Cognitive Amplify-and-Forward Relay Networks with Switch-and-Examine Relaying in Rayleigh Fading Channels

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***Abstract*—This letter introduces cognitive channel-state-information (CSI)-assisted amplify-and-forward (AF) relay networks employing the low-complexity switch-and-examine diversity combining (SEC) relaying scheme. The scheme is based on the SEC diversity combining technique in which a relay out of multiple relays is selected to forward the source message to destination. The selection process is performed such that the end-to-end (e2e) signal-to-noise ratio (SNR) of the selected relay satisfies a predetermined switching threshold. In addition, the letter derives closed-form expressions for the outage probability and symbol error probability (SEP) of the proposed system. The system is also studied at high SNR values where approximate expressions for the outage probability, diversity order, and coding gain are derived. Results show that the diversity order of the studied cognitive AF relaying with SEC selection is the same as its non-cognitive counterpart.**

***Index Terms*—Amplify-and-forward, cognitive relay network, performance analysis, Rayleigh fading, switching threshold.**

I. INTRODUCTION

COGNITIVE radio is an important tool used to improve the spectrum resource utilization efficiency [1]. Several cognitive radio paradigms have been proposed in [2], among which is the underlay scenario. This paradigm allows users in a secondary cell to utilize the frequency bands of users in a primary cell only if the interference is below a certain threshold. Beside cognitive radio networks, a lot of research has been recently done on relay network which is used to solve the problem of multipath fading in wireless systems [3].

A lot of papers studied the performance of cognitive relay networks (CRNs) with single relay and interference constraint [4], [5]. Particularly, in [4] Duong *et al.* evaluated the outage performance of cognitive CSI-assisted AF relay networks over Rayleigh fading channels. Recently, the performance of AF CRNs with optimal power allocation was studied in [6]. Most recently, the distributed switch-and-stay combining (DSSC) scheme was used in [7] to select among the relay path and direct link signals in AF CRNs.

The outage performance of opportunistic decode-and-forward (DF) CRNs was evaluated in [8]. A lot of papers can be found in literature on opportunistic AF CRNs [9], [10]. Particularly, in [10] Bao *et al.* evaluated the outage and error performances of AF CRNs over Rayleigh fading channels. Recently, the outage performance of opportunistic AF CRNs with multiple primary users was studied in [11].

The opportunistic relaying requires a heavy load of channel estimations in selecting among relays. An efficient scheme that can deal with such problem is the low-complexity SEC relaying. It was firstly proposed in [12] as a way for selecting among antennas of secondary transmitter in CRNs. In our paper, the low-complexity SEC scheme is used to select among relays in AF CRNs where once a checked relay satisfies a predetermined switching threshold, it is selected to forward the source message to destination. This reduces the required number of channel estimations and the system complexity.

In this paper, we first introduce the cognitive AF relay networks employing the SEC relaying scheme and then we study the performance of such systems. In the analysis and due to mathematical intricacy in handling the exact e2e SNR of the considered system, a tight lower bound is used to derive closed-form expressions for the outage probability and SEP for the case of generic independent non-identically distributed (i.n.i.d.) relay channels. Furthermore, the outage performance is evaluated at the high SNR regime.

