\lambda_{i,d}\gamma)$, where $\lambda_{i,d}$ is the rate of the channel between the $i$th relay and the destination. Differentiating (31) with respect to $\gamma$ and upon taking the Laplace transform using $\int f_{\rho|h_{SEC,d}|^2}(\gamma) \exp(s\gamma) d\gamma$, and
after some algebraic manipulations, the MGF of \( \rho | C_{L} \|^2 \) can be obtained. As the MRC is used at the destination, the MGF of the total SNR at the MRC output is simply their multiplication \( M_Y(s) = M_{\rho | C_{L} \|^2}(s)M_{\rho | C_{L} \|^2}(s) \).

Upon substituting the MGF of the direct link \( 1 - \frac{\rho}{\lambda_{d}} \) and \( M_{\rho | C_{L} \|^2}(s) \) in \( M_Y(s) \), and after using the operation of partial fraction, the MGF of \( Y \) can be obtained as

\[
M_Y(s) = \sum_{i=0}^{L-1} \sum_{k=0}^{L-1} \prod_{k \neq j} \left(1 - \exp \left(-\lambda_{d}, \lambda_{d} \gamma_{T}\right)\right) \left[ \left(1 - \frac{\rho}{\lambda_{d}}\right)^{-1} \right] \prod_{j=0}^{L-1} \left(1 - \exp \left(-\lambda_{i}, \lambda_{d} \gamma_{T}\right)\right) \left[ \left(1 - \frac{\rho}{\lambda_{d}}\right)^{-1} \right].
\]

(32)

Taking the inverse Laplace transform of (32), the PDF of \( Y \) can be obtained as (30).

Now, the CDF of \( \gamma_{c} \) can be obtained as follows

\[
P_{r}[\gamma_{c} < u| C_{L}] = \int_{0}^{\infty} f_{\gamma}(z) \int_{0}^{\frac{u}{z}} f_{\gamma}(y) dy dz.
\]

(33)

Upon substituting (29) and (30) in (33), and with the help of [20, Eq. (3.351.2)] and after some algebraic manipulations, we get the result in (5).

REFERENCES


Anas Salhab (SM’11) received the B.Sc. degree in Electrical Engineering from Palestine Polytechnic University (PPU), Hebron, Palestine, in 2004. He achieved his M.Sc. degree in Electrical Engineering from Jordan University of Science and Technology (JUST), Irbid, Jordan, in 2007. He received his Ph.D. degree from King Fahd University of Petroleum & Minerals (KFUPM), Dhahran, Saudi Arabia, in May 2013. He is currently a Postdoctoral Fellow at the Electrical Engineering Department in KFUPM. His research interests span special topics in modeling and performance analysis of wireless communication systems including cooperative relay networks, cognitive radio networks, and co-channel interference. Dr. Salhab served as a reviewer for IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY and IEEE COMMUNICATIONS LETTERS.

Salam Zummo received the B.Sc. and M.Sc. degrees in Electrical Engineering from King Fahd University of Petroleum & Minerals (KFUPM), Dhahran, Saudi Arabia, in 1998 and 1999, respectively, and his Ph.D. degree from the University of Michigan at Ann Arbor, USA, in June 2003. He is currently a Professor at the Electrical Engineering Department in KFUPM, and a Senior Member of the IEEE.

Prof. Zummo was awarded Saudi Ambassador Award for early Ph.D. completion in 2003, and the British Council/BAE Research Fellowship Awards in 2004 and 2006. He won the KFUPM Excellence in Research Award for the year 2011 - 2012. He has 6 issued US patents and more than 75 papers published in reputable journals and conference proceedings. His research interests are the area of wireless communications including multihop networks, cooperative diversity, cognitive radio, multiuser diversity, scheduling, MIMO systems, error control coding and interference modeling and analysis in wireless systems.