Published in IET Communications
Received on 16th March 2006
Revised on 27th October 2006
doi: 10.1049/iet-com:20060139

ISSN 1751-8628

Upper bounds on the error probability of coded space–time block coded systems with diversity reception

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Abstract: New union bounds are derived for space–time block coded systems over Rayleigh fading channels. Both maximal ratio combining and generalised selection combining are considered as combining schemes at the receiver. The union bounds are easy to be evaluated using the transfer function of the code. Furthermore, the bounds are general to any coding scheme with a known weight distribution. Results show that the proposed union bounds are tight to simulation results for wide ranges of diversity orders and signal-to-noise ratio values.

1 Introduction

Diversity combining is an effective method to mitigate the effect of fading in wireless communication systems. Diversity can be obtained using frequency, time, space and polarisation. Diversity improves the performance of communication systems by providing M independently faded copies of the transmitted signal such that the probability that all these copies are in a deep fade is low. The diversity gain is obtained by combining the received copies at the receiver. The most known diversity combining schemes are the maximal-ratio combining (MRC), equal-gain combining (EGC), selection combining (SC) and the generalised selection combining (GSC).

Multipath fading is frequently modelled as a Rayleigh distribution. The performance of coded systems with MRC over Rayleigh fading channels with diversity was analysed in [1]. In particular, a union bound was derived in [1] and represented in the product form which allows the use of the transfer function of the code. Several bounds on the error probability of turbo codes over Rayleigh fading channels were presented in [2]. Existing union bounds for coded systems with MRC diversity rely on the use the integral representation of the erfc(·) function, which results in a bound that needs numerical integration to be evaluated, see as an example [3].

Another approach to diversity is to use multiple antennas at the transmitter [4, 5]. Space-time block coding (STBC) was proposed by Alamouti [6] to provide diversity at the transmitter. This idea was soon generalised by Tarokh *et al.* [7] to a general number of transmit antennas. The performance of STBCs with receive diversity was analysed in [8, 9] for Rayleigh fading channels. Union bounds on the bit error probability of coded STBC systems with receive diversity were derived in [10]. The bounds are very loose since they are based on approximating the probability density function (pdf) of the random variable representing the maximum of the receive diversity branches. Therefore it is of interest to derive tight union bounds for the bit error probability of coded STBC systems with receive diversity. In this work, we consider coded STBC systems with MRC or GSC at the receiver.

We derive tight union bounds on the bit error probability of coded STBCs employing MRC or GSC at the receiver over Rayleigh fading channels. The bounds for MRC will be represented in the product form allowing efficient computation of the bound using the transfer function of the code. On the other hand, the bound for GSC will be based on the transfer function of the code and simple to evaluate using the gauss-leguerre integration (GLI) rule [11].

doi: 10.1049/iet-com:20060139

2 System model

The transmitter in a coded STBC system consists of a binary encoder (e.g. convolutional or turbo), an interleaver, a modulator and a STBC. The encoder might be convolutional, turbo, trellis-coded modulation (TCM) or any other coding scheme. The encoder encodes a block of K information bits into a codeword of L symbols. The code rate is denoted as $R_c = K/L$.

The transmitter is equipped with N transmit antennas and M receive antennas. After encoding and interleaving, each group of N signals is mapped into an $N \times N$ transmission matrix $\mathcal{G}$. For the case of $N = 2$, $\mathcal{G}$ is the Alamouti code [6] given by

$$\mathcal{G} = \begin{pmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{pmatrix} \tag{1}$$

where s_0 and s_1 represent two symbols to be transmitted during a it time slot, which is the transmission interval of $\mathcal{G}$ of duration NT, where T is the symbol duration. The ith row of $\mathcal{G}$ is transmitted over the N transmit antennas in the ith time interval of the time slot. More examples of real and complex orthogonal matrices were presented in [7]. In order to be able to detect STBCs, the rows of the transmission matrix have to be orthogonal. For real signal constellations, $N \times N$ matrices that carry N different signals can be constructed. In this case, the resulting STBC is said to be full-rate. However, for complex signal constellations, the construction of orthogonal full-rate matrices is not possible for $N > 2$. Therefore reduced-rate complex STBCs can be constructed for $N > 2$.