II. SYSTEM AND CHANNEL MODELS

Consider a dual-hop spectrum-sharing relay network consisting of one secondary user (SU) source S, K AF relays R_k ($k = 1, \ldots, K$), one SU destination D, and one primary user (PU) receiver P. The communications take place in two phases. In the first phase, the SU source sends its message x to K relays under a transmit power constraint which guarantees that the interference with the PU receiver P does not exceed a threshold $\mathcal{I}_{\mathsf{p}}$. As a result, the SU source S must transmit at a power given by $P_{\mathsf{s}} = \mathcal{I}_{\mathsf{p}}/|h_{\mathsf{s},\mathsf{p}}|^2$, where $h_{\mathsf{s},\mathsf{p}}$ is the channel coefficient of the $\mathsf{S} \rightarrow \mathsf{P}$ link. In the second phase, R_k amplifies the received message from S with a variable gain G_k and forwards the amplified version to the SU destination. The transmit power at R_k must also satisfy PU constraint and is defined as $P_{\mathsf{R}_k} = \mathcal{I}_{\mathsf{p}}/|h_{k,\mathsf{p}}|^2$, where $h_{k,\mathsf{p}}$ is the channel coefficient of the $\mathsf{R}_k \rightarrow \mathsf{P}$ link. Hence, the received message at D from the k^{th} relay R_k is given by $y_{k,\mathsf{d}} = \sqrt{P_{\mathsf{s}}}G_k h_{k,\mathsf{d}} h_{\mathsf{s},k} x + G_k h_{k,\mathsf{d}} n_{\mathsf{s},k} + n_{k,\mathsf{d}}$, where $h_{\mathsf{s},k}$ and $h_{k,\mathsf{d}}$ are the channel coefficients of the $\mathsf{S} \rightarrow \mathsf{R}_k$ and $\mathsf{R}_k \rightarrow \mathsf{D}$ links, respectively, $n_{\mathsf{s},k}$ and $n_{k,\mathsf{d}}$ represent the additive white Gaussian noise (AWGN) terms at R_k and D, respectively, with a power of N_0[1]. As we are using a CSI-assisted AF relaying, the gain G_k can be expressed as $G_k^2 = 1\Big/|h_{k,\mathsf{p}}|^2\left(\frac{|h_{\mathsf{s},k}|^2}{|h_{\mathsf{s},\mathsf{p}}|^2} + \frac{N_0}{\mathcal{I}_{\mathsf{p}}}\right)$. Thus, the e2e SNR of the $\mathsf{S} \rightarrow \mathsf{R}_k \rightarrow \mathsf{D}$ link can be written as [10]

$$\gamma_k = \frac{\frac{\bar{\gamma}|h_{\mathsf{s},k}|^2}{|h_{\mathsf{s},\mathsf{p}}|^2}\frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{|h_{k,\mathsf{p}}|^2}}{\frac{\bar{\gamma}|h_{\mathsf{s},k}|^2}{|h_{\mathsf{s},\mathsf{p}}|^2} + \frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{|h_{k,\mathsf{p}}|^2} + 1} = \frac{\frac{X_{k1}}{Y}X_{k2}}{\frac{X_{k1}}{Y} + X_{k2} + 1}, \qquad (2)$$

where $X_{k1} = \bar{\gamma}|h_{\mathsf{s},k}|^2$, $Y = |h_{\mathsf{s},\mathsf{p}}|^2$, $X_{k2} = \frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{|h_{k,\mathsf{p}}|^2}$, and $\bar{\gamma} = \mathcal{I}_{\mathsf{p}}/N_0$. Using SEC, the first checked relay whose e2e

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[1] It is worthwhile to mention here that the interference from the primary transmitter can be translated into the noise term of the secondary system [13].

