Iterative Decoding Algorithms for I-Q Space-Time Codes

Salam A. Zummo
University of Michigan
e-mail: sazummo@eecs.umich.edu

Wayne E. Stark
University of Michigan
e-mail: stark@eecs.umich.edu

Abstract — **I-Q ST codes have shown the potential to provide high coding gains in rapid fading environments. The optimal decoding of I-Q ST codes involves the use of the super-trellis of the I and Q codes, which is complex in general. This paper proposes two iterative decoding algorithms for I-Q ST codes. The proposed algorithms have similar performance to optimal decoding with much lower complexity.**

I. Introduction

Space-time (ST) codes have shown promise for reliable communications over fading channels by efficiently using transmit antennas [1]. The I-Q encoding technique was used in [2] to design ST codes for rapid fading channels with high coding gains compared to other trellis codes with same complexity. Maximum likelihood decoding (MLD) requires the use of the super-trellis of the I and Q codes, which has high complexity. In this paper, two iterative decoders are proposed to achieve the coding gain provided by I-Q ST codes at low complexity.

II. System Model

The I-Q ST codec operates on blocks of input vectors $\{\mathbf{u}_l\}_{l=1}^{N}$. In every interval, the I-Q encoder accepts a vector of bits $\mathbf{u}_l$ and outputs a signal vector $\mathbf{s}_l$ of length n_t to be transmitted via n_t transmit antennas. Each vector $\mathbf{u}_l$ is split into two equal vectors $\mathbf{u}_{I,l}$ and $\mathbf{u}_{Q,l}$. In the I branch, $\mathbf{u}_{I,l}$ is encoded by the I encoder, resulting in a signal vector $\mathbf{s}_{I,l}$ of length n_t. The same applies to the Q branch. The I and Q encoders are trellis codes employing a 1-D constellation such as M-PAM. The 2-D signal s_l^i to be transmitted over the i^{th} antenna is: $s_l^i = s_{I,l}^i + js_{Q,l}^i$, where $j = \sqrt{-1}$. For simplicity of notations, one receive antenna is assumed. The received signal is

$$y_l = \sqrt{E_s}\sum_{i=1}^{n_t} \alpha_l^i s_l^i + \eta_l, \qquad (1)$$

where E_s is the average energy at each transmit antenna, η_l is additive noise modeled as a zero-mean complex Gaussian random variable with variance $N_0, \mathcal{CN}(0, N_0)$. The coefficient α_l^i is the channel gain from the i^{th} transmit antenna, modeled as an independent samples of $\mathcal{CN}(0,1)$. To avoid the use of the super-trellis, it is required to separate the I and Q 1-D signals from y_l. Hence, two iterative decoding strategies are used in the following.

III. Iterative Demodulation-Decoding (IDD)

In the first algorithm, the I-Q encoding is viewed as the concatenation of two independent stages: the encoding using the I and Q codes, and the I-Q mapping to the 2-D signal constellation. The IDD consists of a detection stage and two soft-input soft-output (SISO) modules for the I and Q codes. The detection stage computes the following probabilities

$$p(\mathbf{s}_{I,l}|y_l, \Omega_l) = Kp(\mathbf{s}_{I,l})\sum_{\forall \mathbf{s}_Q} p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \Omega_l)p(\mathbf{s}_{Q,l}) \qquad (2)$$

$$p(\mathbf{s}_{Q,l}|y_l, \Omega_l) = Kp(\mathbf{s}_{Q,l})\sum_{\forall \mathbf{s}_I} p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \Omega_l)p(\mathbf{s}_{I,l}), \qquad (3)$$

where $\Omega_l = \{\alpha_l^i\}_{i=1}^{n_t}$, K is a normalization constant that makes the probabilities sum to one and the probability $p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \Omega_l)$ is the channel transition probability. The probabilities $p(\mathbf{s}_{I,l})$, $p(\mathbf{s}_{Q,l})$ are the apriori information about the I and Q signal vectors at time l, which are assumed to be equally probable in the first iteration. The detection stage passes the extrinsic information to the I and Q SISO decoders, which update them using the BCJR algorithm in [3]. The algorithm continues iterating between the detection stage and the SISO modules for a number of iterations. I-Q QPSK and 16-QAM ST codes in [2] were simulated with a block length of $N = 500$ and $n_t = 2$. Results show that IDD with 3 iterations achieves a bit error rate (BER) of 10^{-5} within 0.2 dB from the optimal decoding. Similar results were obtained for the case of imperfect channel estimation.

IV. Interference Cancellation Decoder (ICD)

The second algorithm views the decoding the I-Q ST codes as a multiuser detection problem. It consists of a detection stage, I and Q SISO modules and an interference cancellation (IC) stage. The detection stage computes

$$p(y_l|\mathbf{s}_{I,l}, \Omega_l) = \max_{\mathbf{s}_Q \in \mathcal{C}_Q} p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \Omega_l), \qquad \mathbf{s}_I \in \mathcal{C}_I \qquad (4)$$

$$p(y_l|\mathbf{s}_{Q,l}, \Omega_l) = \max_{\mathbf{s}_I \in \mathcal{C}_I} p(y_l|\mathbf{s}_{I,l}, \mathbf{s}_{Q,l}, \Omega_l), \qquad \mathbf{s}_Q \in \mathcal{C}_Q, \qquad (5)$$

where $\mathcal{C}_I$ and $\mathcal{C}_Q$ are the codebooks at the output of the I and Q encoders, respectively. These probabilities are used in the I and Q SISO modules as the channel observations. They compute soft information about the I and Q signal vectors using BCJR algorithm and pass the extrinsic infotrmation to the IC stage, which forms new estimates of the I and Q faded signals using

$$z_{I,l} = y_l - \sqrt{E_s}\sum_{\forall \mathbf{s}_Q} p(\mathbf{s}_{Q,l}) \sum_{i=1}^{n_t} s_{Q,l}^i(-\alpha_{Q,l}^i + j\alpha_{I,l}^i) \qquad (6)$$

$$z_{Q,l} = y_l - \sqrt{E_s}\sum_{\forall \mathbf{s}_I} p(\mathbf{s}_{I,l}) \sum_{i=1}^{n_t} s_{I,l}^i(\alpha_{I,l}^i + j\alpha_{Q,l}^i), \qquad (7)$$

where $\alpha_l^i = \alpha_{I,l}^i + j\alpha_{Q,l}^i$. In the proceeding iterations, the SISO modules use the vectors $\{z_{I,l}\}_{l=1}^{N}$ and $\{z_{Q,l}\}_{l=1}^{N}$ as their channel observations and the extrinsic information $\{p(\mathbf{s}_{I,l}), p(\mathbf{s}_{Q,l})\}_{l=1}^{N}$ as the apriori information. The algorithm continues iterating between the SISO modules and the IC stage for a number of iterations. For constant-envelop constellations such as QPSK, ICD with 3 iterations performs 0.7 dB from the optimal decoding at a BER of 10^{-5}.

References

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