To be able to detect STBCs, the fading process from each transmit antenna should remain constant for at least one time slot, that is, NT seconds. Let $\boldsymbol{G}_l$ be the transmission matrix in the lth time slot of the codeword. The corresponding received vector at the jth receive antenna is

$$\boldsymbol{y}_{l,j} = \sqrt{E_s}\mathcal{G}_l \boldsymbol{h}_{l,j} + \boldsymbol{z}_{l,j} \tag{2}$$

where E_s is the average received signal energy at each receive antenna, $\boldsymbol{z}_{l,j}$ is a length-N column noise vector at the jth receive antenna with a distribution $\mathcal{CN}(0, N_0\boldsymbol{I})$ and $\boldsymbol{I}$ denotes the $N \times N$ identity matrix. The vector $\boldsymbol{h}_{l,j}$ contains the channel gains from the transmit antennas in time slot l to the jth receive antenna and is modelled as $\mathcal{CN}(0, \boldsymbol{I})$. The fading processes between different pairs of transmit–receive antennas are assumed to be independent and identically distributed (i.i.d).

The decoder chooses the codeword $\boldsymbol{S}$ that maximises the metric

$$\boldsymbol{m}(\boldsymbol{Y}, \boldsymbol{S}) = \sum_{l=1}^{L/N}\sum_{j=1}^{M} \mathrm{Re}\{\boldsymbol{y}_{l,j}^* \mathcal{G}_{l,j}\boldsymbol{h}_{l,j}\} \tag{3}$$

where $(\cdot)^*$ denotes the complex conjugate of a complex vector. Signal vectors received at different receive antennas within a time slot are combined such that the performance is improved. In MRC, the received signal vectors at different diversity branches are weighted by the corresponding channel gains. The resultant signal-to-noise ratio (SNR) at time slot l of the codeword is given by $\Gamma_l = E_s \sum_{i=1}^{M} a_{l,i}/N_0$, where $a_{l,i} = \|\boldsymbol{h}_{l,i}\|^2$ is the norm of the vector $\boldsymbol{h}_{l,i}$. In GSC, the receiver selects the largest M_c receive diversity branches among the M branches and combines them using MRC. If we arrange the norms $a_{l,1}, \ldots, a_{l,M}$ in a descending order $a_{l,(1)} \geq a_{l,(2)} \geq \cdots \geq a_{l,(M)}$, then SNR at the output of the GSC is given by $\Lambda_l = E_s \sum_{i=1}^{M_c} a_{l,(i)}^2/N_0$.

The pairwise error probability (PEP) is defined as the probability of decoding a codeword $\boldsymbol{S}$ as another codeword $\hat{\boldsymbol{S}}$. In the following, the PEP is written in the product form as

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) \leq K_c \times \prod_{l=1}^{L} W(s_l, \hat{s}_l) \tag{4}$$

where K_c is a tightening constant that does not depend on the error sequence and $W(s_l, \hat{s}_l)$ is the error weight profile between a decoded symbol $\hat{s}_l$ and the transmitted symbol s_l. $\hat{s}_l$. This form enables the use of the transfer function of the code to evaluate the union bound on the bit error probability. It is worth mentioning that the results derived in this paper apply to STBCs drawn from orthogonal designs [7]. In the case of a non-full-rate STBC, there will be a reduction in the SNR because of the reduction in the STBC rate.

3 Maximal-ratio combining

The conditional PEP for MRC can be written as

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}|\boldsymbol{H}) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\sum_{l=1}^{L} d_l \gamma_l}\right) \tag{5}$$

where $\boldsymbol{H}$ is a matrix that contains the fading gains affecting a codeword, $d_l = E_s|s_l - \hat{s}_l|^2/4N_0$ and $\gamma_l = \sum_{i=1}^{M} \boldsymbol{h}_{l,i}^* \boldsymbol{h}_{l,i}$, where $*$ denotes the transpose conjugate of a vector. Since STBC is used with the simple detection scheme in [6], the random variable γ_l is an NM-Erlang random variable with a pdf [12] given by

$$f_{\gamma_l}(\gamma) = \frac{1}{(NM-1)!}\gamma^{NM-1}\mathrm{e}^{-\gamma} \quad \gamma \geq 0 \tag{6}$$