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$$
\begin{aligned}
P_{\mathsf{out}} \simeq{}& \sum_{i=0}^{K-1} \pi_i \lambda_{\mathsf{s,p}} \left\{ \frac{\left(1-\exp\left(-\lambda_{\mathsf{s,p}}\gamma_{\mathsf{T}}\right)\right)}{\lambda_{\mathsf{s,p}}} + \sum_{k=0}^{K-1}(-1)^{k+1} \sum_{\substack{n_0<\ldots<n_k\\ n(.)\neq i}}^{K-1} \prod_{t=0}^{k} \frac{\left(1+\lambda_{n_t 2}\gamma_{\mathsf{T}}\right)^{-1}}{\Delta_1} - \left(1+\lambda_{i2}\gamma_{\mathsf{out}}\right)^{-1} \right.\\
&\left. \times \left[\frac{\left(1-\exp\left(-\left(\lambda_{i1}\gamma_{\mathsf{out}}+\lambda_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)\right)}{\lambda_{i1}\gamma_{\mathsf{out}}+\lambda_{\mathsf{s,p}}} + \sum_{k=0}^{K-1}(-1)^{k+1} \sum_{\substack{n_0<\ldots<n_k\\ n(.)\neq i}}^{K-1} \prod_{t=0}^{k} \frac{\left(1-\exp\left(-\Delta_1\gamma_{\mathsf{T}}\right)\right)}{\left(1+\lambda_{n_t 2}\gamma_{\mathsf{T}}\right)\Delta_1} \right] \right\} + \sum_{l=0}^{K-1} \pi_l \lambda_{\mathsf{s,p}} \left(\sum_{q=0}^{K} \frac{(-1)^q}{q!} \right.\\
&\times \sum_{m_1,\ldots,m_q}^{K} \prod_{z=1}^{q} \frac{\left(1-\exp\left(-\Delta_2\gamma_{\mathsf{T}}\right)\right)}{\left(1+\lambda_{m_z 2}\gamma_{\mathsf{T}}\right)\Delta_2} + \sum_{w=0}^{K-1} \pi_{((l-w))_K} \left[\left(1+\lambda_{l2}\gamma_{\mathsf{T}}\right)^{-1} \left\{ \frac{\exp\left(-\left(\lambda_{l1}\gamma_{\mathsf{T}}+\lambda_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)}{\lambda_{l1}\gamma_{\mathsf{T}}+\lambda_{\mathsf{s,p}}} + \sum_{p=0}^{w-1}(-1)^{p+1} \right.\right.\\
&\left.\left.\left. \times \sum_{v_0<\ldots<v_p}^{w-1} \prod_{g=0}^{p} \frac{\exp\left(-\Delta_3\gamma_{\mathsf{T}}\right)}{\Delta_4\Delta_3} \right\} - \left(1+\lambda_{l2}\gamma_{\mathsf{out}}\right)^{-1} \left\{ \frac{\exp\left(-\left(\lambda_{l1}\gamma_{\mathsf{out}}+\lambda_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)}{\lambda_{l1}\gamma_{\mathsf{out}}+\lambda_{\mathsf{s,p}}} + \sum_{p=0}^{w-1}(-1)^{p+1} \sum_{v_0<\ldots<v_p}^{w-1} \prod_{g=0}^{p} \frac{\exp\left(-\Delta_5\gamma_{\mathsf{T}}\right)}{\Delta_4\Delta_5} \right\} \right] \right),
\end{aligned}
\tag{1}
$$

where $\Delta_1 = \sum_{s=0}^{k} \lambda_{n_s 1}\gamma_{\mathsf{T}} + \lambda_{\mathsf{s,p}}$, $\Delta_2 = \sum_{r=1}^{q} \lambda_{m_r 1}\gamma_{\mathsf{T}} + \lambda_{\mathsf{s,p}}$, $\Delta_3 = \sum_{u=0}^{p} \lambda_{((l-w+v_u))_K 2} + \lambda_{l1}\gamma_{\mathsf{T}} + \lambda_{\mathsf{s,p}}$, $\Delta_4 = 1 + \lambda_{((l-w+v_g))_K 2}\gamma_{\mathsf{T}}$, and $\Delta_5 = \sum_{u=0}^{p} \lambda_{((l-w+v_u))_K 2} + \lambda_{l1}\gamma_{\mathsf{out}} + \lambda_{\mathsf{s,p}}$.

SNR γ_k satisfies a switching threshold is selected to forward the source message to destination. To simplify analysis, an upper bound on γ_k in (2) is used as follows [10], [13]

$$
\gamma_k \le \gamma_k^{\mathsf{up}} = \min\left(\frac{X_{k1}}{|h_{\mathsf{s,p}}|^2}, \frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{|h_{k,\mathsf{p}}|^2}\right) = \min(\gamma_{k1}, \gamma_{k2}). \tag{3}
$$

With Rayleigh distributed channels, the gains $|h_{\mathsf{s,p}}|^2$, $|h_{\mathsf{s},k}|^2$, $|h_{k,\mathsf{d}}|^2$, and $|h_{k,\mathsf{p}}|^2$ will follow exponential distribution with mean powers $\Omega_{\mathsf{s,p}}$, $\Omega_{\mathsf{s},k}$, $\Omega_{k,\mathsf{d}}$, and $\Omega_{k,\mathsf{p}}$, respectively. It is assumed that perfect channel information including the interference channel is available at the secondary users[2]. The SEC relaying scheme works as follows: 1) At the beginning of each transmission time, the SNR γ_k of the current active relay is checked against the switching threshold. If it is still above the threshold, this relay sends an acknowledgement (ACK) to the destination and keeps forwarding the source massage to it. 2) If the SNR of the checked relay is below the switching threshold, the relay sends a negative acknowledgement (NACK) to the destination, which in turn switches to other relay. 3) Through the channel estimation techniques such as the ones provided in [14], the second relay estimates its channels and goes to step 1. This process continues till an acceptable relay is found or the last relay is reached where the scheme sticks to it.

III. PERFORMANCE ANALYSIS

A. *Outage Probability*

Lemma 1: The outage probability for CRNs employing CSI-assisted AF SEC relaying over Rayleigh fading channels is given as in (1) on the top of this page.

Proof: To evaluate the outage probability, the CDF of SNR at D γ_{D} is required to be obtained first. The CDF of γ_k^{up} conditioned on $h_{\mathsf{s,p}}$ can be written as

$$
F_{\gamma_k^{\mathsf{up}}}\left(\gamma|h_{\mathsf{s,p}}\right) = 1 - \left(1 - F_{\gamma_{k1}}\left(\gamma|h_{\mathsf{s,p}}\right)\right)\left(1 - F_{\gamma_{k2}}\left(\gamma|h_{\mathsf{s,p}}\right)\right). \tag{4}
$$

It is easy to see that

$$
\begin{aligned}
F_{\gamma_{k1}}\left(\gamma|h_{\mathsf{s,p}}\right) ={}& 1 - \exp\left(-\lambda_{k1}\gamma|h_{\mathsf{s,p}}|^2\right), && (5)\\
F_{\gamma_{k2}}\left(\gamma|h_{\mathsf{s,p}}\right) ={}& \int_0^\infty F_{|h_{k,\mathsf{d}}|^2}\left(\frac{\gamma}{\bar{\gamma}}x\right) f_{|h_{k,\mathsf{p}}|^2}(x)\,dx \\
={}& 1 - \left(1+\lambda_{k2}\gamma\right)^{-1}, && (6)
\end{aligned}
$$

where $\lambda_{k1} = 1/\left(\Omega_{\mathsf{s},k}\bar{\gamma}\right)$ and $\lambda_{k2} = \Omega_{k,\mathsf{p}}/\left(\Omega_{k,\mathsf{d}}\bar{\gamma}\right)$. Upon substituting (5) and (6) in (4), we get

$$
F_{\gamma_k^{\mathsf{up}}}\left(\gamma|h_{\mathsf{s,p}}\right) = 1 - \frac{\exp\left(-\lambda_{k1}\gamma|h_{\mathsf{s,p}}|^2\right)}{\left(1+\lambda_{k2}\gamma\right)}. \tag{7}
$$

The conditional CDF at the output of the SEC relay selection scheme is given by [15]

$$
F_{\gamma_{\mathsf{D}}}\left(\gamma|h_{\mathsf{s,p}}\right) =
\begin{cases}
\sum_{i=0}^{K-1} \pi_i F_{\gamma_i^{\mathsf{up}}}(\gamma|h_{\mathsf{s,p}}) \prod_{\substack{k=0\\ k\neq i}}^{K-1} F_{\gamma_k^{\mathsf{up}}}(\gamma_{\mathsf{T}}|h_{\mathsf{s,p}}), & \gamma < \gamma_{\mathsf{T}};\\
\sum_{l=0}^{K-1} \pi_l \Big(\prod_{q=1}^{K} F_{\gamma_q^{\mathsf{up}}}(\gamma_{\mathsf{T}}|h_{\mathsf{s,p}}) + \sum_{w=0}^{K-1} \pi_\kappa & \\
\times \left[F_{\gamma_l^{\mathsf{up}}}(\gamma|h_{\mathsf{s,p}}) - F_{\gamma_l^{\mathsf{up}}}(\gamma_{\mathsf{T}}|h_{\mathsf{s,p}}) \right] & \\
\times \prod_{p=0}^{w-1} F_{\gamma^{\mathsf{up}}_{((l-w+p))_K}}(\gamma_{\mathsf{T}}|h_{\mathsf{s,p}}) \Big), & \gamma \ge \gamma_{\mathsf{T}},
\end{cases}
\tag{8}
$$

where K is the number of relays, γ_{T} is the switching threshold, $\pi_i, i = 0, \ldots, K-1$ is the probability that the i^{th} relay is chosen [15], and $\kappa = ((l-w))_K$, where $((l-w))_K$ is $l-w$ modulo K. Upon substituting (7) in (8) and with the help of the product identities in [16] and [10], we get (1). ■

B. *Symbol Error Probability*

The SEP is expressed in terms of $F_{\gamma_{\mathsf{D}}}(\gamma)$ as

$$
\mathrm{SEP} \simeq \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{\exp(-b\gamma)}{\sqrt{\gamma}} F_{\gamma_{\mathsf{D}}}(\gamma)\,d\gamma, \tag{9}
$$

where a and b are modulation-specific constants.