The unconditional PEP is found by averaging (5) over the statistics of γ_l's as

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) = \frac{1}{2}\int_0^\infty \cdots \int_0^\infty \operatorname{erfc}\left(\sqrt{\sum_{l=1}^{L} d_l \gamma_l}\right) f_\gamma(\gamma_1)\cdots f_\gamma(\gamma_L)\, \mathrm{d}\gamma_1 \cdots \mathrm{d}\gamma_L \quad (7)$$

Define

$$\delta_l = \frac{d_l}{1+d_l} \quad \text{and} \quad \omega_l = \gamma_l(1+d_l) \quad (8)$$

By using the change of variables in (8) and re-arranging terms inside the integral [1], the PEP becomes

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) = \frac{1}{2}\left[\prod_{l\in\eta}\frac{1}{(1+d_l)^{NM}}\right] \times \int_0^\infty \cdots \int_0^\infty \operatorname{erfc}\left(\sqrt{\sum_{l=1}^{L_\eta}\delta_l\omega_l}\right) \times \exp\left[\sum_{l=1}^{L_\eta}\delta_l\omega_l\right] \times f_\omega(\omega_1)\cdots f_\omega(\omega_{L_\eta})\, \mathrm{d}\omega_1\cdots \mathrm{d}\omega_{L_\eta} \quad (9)$$

where $f_\omega(\omega_l)$ is the pdf of the variable ω_l, $\eta = \{l{:}s_l \neq \hat{s}_l\}$ and $L_\eta = |\eta|$ is the minimum time diversity of the code. Note that in (9), we assumed, without the loss of generality, that the first L_η elements of the error codeword belong to the set η defined above. Furthermore, in (9), the pdfs $f_\omega(\omega_l)$ follow the same form of(6) with ω_l replacing γ. Note that the variables $\{\omega_l\}$ that appeared in (9) are different from the variables $\{\omega_l\}$ defined in (8) because terms were re-arranged to yield (9). Define $\delta_m = \min\{\delta_l,\ l \in \eta\}$, and note that $\sum_{l\in\eta}\delta_l\omega_l \geq \delta_m\sum_{l\in\eta}\omega_l$. Since $\operatorname{erfc}(x)\mathrm{e}^{x^2}$ is monotonically decreasing function for $x \geq 0$, then the PEP can be upper bounded as

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) \leq \frac{1}{2}\left[\prod_{l\in\eta}\frac{1}{(1+d_l)^{NM}}\right] \times \int_0^\infty \operatorname{erfc}\left(\sqrt{\delta_m\Omega}\right)\mathrm{e}^{\delta_m\Omega} f_\Omega(\Omega)\,\mathrm{d}\Omega \quad (10)$$

where $\Omega = \sum_{l=1}^{L_\eta}\omega_l$ is an NML_η-Erlang random variable with a pdf given by

$$f_\Omega(\Omega) = \frac{1}{(NML_\eta - 1)!}\Omega^{NML_\eta - 1}\mathrm{e}^{-\Omega} \quad \Omega \geq 0 \quad (11)$$

Substituting (11) in (10) results in

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) \leq \frac{J}{2}\left[\prod_{l=1}^{L_\eta}\frac{1}{(1+d_l)^{NM}}\right] \quad (12)$$

where

$$J = \frac{1}{(NML_\eta - 1)!}\int_0^\infty \operatorname{erfc}\left(\sqrt{\delta_m\Omega}\right) \times \Omega^{NML_\eta - 1}\mathrm{e}^{\Omega(\delta_m - 1)}\,\mathrm{d}\Omega \quad (13)$$

In the following, the integral in (13) is simplified using two approaches resulting in two upper bounds on the PEP.