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[2] Secondary users can know the channel information of primary user by either a direct reception of pilot signals from a primary user [14].

$$
\begin{aligned}
\mathrm{SEP} \simeq{}& \frac{a\sqrt{b}}{2\sqrt{\pi}}\lambda_{\mathsf{s,p}} \Bigg\{ \sum_{i=0}^{K-1}\pi_i \Bigg(\frac{\sqrt{\pi}}{\sqrt{b}\lambda_{\mathsf{s,p}}} + \sum_{k=0}^{K-1}(-1)^{k+1} \sum_{\substack{n_0<\ldots<n_k\\ n(.)\neq i}}^{K-1} \prod_{t=0}^{k} \frac{(\chi_1)^{-1}}{\vartheta_1} - \Bigg[\left(-\frac{\lambda_{i1}}{\lambda_{i2}}+\lambda_{\mathsf{s,p}}\right)^{-1}\beta_1 \Bigg\{ \exp\left(\frac{b}{\lambda_{i2}}\right) \\
&\times \Bigg(\Gamma\left(1/2, b/\lambda_{i2}\right) - \frac{\Gamma\left(1/2,(\lambda_{i1}\gamma_{\mathsf{T}}+b)/\lambda_{i2}\right)}{\exp\left(\left(\lambda_{\mathsf{s,p}}-\frac{\lambda_{i1}}{\lambda_{i2}}\right)\gamma_{\mathsf{T}}\right)} \Bigg) + \mu_1\exp\left(\frac{b\lambda_{\mathsf{s,p}}}{\lambda_{i1}}\right) \Bigg(\Gamma\left(1/2,(\lambda_{i1}\gamma_{\mathsf{T}}+b)\lambda_{\mathsf{s,p}}/\lambda_{i1}\right) - \Gamma\left(1/2, b\lambda_{\mathsf{s,p}}/\lambda_{i1}\right) \Bigg) \Bigg\} \\
&+ \sum_{k=0}^{K-1}(-1)^{k+1} \sum_{\substack{n_0<\ldots<n_k\\ n(.)\neq i}}^{K-1}\prod_{t=0}^{k}\frac{\left(1-\exp\left(-\vartheta_1\gamma_{\mathsf{T}}\right)\right)}{\chi_1\vartheta_1} \Bigg]\Bigg) + \sum_{l=0}^{K-1}\pi_l \Bigg(\sum_{q=0}^{K}\frac{(-1)^q}{q!}\sum_{m_1,\ldots,m_q}^{K}\prod_{z=1}^{q}\frac{\exp\left(-\vartheta_2\gamma_{\mathsf{T}}\right)}{\chi_2\vartheta_2} + \sum_{w=0}^{K-1}\pi_{((l-w))_K} \\
&\times \Bigg[(1+\lambda_{l2}\gamma_{\mathsf{T}})^{-1}\Bigg\{ \frac{\exp\left(-\left(\lambda_{l1}\gamma_{\mathsf{T}}+\lambda_{\mathsf{s,p}}\right)\right)}{(\lambda_{l1}\gamma_{\mathsf{T}}+\lambda_{\mathsf{s,p}})} + \sum_{p=0}^{w-1}(-1)^{p+1}\sum_{v_0,\ldots,v_p}^{w-1}\prod_{g=0}^{p}\frac{\exp\left(-\vartheta_3\gamma_{\mathsf{T}}\right)}{\chi_3\vartheta_3}\Bigg\} - \Bigg\{ \frac{\exp\left(-\lambda_{\mathsf{s,p}}\gamma_{\mathsf{T}}\right)}{(\mu_3\beta_2)^{-1}} \Bigg(\exp\left(\phi_2\right) \\
&\times \Gamma\left(1/2,\left(\lambda_{l1}\gamma_{\mathsf{T}}+b\right)/\lambda_{l2}\right) - \mu_2\exp\left(\phi_1\right)\Gamma\left(1/2,(\lambda_{l1}\gamma_{\mathsf{T}}+b)\lambda_{\mathsf{s,p}}/\lambda_{l1}\right) \Bigg)\Bigg\} + \sum_{p=0}^{w-1}(-1)^{p+1}\prod_{g=0}^{p}\frac{\exp\left(-\vartheta_4\gamma_{\mathsf{T}}\right)}{1+\lambda_{((l-w+v_g))_K2}\gamma_{\mathsf{T}}} \\
&\times \Bigg\{ \mu_3\beta_2 \Bigg(\exp\left(\phi_2\right)\Gamma\left(1/2,\left(\lambda_{l1}\gamma_{\mathsf{T}}+b\right)/\lambda_{l2}\right) - \mu_2\exp\left(\phi_1\right)\Gamma\left(1/2,(\lambda_{l1}\gamma_{\mathsf{T}}+b)\lambda_{\mathsf{s,p}}/\lambda_{l1}\right) \Bigg)\Bigg\}\Bigg]\Bigg)\Bigg\},
\end{aligned}
\tag{10}
$$

where $\vartheta_1 = \sum_{s=0}^{k}\lambda_{n_s1}\gamma_{\mathsf{T}} + \lambda_{\mathsf{s,p}}$, $\vartheta_2 = \sum_{r=0}^{q}\lambda_{m_r1}\gamma_{\mathsf{T}} + \lambda_{\mathsf{s,p}}$, and $\vartheta_3 = \left(\sum_{u=0}^{p}\lambda_{((l-w+v_u))_K1} + \lambda_{l1}\right)\gamma_{\mathsf{T}} + \lambda_{\mathsf{s,p}}$, $\vartheta_4 = \sum_{u=0}^{p}\lambda_{((l-w+v_u))_K1}\gamma_{\mathsf{T}} + \lambda_{\mathsf{s,p}}$, $\chi_1 = 1 + \lambda_{n_t2}\gamma_{\mathsf{T}}$, $\chi_2 = 1 + \lambda_{m_z2}\gamma_{\mathsf{T}}$, $\chi_3 = 1 + \lambda_{((l-w+v_g))_K2}\gamma_{\mathsf{T}}$, $\beta_1 = \Gamma(1/2)(\lambda_{i2})^{-1/2}$, $\beta_2 = \Gamma(1/2)(\lambda_{l2})^{-1/2}$, $\mu_1 = \left(\frac{\lambda_{i2}\lambda_{\mathsf{s,p}}}{\lambda_{i1}}\right)^{-1/2}$, $\mu_2 = \left(\frac{\lambda_{l2}\lambda_{\mathsf{s,p}}}{\lambda_{l1}}\right)^{-1/2}$, $\mu_3 = \left(-\frac{\lambda_{l1}}{\lambda_{l2}} + \lambda_{\mathsf{s,p}}\right)^{-1}$, $\phi_1 = \frac{(\lambda_{l1}\gamma_{\mathsf{T}}+b)\lambda_{\mathsf{s,p}}}{\lambda_{l1}}$, $\phi_2 = \frac{\lambda_{l1}\gamma_{\mathsf{T}}+b}{\lambda_{l2}}$.

Lemma 2: The SEP for CRNs employing CSI-assisted AF SEC relaying is given as in (10) on the top of this page.

Proof: By replacing γ_{out} with γ in (1) and using the partial fraction and (9) and with the help of [17, Eq. (3.361.2)] and [17, Eq. (3.383.10)], we get (10). ■

C. Asymptotic Outage Probability

The conditional CDF of γ_{D} is given for the case of identical first and second hop channels by [15]

$$
F_{\gamma_{\mathsf{D}}}\left(\gamma|h_{\mathsf{s,p}}\right) = \begin{cases} \left[F_{\gamma^{\mathsf{up}}}(\gamma|h_{\mathsf{s,p}})\right]^{K-1}, & \gamma < \gamma_{\mathsf{T}}; \\ 1 - \sum_{j=0}^{K-1}\left[F_{\gamma^{\mathsf{up}}}(\gamma_{\mathsf{T}}|h_{\mathsf{s,p}})\right]^{j} & \\ \times\left[1 - F_{\gamma^{\mathsf{up}}}(\gamma|h_{\mathsf{s,p}})\right], & \gamma \geq \gamma_{\mathsf{T}}. \end{cases} \tag{11}
$$