3.1 Bound 1

Using (6.286) of [13], the integral in (13) can be evaluated as

$$J = \frac{\Gamma(NML_\eta + 0.5)}{\sqrt{\pi}NML_\eta\delta_m^{NML_\eta}(NML_\eta - 1)!}\,{}_2F_1 \times \left(NML_\eta, NML_\eta + 0.5; NML_\eta + 1; 1 - \frac{1}{\delta_m}\right) \quad (14)$$

where $\Gamma(\cdot)$ is the gamma function and ${}_2F_1(.,.;.;.)$ is the Gaussian confluent hypergeometric function defined in [13]. Defining $x = 1 - (1/\delta_m)$ and using the relation ${}_2F_1(\alpha, \beta; \gamma; z) = (1-z)^{-\alpha}\,{}_2F_1(\alpha, \gamma-\beta; \gamma; z/(z-1))$ results in

$$J = \frac{\Gamma(NML_\eta + 0.5)}{\sqrt{\pi}\delta_m^{NML_\eta}(NML_\eta)!}(1-x)^{-NML_\eta} \times {}_2F_1\left(NML_\eta, 0.5; NML_\eta + 1; \frac{x}{x-1}\right) \quad (15)$$

Using the relation ${}_2F_1(NML_\eta, 0.5; NML_\eta + 1; x/x-1) = NML_\eta(x/(x-1))^{-NML_\eta}B_{x/(x-1)}(NML_\eta, 0.5)$ and substituting (15) in (12), the PEP can be finally simplified to

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) \leq \frac{\Gamma(NML_\eta + 0.5)}{2\sqrt{\pi}(NML_\eta - 1)!}(1-\delta_m)^{-NML_\eta} \times B_{x/(x-1)}(NML_\eta, 0.5)\prod_{l=1}^{L_\eta}\frac{1}{(1+d_l)^{NM}} \quad (16)$$

where $B_x(.,.)$ is the incomplete beta function defined in[13]. Using the transfer function of the code under consideration, the union bound on the bit error probability is finally written as

$$P_b \leq \frac{1}{k}\frac{\Gamma(NML_\eta + 0.5)}{2\sqrt{\pi}(NML_\eta - 1)!}(1-\delta_m)^{-NML_\eta}B_{x/(x-1)} \times (NML_\eta, 0.5)\left.\frac{\partial T(D, I)}{\partial I}\right|_{I=1, D=(1+d_l)^{-NM}} \quad (17)$$

where k is the number of input bits to the encoder at each trellis transition and $T(D, I)$ is the transfer function of the code. Here, at each transition in the code trellis, the exponentof D represents the distance from the zero symbol, whereas the exponent of I represents the weight of the information sequence that caused this trellis transition.

doi: 10.1049/iet-com:20060139

3.2 Bound 2

Making the change of variable $\xi = \Omega(1-\delta_m)$ and using the integral form of the erfc(·) function, the integral in (13) can be written as

$$J = \frac{1}{(1-\delta_m)^{NML_\eta}} \int_0^\infty \left(\sqrt{\frac{2}{\pi}} \int_{\sqrt{2\nu\xi}}^\infty \mathrm{e}^{-\tau^2/2}\, \mathrm{d}\tau \right) \times \frac{\xi^{NML_\eta - 1} \mathrm{e}^{-\xi}}{(NML_\eta - 1)!}\, \mathrm{d}\xi \tag{18}$$

where $\nu = \delta_m/1-\delta_m$. Changing the order of integration and using the properties of the number of arrivals in a Poisson random process as in [2], (18) simplifies to

$$J = \frac{\sqrt{2}}{\sqrt{\pi}(1-\delta_m)^{NML_\eta}} \int_0^\infty \mathrm{e}^{-\tau^2/2} \times \left[\sum_{r=NML_\eta}^{\infty} \frac{1}{r!} \mathrm{e}^{-\tau^2/2\nu} \left(\frac{\tau^2}{2\nu} \right)^r \right] \mathrm{d}\tau \tag{19}$$

which can be evaluated as

$$J = \frac{\sqrt{\delta_m}}{(1-\delta_m)^{NML_\eta}} \sum_{r=NML_\eta}^{\infty} \left(\frac{1-\delta_m}{4} \right)^r \binom{2r}{r} \tag{20}$$

Following [2] and substituting (20) in (10), the PEP can be finally upper bounded as

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) \le \frac{4^{-NML_\eta}}{2\sqrt{\delta_m}} \binom{2NML_\eta}{NML_\eta} \prod_{l=1}^{L_\eta} \frac{1}{(1+d_l)^{NM}} \tag{21}$$

Using the transfer function of the code, the bit error probability an be upper bounded by

$$P_b \le \frac{1}{k} \frac{4^{-NML_\eta}}{2\sqrt{\delta_m}} \binom{2NML_\eta}{NML_\eta} \left. \frac{\partial T(D, I)}{\partial I} \right|_{I=1, D=(1+d_l)^{-NM}} \tag{22}$$

For illustration, the proposed union bounds were evaluated for two STBCs systems. The first one employs a rate-1/2 (5,7) convolutional code, whereas the second uses a eight-state eight PSK TCM system presented in [14]. Note that the bound is applicable to any coding scheme with a known transfer function such as turbo codes and product codes. Fig. 1 shows the simulation and analytical results for the convolutionally coded STBC with two transmit antennas, $N = 2$. We can see in the figure that the proposed bounds are tight to simulation results for a wide range of SNR values. In Fig. 2, the same information is shown for eight PSK TCM coded STBCs with four transmit antennas, $N = 4$. Obviously, since the constellation used is complex, the resulting STBC has a rate of 3/4. Again, the bound is tight to simulation results.

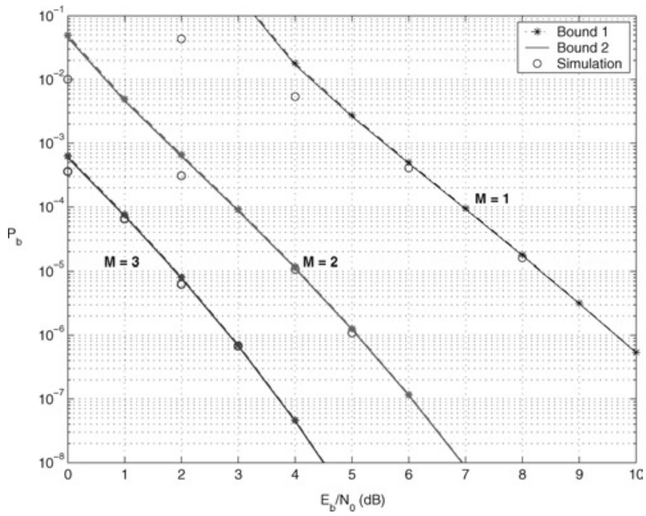

Figure 1 *Bit error probability of convolutionally coded STBCs with $n_t = 2$ and MRC with numbers of diversity branches*

4 Generalised selection combining

The conditional PEP for GSC is given by (5) with the variables $\{\gamma_l\}$ being replaced with the variables $\{\beta_l\}$, where $\beta_l = \sum_{i=1}^{M} a_{l,(i)}^2$. Using the integral expression $\mathrm{erfc}(x) = 2/\pi \int_0^{\pi/2} \mathrm{e}^{(-x^2/\sin^2\theta)}\, \mathrm{d}\theta$ [15], the PEP is written as

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{l=1}^{L} \Phi_\beta(d_l \alpha_\theta)\, \mathrm{d}\theta \tag{23}$$

where $\alpha_\theta = 1/\sin^2\theta$ and the product is because of the independence of the fading variables affecting different symbols. In (23), $\Phi_\beta(d_l\alpha_\theta)$ is the moment generating

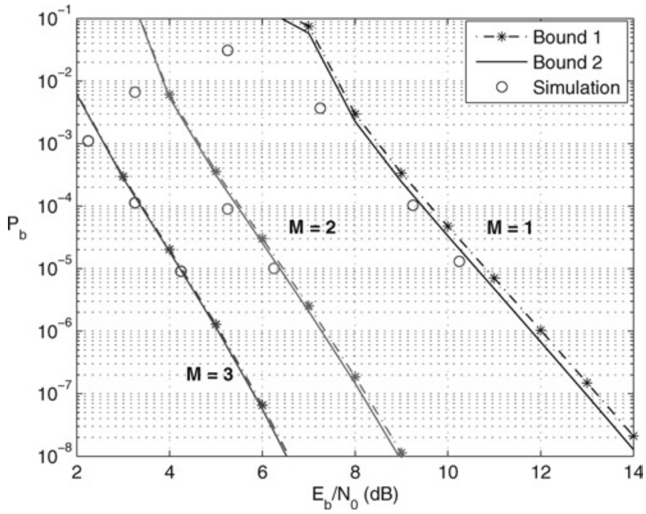

Figure 2 *Bit error probability of eight PSK TCM combined with rate-3/4, $n_t = 4$ STBC and MRC with different numbers of diversity branches*