As $\bar{\gamma} \to \infty$, the CDF in (7) simplifies to $F_{\gamma_k^{\mathsf{up}}}\left(\gamma|h_{\mathsf{s,p}}\right) \approx \lambda_1|h_{\mathsf{s,p}}|^2\gamma$. Substitute this CDF in (11) and follow the same method as in proof of lemma 1, and with the help of [17, Eq. (3.351.1)] and [17, Eq. (3.351.2)], P_{out} at high SNR equals

$$
\begin{aligned}
P_{\mathsf{out}}^{\infty} ={}& \left(\frac{\lambda_1\gamma_{\mathsf{T}}}{\lambda_{\mathsf{s,p}}}\right)^{K}\Bigg[\frac{\gamma_{\mathsf{out}}\ \gamma\left(K+1,\lambda_{\mathsf{s,p}}\gamma_{\mathsf{T}}\right)}{\gamma_{\mathsf{T}}} + \sum_{j=0}^{K-1}\frac{\left(\frac{\lambda_1}{\lambda_{\mathsf{s,p}}}\right)^{j+1}}{(\gamma_{\mathsf{T}})^{-j}} \\
&\times\frac{\Gamma\left(j+2,\lambda_{\mathsf{s,p}}\gamma_{\mathsf{T}}\right)}{\left(\gamma_{\mathsf{out}}-\gamma_{\mathsf{T}}\right)^{-1}} + \Gamma\left(K+1,\lambda_{\mathsf{s,p}}\gamma_{\mathsf{T}}\right)\Bigg].
\end{aligned}
\tag{12}
$$

The result in (12) is dominated by the second term which is still dominant when $j = 0$. Finally, (12) can be written as

$$
P_{\mathsf{out}}^{\infty} = \left\{\Omega_{\mathsf{s,r}}\lambda_{\mathsf{s,p}}(\Gamma\left(2,\lambda_{\mathsf{s,p}}\gamma_{\mathsf{T}}\right)\left(\gamma_{\mathsf{out}}-\gamma_{\mathsf{T}}\right))^{-1}\bar{\gamma}\right\}^{-1}. \tag{13}
$$

It is clear from (13) that the coding gain is affected by several parameters such as, γ_{out}, γ_{T}, and the diversity order of the cognitive AF SEC relaying system is similar to that of its non-cognitive counterpart, see, e.g., [18] with no direct link. Specifically, it is equal to 1 and is independent of the primary network. In general, the switching threshold can be chosen to be near the average value of the e2e SNR which can be upper bounded by the minimum of its two hops. Hence, $\gamma_{\mathsf{T-Opt.}}$ can be simply considered to be $\min\left(\frac{\bar{\gamma}|h_{\mathsf{s},k}|^2}{|h_{\mathsf{s,p}}|^2}, \frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{|h_{k,\mathsf{p}}|^2}\right)$. Table I shows the results for $K = 4$, $\gamma_{\mathsf{out}} = 13.01$ dB, $\Omega_{\mathsf{s,p}} = 35$, $\Omega_{\mathsf{s},k} = 0.91$, $\Omega_{k,\mathsf{d}} = 0.82$, $\Omega_{k,\mathsf{p}} = 0.73$ for $k = 1, \ldots, 4$. It is clear from Table 1 that this method gives excellent results over a wide range of SNR values.