doi: 10.1049/iet-com:20060139

function (MGF) of the random variable β defined as

$$\Phi_\beta(d) = \mathrm{E}_\beta[\mathrm{e}^{-\beta d}] \tag{24}$$

Using the result of [16], the MGF in (24) can be expressed as

$$\Phi_\beta(d) = M_c\binom{M}{M_c}\int_0^\infty \mathrm{e}^{-dx} f_{a^2}(x) \times [F_{a^2}(x)]^{M-M_c}[\phi_{a^2}(d,x)]^{M_c-1}\,\mathrm{d}x \tag{25}$$

where $f_{a^2}(x)$ and $F_{a^2}(x)$ are the pdf and cumulative distribution function (CDF) of the SNR of each diversity branch, and $\phi_{a^2}(d, x)$ is the marginal MGF given by

$$\phi_{a^2}(d,x) = \int_x^\infty \mathrm{e}^{-dt} f_{a^2}(t)\,\mathrm{d}t \tag{26}$$

For STBCs employing the simple detection scheme presented in[6] over Rayleigh fading channels, the pdf and CDF of fading on each receive diversity branch are given, respectively, by

$$f_{a^2}(x) = \frac{1}{(N-1)!}x^{N-1}\mathrm{e}^{-x} \quad x \geq 0 \tag{27}$$

$$F_{a^2}(x) = \gamma(N, x) \quad x \geq 0 \tag{28}$$

where $\gamma(a, y) = 1/\Gamma(a)\int_0^y \mathrm{e}^{-t}t^{a-1}\,\mathrm{d}t$ is the incomplete gamma function. The marginal MGF is written as

$$\phi_{a^2}(d,x) = \frac{1}{(N-1)!}\frac{1}{(1+d)^N} \times [1-\gamma(N, x(1+d))] \tag{29}$$

Substituting (27)–(29) into (25), we obtain

$$\Phi_\beta(d) = \frac{M_c\binom{M}{M_c}}{\Gamma(m)^{M_c}(1+d)^{N(M_c-1)}} \times \int_0^\infty \exp(-x(1+d))x^{N-1} \times [\gamma(N,x)]^{M-M_c} \times [1-\gamma(N, x(1+d))]^{M_c-1}\,\mathrm{d}x \tag{30}$$

Making the change of variable $y = x(1 + d\alpha_\theta)$, the PEP becomes

$$P(\boldsymbol{S} \to \hat{\boldsymbol{S}}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{l=1}^{L} \times \left\{\frac{M_c\binom{M}{M_c}}{(N-1)!^{M_c}(1+d_l)^{N(M_c-1)}(1+d_l\alpha_\theta)^N} \times \int_0^\infty \mathrm{e}^{-y}y^{N-1}g(y)\,\mathrm{d}y\right\}\mathrm{d}\theta \tag{31}$$

where $g(y)$ is given by

$$g(y) = \left[\gamma\left(N, \frac{y}{1+d}\right)\right]^{M-M_c} \times \left[1-\gamma\left(N, \frac{y(1+d_l)}{(1+d_l\alpha_\theta)}\right)\right]^{M_c-1} \tag{32}$$

Define $\eta = \{l : s_l \neq \hat{s}_l\}$, then $L_\eta = |\eta|$ represents the minimum time diversity of the code. Using the transfer function of the code, the union bound on the bit error probability is finally given by

$$P_b \leq \frac{1}{k\pi}\int_0^{\pi/2}\left\{\left.\frac{\partial T(\overline{D(\theta)}, I)}{\partial I}\right|_{I=1, D=\mathrm{e}^{-E_s/4N_0}}\right\}\mathrm{d}\theta \tag{33}$$

where $T(\overline{D(\theta)}, I)$ is the transfer function of the code evaluated at $\overline{D(\theta)}$ that is given by

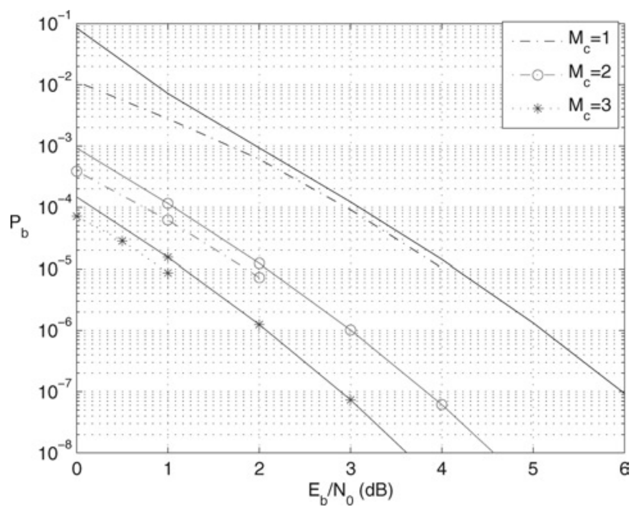

Figure 3 *Bit error probability of convolutionally coded STBCs with $n_t = 2$ and GSC with $M = 4$ (solid: bound, dashed: simulation)*

doi: 10.1049/iet-com:20060139

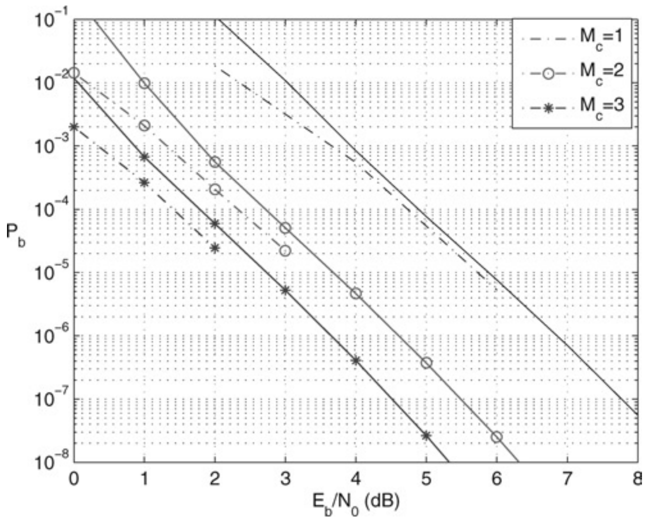

Figure 4 *Bit error probability of eight PSK TCM coded STBCs with $n_t = 2$ and GSC with $M = 4$ (solid: bound, dashed: simulation)*

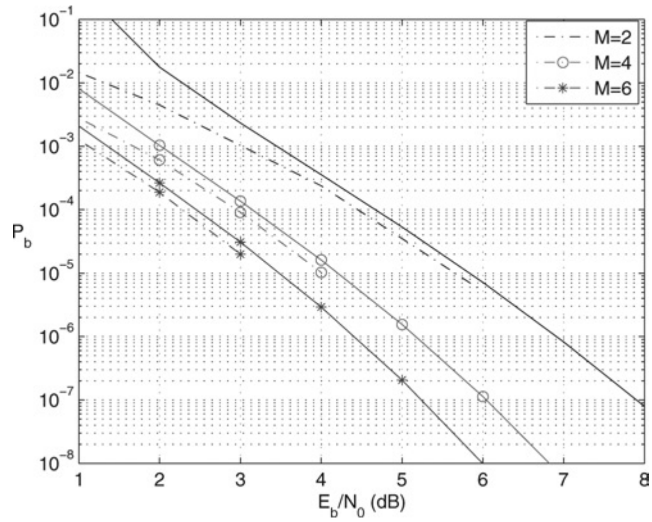

Figure 6 *Bit error probability of convolutionally coded STBCs with $n_t = 2$ and SC with different numbers of diversity branches (solid: bound, dashed: simulation)*

$$\overline{D(\theta)}|_{D=\mathrm{e}^{-E_s/4N_0}} = \frac{M_c\binom{M}{M_c}}{(N-1)!^{M_c}(1+d_l)^{N(M_c-1)}(1+d_l\alpha_\theta)^N} \times \int_0^\infty \mathrm{e}^{-y} y^{N-1} g(y)\ \mathrm{d}y \qquad (34)$$

where $g(y)$ is defined in (32). The expression in (34) can be evaluated using the GLI rule, which states the following

$$\int_0^\infty \mathrm{e}^{-y} y^{N-1} g(y)\,\mathrm{d}y \simeq \sum_{p=1}^{P} w_N(p) g(y_N(p)) \qquad (35)$$

where $w_N(p)$ and $y_N(p)$ are the pth weight and abscissa, respectively, computed according to the GLI rule as in [11].