IV. SIMULATION AND NUMERICAL RESULTS

It is clear from Figure 1 that the asymptotic results perfectly converge to the analytical results as well as the exact ones. Also, it is obvious that the used bound of the e2e SNR is indeed very tight; especially, in the high SNR region. Furthermore, we can see that the SEC relaying scheme has nearly the same performance as the best-relay selection scheme for very low SNR region; whereas, with increasing SNR, the best-relay selection clearly outperforms the SEC relaying. In addition, we can see that for the SEC relaying, the performance improves as K increases; especially, at the range of SNR values that are comparable to the switching threshold γ_{T}. More importantly, for $K = 2$, 3, and 4, it is obvious that at both low and high SNR values, all curves asymptotically converge to the same behavior and no gain is achieved in system performance with

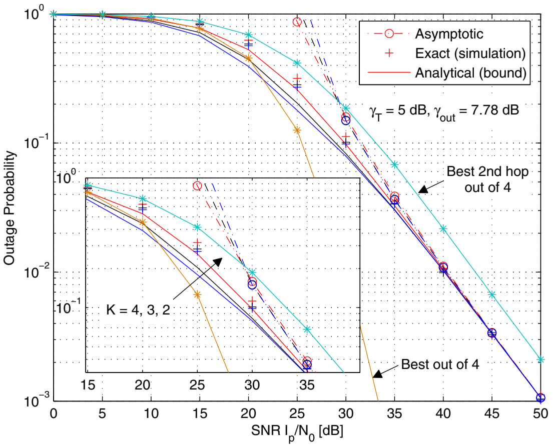

Fig. 1. P_{out} vs SNR for different values of K and $\Omega_{\text{s,p}} = 30$, and $\Omega_{\text{s},k} = 0.8$, $\Omega_{k,\text{d}} = 0.7$, $\Omega_{k,\text{p}} = 0.1$ for $k = 1, \ldots, 4$.

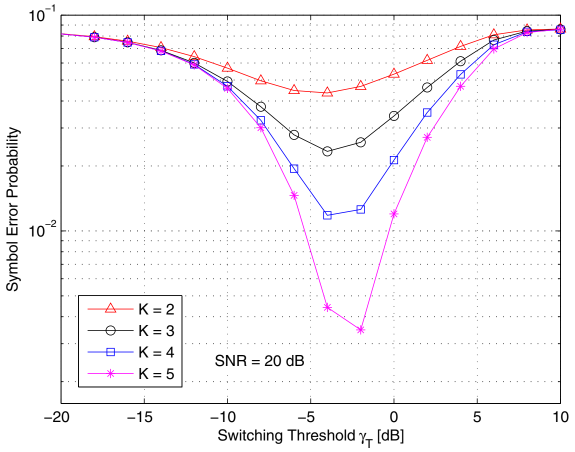

Fig. 2. SEP vs γ_{T} for different values of K and $\Omega_{\text{s,p}} = 20$, and $\Omega_{\text{s},k} = 0.1$, $\Omega_{k,\text{d}} = 0.2$, $\Omega_{k,\text{p}} = 0.3$ for $k = 1, \ldots, 5$.

TABLE I
APPROXIMATE OPTIMUM SWITCHING THRESHOLD IN dB

$\mathcal{I}_{\text{p}}/N_0$ [dB]	0	5	10	15	20
$\gamma_{\text{T}-\text{Opt.}}$	-20	-11.3	-5.95	-0.73	4.75
Approx. $\gamma_{\text{T}-\text{Opt.}}$	-15.9	-10.7	-5.85	-0.85	4.15

adding more relays. This is expected since when γ_{T} takes values that are much smaller or larger than the average SNR, the system asymptotically converges to the case of two-relay system and hence, adding more relays will add no gain to the system behavior. Finally, it is obvious that the SEC relaying outperforms the partial-relay selection over the whole range of SNR as this scheme considers only one hop of relay channels in the selection process.

Figure 2 shows that increasing K leads to a significant gain in the system performance; especially, in the range of γ_{T} that is comparable to the average value of γ_k^{up}. Also, when the average value of γ_k^{up} is very small compared to γ_{T}, all relays will be unacceptable most of the time. On the other hand, if it is very high compared to γ_{T}, all relays will be acceptable and one relay will be used most of the time. Thus, in both cases, adding more relays will add no gain to system behavior.

V. CONCLUSION

In this letter, we introduced cognitive CSI-assisted AF relay network employing the SEC relaying scheme. Closed-form expressions for the outage probability and symbol error probability were derived. Furthermore, the asymptotic outage performance was evaluated resulting in approximate expressions for the diversity order and coding gain. Results show that the diversity order of the studied cognitive AF relaying with SEC selection is the same as its non-cognitive counterpart.

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