The proposed bound was evaluated for a rate-1/2 (5,7) convolutional code and an eight-state eight PSK TCM system presented in [14]. Nevertheless, the bound is applicable to any coding scheme with a known transfer function such as turbo codes and product codes. Figs. 3–5, show the simulation and analytical results for convolutionally and eight PSK TCM coded STBC systems, respectively, with different numbers of transmit antennas N and with different selected diversity branches out of $M = 4$. We observe that the bound is tight to simulation results for a wide range of SNR and diversity

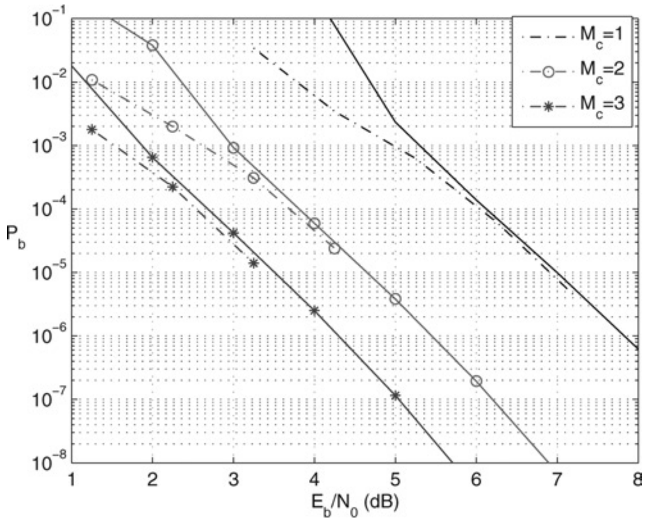

Figure 5 *Bit error probability of eight PSK TCM combined with rate-3/4, $n_t = 4$ STBC and GSC with $M = 4$ (solid: bound, dashed: simulation)*

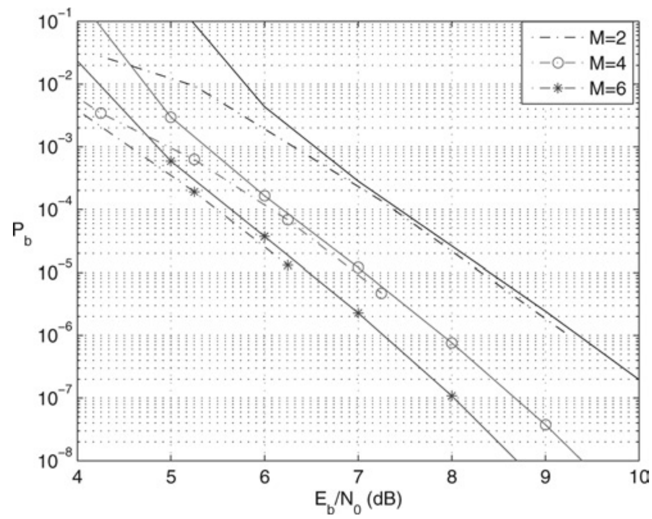

Figure 7 *Bit error probability of eight PSK TCM coded STBCs with $n_t = 4$ and SC with different numbers of diversity branches (solid: bound, dashed: simulation)*

orders. In addition, we note that the bound is simple to evaluate using the GLI rule. Figs. 6 and 7 show the performance of convolutionally and eight PSK TCM coded STBC with SC, respectively. From the figures, we observe that the bound is tight to simulation results for a wide range of diversity orders.

5 Conclusion

Union bounds on the bit error probability of coded STBC systems with receiver combining using MRC and GSC over Rayleigh fading channel were derived. Results show that the bounds are tight to simulation results for a wide range of diversity orders. Furthermore, the proposed bounds are easily evaluated using the transfer function of the code.

6 Acknowledgment

The author would like to acknowledge the support provided by KFUPM under grant FT060027.

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doi: 10.1049/iet-com:20060139